FINITE ELEMENT ANALYSIS OF FLUID-STRUCTURE INTERACTION PROBLEMS

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Abstract

Interaction between solid and fluid has been recognized to be an important factor for the design of nuclear reactor components. The assumed load cases such as earthquake, aircraft crash, the loss-of-coolant accident are of special interest. Fatigue calculations resulting from small deterministic or probabilistic pressure loads have also to be seen as a fluid-structure interaction problem.

To calculate fluid-structure interactions, the finite element method is used to obtain the small pressure distribution in the fluid as well as the small displacements in the solid.

Using this method, two simultaneous differential equations exist for the coupled fluid-structure phenomenon:

\[ \dot{M}u + C\ddot{u} + Ku - \frac{1}{\delta} ST^T p = f \]

\[ \ddot{G} \ddot{p} + L\ddot{p} + Hp + S\dot{u} = g \]

(1)

In some cases, considering the fluid to be incompressible is sufficient. Then, the solution of eq. (1) becomes very simple. The matrices G and L vanish and the second equation in eq. (1) can be incorporated into the first one. For this case, the fluid acts as an additional mass only.

The assumption of a compressible fluid demands simultaneous solution of eq. (1). A very effective method is achieved by transforming the deflection vector, \( u \), and the pressure vector, \( p \), into a state vector, \( x \), which also includes the time derivations \( \dot{u} \) and \( \ddot{p} \). This method is well known from the control theory and leads to an ordinary differential equation of first order

\[ \dot{x} = Ax + Bw \]

(2)

which can be solved very accurately in both the time and the frequency domain.

For first test calculations, a cylindrical barrel with a flat bottom, which is partially filled with water, was investigated. Rotationally symmetric structural deflections and the pressure fluctuations were computed in the time domain as well as in the frequency domain. Calculations were made with and without compressible and incompressible water.
1. Introduction

Fluid-structure-interaction in dynamic problems has found a growing world-wide interest in recent years. This is especially true for the structural design of nuclear reactor components under various loads.

In this paper a contribution is made to the numerical treatment of "small" pressure pulses in the fluid and "small" deflections in the structure wet by the fluid. The representation of fluid and structure by linear finite elements is very suitable for this problem as considerable experience has been gained with structure which can also be applied to fluids.

2. FEM-Formulation for the linear structure

Fluid and structure are modelled by linear elements. The pressure of the fluid acts on the structure by discrete point loads calculated according to the theory of the finite elements /1/. This type of coupling leads to a coupling matrix [S]. The structure is described by the equation

\[ [M] \{u\} + [C] \{\dot{u}\} + [K] \{u\} = \frac{1}{\rho} [S^T] \{p\} + \{f\} \]  

with the mass matrix [M], the damping matrix [C], the stiffness matrix [K] and the displacement vector \( \{u\} \). The term \( \frac{1}{\rho} [S^T] \{p\} \) stands for the load on the structure caused by the pressure field \( \{p\} \); \( \{f\} \) is an additional load on the structure from sources other than the fluid.

In the two-dimensional sample problem, which will be described later, axisymmetric thin shell elements and plate elements of the same type have been used to model a circular cylinder with a flat bottom plate. The pressure distribution over the element is linear to comply with the linear pressure distribution of the fluid elements.

3. FEM-Formulation for the linear fluid

A fluid conforming to the following limitations is called a "linear fluid":

- The velocities \( \{v\} \) of the fluid are small so that the product \( \{v\} \cdot \{\text{grad} v\} \) can be neglected.
- The shear tensions in the fluid are proportional to the velocities.
- The state of the fluid may be described by the pressure only.

With these assumptions the usual conservation laws may be condensed to one equation:

\[ \Delta p = 0 \] for the incompressible fluid

\[ \Delta p = \frac{1}{c^2} \left( \frac{\partial^2 p}{\partial t^2} + k \frac{\partial p}{\partial t} \right) \] for the compressible fluid

(2)

where \( c \) is the speed of sound and \( k \) the viscous damping coefficient. Boundary conditions for this problem are as follows:

\[ d_p + \beta \frac{\partial p}{\partial n} = 0 \] (3)

If \( d = 0 \) is true for all boundaries, the problem is not well posed and the solution is only determined up to an additive pressure \( p_0 \).

On the fluid structure boundary,
\[
\frac{\partial p}{\partial n} = -\mathbf{g}_n \tag{4}
\]

applies, where \(\mathbf{g}_n\) is the normal acceleration.

This problem can be treated with the help of finite elements /4/. The problem has the discrete form:

\[
\begin{bmatrix} [C] \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} + \begin{bmatrix} [L] \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} + \begin{bmatrix} [H] \end{bmatrix} \begin{bmatrix} p \end{bmatrix} = \begin{bmatrix} \mathbf{g} \end{bmatrix} - \begin{bmatrix} [S] \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \tag{5}
\]

where \([C]\) is the inertia matrix, \([L]\) the viscous damping matrix, \([H]\) is the volumetric fluid matrix. \([S][\mathbf{u}]\) are the loads from the structure, while all other loads from boundary conditions, sinks and sources give a vector \([\mathbf{g}]\).

The boundary condition \(d \neq 0\) adds to the matrix \([H]\), which without this would be singular - as would be expected.

The computer code LINFLU /5/ uses a triangular element with linear shape functions to remain compatible with the structural element. Contrary to statements in literature /6/ the element shows satisfactory results and short running times.

