PLASTIC INSTABILITY OF ZIRCALOY CLADDING SUBMITTED TO LOCA CONDITIONS

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SUMMARY

The ballooning of Zircaloy tubes used as fuel cladding in pressurised Water Reactor has focused attention from many workers with respect to the loss of coolant accident.

This paper gives an other approach of the plastic instability criterion of a tube submitted to internal pressure and temperature whatever the rate of variation of both parameters.

For that purpose, it is assumed that in each temperature zone corresponding to three metallurgical states (α phase, α + β, β phase) a general "consititution law" can be written, in which equivalent strain rate $\dot{\varepsilon}$, equivalent stress $\sigma$, time and temperature appear, say:

$$\dot{\varepsilon} = A (\sigma)^n \exp \left(-\frac{Q}{RT}\right) (t)^m$$

the parameters A, n, Q, m are constant.

In that case, it is assumed that one is able to compute at each time step the following integral

$$F(t) = \int_{t_0}^{t} (\Delta P)^n \left(\frac{r_0}{c_0}\right)^n \exp \left(-\frac{Q}{RT}\right) (t - t_0)^m \, dt$$

where $t_0 =$ origin of the plasticity ($\sigma^\mu = \sigma_{yield}$)
$\Delta P =$ internal minus external pressure
$r_0$, $c_0 =$ cladding mean radius and thickness

One shows that the instability criterion reduces to

$$F(t) < 1 / A \cdot n \cdot \sqrt{3}.$$  

This criterion is used first for the interpretation of burst tests experimental results and the fit of A, n, m. The Q value might also be evaluated by the same way but it is possible to get it from a lot of literature data on the diffusion activation energy in Zircaloy α phase, α + β, and β phase only.

Regarding the experimental fit of the stress and time parameters, the experiments at constant pressure and transient temperature for one part and constant temperature and transient pressure for an other part, give different approaches which reduces substantially the uncertainties of n, m, A.

The set of constant parameters being evaluated, one is able to analyse the deformation of zircaloy tube in any condition of temperature and pressure rate.
Mechanical instability phenomena generally involve a tendency towards eigenvalues: this is true, for example, of both elastic and plastic buckling (cf. reference /1/). From a physical standpoint this implies that, irrespective of the process involved, a loading exists at which "instantaneous" structural failure occurs.

When the material is subjected to a deformation law involving time, the definition of the instability is not always this simple. Is there, for example, a deformation threshold beyond which the structural failure is unavoidable irrespective of subsequent loading? A number of authors have attempted to answer this question for simple loading. As will become evident in the following discussion, however, the criteria proposed generally do not resolve this basic question and thus do not constitute true instability criteria.

After an analysis of French and foreign experimental results it was decided to attempt a new approach to the conventional internally pressurized tube problem. This study covers an analysis of the behavior of a PWR fuel rod cladding tube during a loss-of-coolant accident (LOCA), and is intended to fit a LOCA fuel code simple enough to be coupled with a thermohydraulic code.

1. **COMMONLY PROPOSED INSTABILITY CRITERIA**

In order to simplify the formulation of the problem and to facilitate comparisons among various theories, the following hypotheses will be adopted here:

- A closed, thin-wall cylinder is subjected to variable internal pressures and temperatures which constitute the only stresses on the tube. No other axial loads, in particular, are applied.
- The material is isotropic and incompressible during plastic deformations.
- Only plastic deformations are examined. Elastic deformations and thermal expansion are disregarded.

A stability criterion often mentioned in the literature /2,7/ /3,7/ /4,7/ /5,7/ was first introduced by HART /6,7/ for tensile testing: it simply proposes that under a constant load instability occurs when the deformation rate of a previously thinned zone exceeds that of the remainder of the test specimen, i.e.:

\[(\delta A/\delta A)_p > 0\]

where \(A\) = specimen diameter.

This criterion is meaningless when the load \(P\) is not constant.

In the case of a pressurized tube, FRANKLIN /7,7/ proposes by analogy that instability occurs if there are local clad thinning rate peaks, i.e.:

\[\frac{\delta W}{\delta W} > 0\]

where \(W\) = cladding wall thickness; this relation holds only when the circumferential stress is constant.
LIN /-57 advocates the same principle, stating that instability occurs when, for two tube zones of radii \( r_1 \) and \( r_2 \), the following relation is true:

\[
\frac{r_2 - r_1}{r_2/r_1 - 1} > 0
\]  

(1)

Assuming that this criterion is valid, and if, for the material in question, the relation between strain, strain rate, stress and temperature is known at each time, it should then be possible to locate the instability threshold for a pressurized tube.

This point is discussed by LIN /-67/, who proposes a very simple relation for an axially unstressed tube:

\[
\dot{\varepsilon} > \frac{N}{\sqrt{\dot{\varepsilon}}}
\]  

(2)

provided \( \sigma = K (\dot{\varepsilon})^N (\varepsilon)^M \)

The second equation in (2) represents the zircaloy constitution equation. With the values cited by LIN, the limit on \( \varepsilon \) is 3.31.

Correlations with tube bursting experiments show that, although this criterion is perfectly valid, it is of limited practical use for the following reasons:

- The experiments do not reveal the special nature of this threshold: thus, while it does in fact correspond to a change of sign of the relative deformation rate, this change is by no means irreversible particularly when the load varies.

- The threshold, moreover, is located in a zone of sufficient deformation that the constant load hypothesis is no longer valid.

2. DEVELOPING A MODEL INDEPENDENT OF THE TYPE OF STRESS LOADING

In the following discussion the temperature and stress are assumed to vary in random fashion.

Initially let us consider a thermoplastic deformation model of a sufficiently general nature to account for the experimental results while maintaining constant parameter values in the following equation over wide temperature ranges:

\[
\dot{\varepsilon} = A \exp \left( \frac{-Q}{RT} \right) (\sigma)^n (t)^m
\]  

(3)

where \( \sigma \) and \( \varepsilon \) represent the equivalent plastic stress and strain above the elastic limit.

As an initial hypothesis it shall be assumed that for each of the three metallurgical states (pure \( \alpha \), \( \alpha + \beta \) and pure \( \beta \)) there is a set of constant coefficients \( A, n, m \) and \( Q \). The transition temperatures for the 3 states are situated at about 840°C and 900°C. It must be noted that these limits are not really independent of the deformation rates imposed on the material.
This study is based essentially on a one-dimensional mechanical analysis, although paragraph 4 discusses the contribution of a two-dimensional analysis. For application to a complete fuel rod, the rod may be divided into isothermal zones, each of which is considered mechanically independent although the loads are dependent by virtue of the internal pressure.

Under these conditions, a number of well known relations exist between the equivalent elongation and stress; cf. /1,6,7/, for example.

Let us assume that:
\[ r_0 = \text{initial mean clad radius} \]
\[ W_0 = \text{initial wall thickness} \]
\[ P_i = \text{internal pressure (time-dependent)} \]
\[ P_e = \text{external pressure (time-dependent)} \]

The equivalent stress is then:
\[ \sigma = \sqrt{\frac{3}{2}} \cdot \sigma_c \]  \hspace{1cm} (4)

and the equivalent strain:
\[ \varepsilon = 2\sqrt{\frac{3}{3}} \cdot \varepsilon_c \]  \hspace{1cm} (5)

where \( \sigma_c \) and \( \varepsilon_c \) represent the circumferential stress and strain. Moreover:
\[ \frac{d \sigma_c}{\sigma_c} = \frac{d (P_i - P_e)}{P_i - P_e} + \frac{dr}{r} \cdot \frac{dW}{W} \]  \hspace{1cm} (6)

If the deformation occurs at constant volume, then:
\[ \frac{dr}{r} = - \frac{dW}{W} \]

since \( \frac{dz}{z} = 0 \).