4. Solution of the differential equation for fluid and structure

4.1 Incompressible fluid

In many applications it is sufficient to consider the fluid as incompressible. This is especially true if the fluid only participates "passively" in the dynamics, that means the loads on the system do not arise from the fluid as is the case in earthquake conditions. Then \([C]\) and \([L]\) in eq. (5) are \([\mathbf{0}]\), since the speed of sound is infinite. Eq. (5) may be modified and be inserted in eq. (1). This gives:

\[
\begin{bmatrix} \mathbf{M} + \frac{1}{\rho} \mathbf{S}^{T} \mathbf{H}^{-1} \mathbf{S} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{v}} \end{bmatrix} + \begin{bmatrix} [C] \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{v}} \end{bmatrix} + \begin{bmatrix} [K] \end{bmatrix} \begin{bmatrix} \mathbf{u} \end{bmatrix} = \begin{bmatrix} \frac{1}{\rho} \mathbf{S}^{T} \mathbf{H}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{g} \end{bmatrix} + \begin{bmatrix} \mathbf{f} \end{bmatrix} \tag{6}
\]

where \(\frac{1}{\rho} \mathbf{S}^{T} \mathbf{H}^{-1} \mathbf{S}\) is the so-called "additive mass" matrix and \(\frac{1}{\rho} \mathbf{S}^{T} \mathbf{H}^{-1} \mathbf{g}\) is an additional load on the structure.

4.2 Compressible fluid

Several methods have been proposed to solve the coupled differential equations eq. (1) and eq. (5). One group of methods forms a new system of equations by combining the vectors \([\mathbf{u}]\) und \([\mathbf{p}]\) to a new vector. The resulting system can be solved by standard methods. In general one does not arrive at symmetric matrices except in the case when the variable \([\mathbf{v}] = [\dot{u}]\) is introduced instead of \([\mathbf{u}]\) for the structure, which is not satisfactory for structural calculations. Another group is based on iterative solution techniques for eq. (1) and eq. (5). Such solutions start e.g. with a prediction of pressures which controls the solutions for the fluid and the structure and which can be modified if necessary.

The method used here is based on a simultaneous solution of both equations:

If the variables \([\mathbf{u}], [\dot{\mathbf{u}}], [\mathbf{p}], [\dot{\mathbf{p}}]\) are combined to form a vector of state

\[
[\mathbf{x}] = [\mathbf{u}, \dot{\mathbf{u}}, \mathbf{p}, \dot{\mathbf{p}}]^{T} \tag{7}
\]
then
\[
\{ \dot{x} \} = [A] \{ x \} + [B] \{ w \}
\]
follows for eq. (1) and eq. (5) where
\[
A = \begin{bmatrix}
\mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\
\mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\
-M^{-1}K & -G^{-1}1S^T & G^{-1}1 S^{-1} & \mathbf{0} \\
-G^{-1}1 K & -G^{-1}1 S^{-1} & G^{-1}1 S^{-1} & -G^{-1}1 L
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
\mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} \\
-M^{-1}1S^{-1} & \mathbf{0} \\
-G^{-1}1S^{-1} & \mathbf{I}
\end{bmatrix}
\]
\[
\{ w \} = [\mathbf{f}, \mathbf{g}]^T
\]
\[
[I] \text{ unit matrix} \\
[0] \text{ zero matrix}
\]

Since in reactor technology calculations in the time and frequency domain are of interest, eq. (8) is solved by modal superposition, that is eigenvalues and eigenvectors of A are calculated once and stored. The matrix of eigenvectors decouples eq. (8) and due to this fact eq. (8) can be solved in both the time and the frequency domain fast and accurately.

5. Test calculations

As a test, a cylindrical vessel with a flat bottom plate and partially filled with water was used. The axisymmetric behaviour was calculated for the dry vessel and the vessel filled with compressible and incompressible water. Calculations for all cases have been performed in both the time and the frequency domain and have been compared. The loads were applied in the form of a step function on the bottom plate. No damping was used. Fig. 1 shows displacements and pressures from 0 to 4 msec in different positions of the vessel. From the motion of the bottom plate the pressure wave, moving from the bottom to the free surface and back again, can be seen from the motion of the first minimum. Fig. 2 shows the same, but from 0 to 40 msec. It can be seen that pressure close to the walls depends on the structure, (pressure \( P_B \)), further away from the walls it shows primarily its own eigenfrequencies (pressure \( P_{13} \)). Fig. 3 shows the displacement \( \mathbf{U}_{16} \), from 0 to 40 ms for a dry structure, incompressible fluid and compressible fluid. There is a big difference between the dry structure and the incompressible fluid, but between the compressible and incompressible fluid there is almost none. Fig. 4 shows the same displacement in the frequency domain. Here, too, the big difference lies in neglecting the fluid.

It can be deduced, that the fluid gives lower frequencies since the fluid acts mainly as an "additive mass" and motions of the structure influences the pressure close to the walls only. There is only a small difference when compressible fluid is considered instead of incompressible.

6. Conclusion

Fluid structure interaction problems have been solved by a finite element method. The differential equations of fluid and structure have been
solved simultaneously unlike the usual techniques. This method has the dis-
advantage that due to limited storage capabilities only small and medium-
sized problems can be solved on the computers presently available. As, in
the future, storage capabilities will be offered on a large scale and at
smaller costs, the proposed method seems to be especially promising for
future applications.

7. References

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Fig. 1 Pressures and displacements from A to A up to 4 ms

Fig. 2 Pressures and displacements from A to A up to 40 ms
**Fig. 3** Displacement $U_{16}$ with and without fluid

**Fig. 4** Amplitude of displacement $U_{16}$ with and without fluid