Equation (5) then becomes:
\[ \frac{d \sigma_c}{\sigma_c} = \frac{d (P_i - P_e)}{P_i - P_e} + 2 \frac{dr}{r} \]  \hspace{1cm} (7)

with
\[ \frac{dr}{r} = \varepsilon_c = \sqrt{\frac{3}{2}} \cdot \varepsilon \]  \hspace{1cm} (8)

therefore:
\[ \frac{d \sigma_c}{\sigma_c} = \frac{d (P_i - P_e)}{P_i - P_e} + \sqrt{\frac{3}{2}} \cdot \varepsilon \]  \hspace{1cm} (9)

i.e., after integration:
\[ \sigma_c = \frac{(P_i - P_e) \cdot r_0}{W_0} \exp \left( \sqrt{\frac{3}{2}} \cdot \varepsilon \right) \]  \hspace{1cm} (10)

and
\[ \sigma = \sqrt{\frac{3}{2}} \cdot (P_i - P_e) \cdot \frac{r_0}{W_0} \exp \left( \sqrt{3} \cdot \varepsilon \right) \]

After transposition of equation (10) into equation (3) and separation of variable \( \varepsilon \):
\[ \exp \left( - n\sqrt{3} \cdot \varepsilon \right) = A \left( \sqrt{\frac{3}{2}} \cdot \frac{n}{P_i - P_e} \right)^{\frac{n}{W_0}} \exp \left( - Q/RT \right) \left( t - t_0 \right)^m \]  \hspace{1cm} (11)

where \( t_0 \) = initial instant of plasticity.
As both terms contain continuous functions, the left-hand term may be integrated from 0 to $\varepsilon$ and the right-hand term from $t_0$ to $t$ without modifying the equality. Hence:

$$\frac{1}{n\sqrt{3}} \left[ 1 - \exp \left( - n\sqrt{3} \varepsilon \right) \right] = A \int_{t_0}^{t} \left( \frac{\sqrt{3}}{2} \right)^n (P_i - P_e) \left( \frac{r_0}{r_0^n \exp (- Q/RT)} \right) (t - t_0)^m dt.$$  \hspace{1cm} (12)

If:

$$F(t) = \int_{t_0}^{t} \left( \frac{\sqrt{3}}{2} \right)^n (P_i - P_e) \left( \frac{r_0}{r_0^n \exp (- Q/RT)} \right) (t - t_0)^m dt \hspace{1cm} (13)$$

then:

$$1 - \exp \left( - n\sqrt{3} \varepsilon \right) = n\sqrt{3} \cdot A \cdot F(t) \hspace{1cm} (14)$$

Equation (14) gives the instability criterion. Thus if a tube is subjected to non-decreasing temperature and pressure ramps, $F(t)$ increases indefinitely provided that $m > -1$, which is always true. However, the first term cannot exceed 1, and when the product $n\sqrt{3} \cdot A \cdot F(t)$ reaches this value, $\varepsilon$ tends towards infinity. This is therefore a matter of irreversible instability such as buckling, and is completely unlike a simple reversal of the relative deformation acceleration in a previously thinned area.

As a general rule, therefore, the instability criterion is expressed as follows:

$$F(t) \geq 1/n\sqrt{3} \cdot A \hspace{1cm} (15)$$

It must pointed out that $F(t)$ depends both on the stress history and on the physical properties of zircaloy in this example.

As long as this inequality is not confirmed the engineering circumferential strain may be calculated:

$$\varepsilon = \exp \varepsilon_c - 1 = \exp \left( \sqrt{3}/2 \cdot \varepsilon \right) - 1$$

Equation (14) may be rewritten:

$$\exp (- n\sqrt{3} \cdot \varepsilon) = 1 - n\sqrt{3} \cdot A \cdot F(t)$$

so that:

$$\varepsilon = \left[ 1 - A \cdot n\sqrt{3} \cdot F(t) \right]^{-1/2n} - 1 \hspace{1cm} (16)$$

In most cases $F(t)$ cannot be calculated analytically.

It remains to be shown whether temperature zones exist in which the constant parameter hypothesis is true.

3. INTERPRETING EXPERIMENTAL RESULTS

It is clear that parameter evaluation can be simplified by limiting the number of experimental variables. Two types of tube burst experiments may thus be considered: temperature ramps under constant pressure, or pressure ramps at constant temperature.

Let us consider what happens to the integral $F(t)$ in each case.
3.1 - Temperature Ramp under Constant Pressure

After eliminating the pressure terms from the integral, it becomes:

\[ F(t) = (\sigma_0)^n \int_{t_0}^{t} \exp(-Q/RT) (t - t_0)^m \, dt \quad (17) \]

in which \( \sigma_0 \) represents the equivalent stress in the initial geometry. Given the temperature \( T \) as a function of time \( t \), it is enough to know \( m \) and \( Q \) to calculate the integral.

The instability occurs when the temperature reaches a value such that:

\[ \int_{t_0}^{t} \exp(-Q/RT) (t - t_0)^m \, dt = \frac{1}{n \sqrt{3}} \cdot A \left(\sigma_0\right)^n \quad (18) \]

If \( Q \) is determined either by the strain rate for relatively small deformation (< 10%) or by more basic experiments, \( m \) is the only remaining parameter on the basis of which a set of experiments may be fitted, assuming that the heating rate is considered as the variable parameter. These experiments thus provide a simple approach to \( m \).

3.2 - Pressure Ramp at Constant Temperature

The integral \( F(t) \) becomes:

\[ F(t) = \exp(-Q/RT) \int_{0}^{t} (vt + \sigma_{eq})^n \, dt \quad (19) \]

where \( v \) represents the stress rate of \( \sigma_0 \) calculated in the initial geometry. The instant \( t = 0 \) corresponds to the transition \( \sigma_0 = \sigma_{eq} \) in the initial geometry. It may be seen that \( m \) and \( n \) must be known in order to calculate \( F(t) \). At instability the expression becomes:

\[ F(t) = \frac{1}{n \sqrt{3}} A \exp(-Q/RT) \quad (20) \]

which provides another relation between \( n \), \( A \) and \( Q \).

It now becomes clear that it is highly advantageous to run a set of experiments at constant temperature and another at constant pressure.

3.3 - Measured Values

For the burst experiments the instant of tube fracture was assimilated with the instant of instability. This is not entirely exact since the instant of rupture corresponds to a finite elongation, and the purpose of this study is not to define a rupture criterion. Nevertheless, when the elongation exceeds 40% the process occurs so quickly that this assimilation of instability and fracture is not far wrong. The 40% value is the lower limit of rupture elongation for zircaloy at temperatures above 700°C.

Up to now, the work has been limited to experiments in which only one of the parameters (i.e. temperature or pressure) is allowed to vary and under simple measurement conditions: relatively uniform temperatures and well defined internal pressure.
The EDGAR experiments \(^7\) run by the CEA at Saclay are of interest from this standpoint in that they involve empty tubes. Filled tubes, such as reactor fuel rods, create measurement problems for monitoring the internal pressure and its variations during ballooning.

In these experiments, the temperature parameter must unquestionably be measured with the greatest precision. It is the cladding temperature uniformity which determines short or long ballooning of the tube. The latter type of deformation can only occur when the temperature is extremely uniform over the length of the tube and held constant throughout the plastic deformation phase. The Springfields experiments \(^8\) are fully characteristic of this phenomenon: "long ballooning" is never obtained when the tube bursts occurs during a temperature ramp; this is because perfect uniformity is much more difficult to achieve during the ramp than under steady-state conditions.

During a temperature rise at constant pressure the most sensitive parameter is \(Q\), since it affects the experimental term. \(Q\) was assigned a value of 283 kJ on the basis of the EDGAR experiments; this is coherent with the results reported by HOOD & SCHULTZ \(^11\), NAIK & ARGAWALA \(^12\) and HINDE \(^8\).

With this parameter determined, \(F(t)\) may be computed for various values of \(m\) and different heating rates \(v\) for a set of constant-temperature experiments. If \(m\) is defined and \(F(t)\) is calculated for different values of \(v\) corresponding to actual experiments, the various measured burst temperatures should correspond to the same \(F(t)\) limit value since it depends only on the constant parameters \(A\) and \(n\). This allows \(m\) to be adjusted (cf. Figure 1), giving the value \(m = 0.3\).

Given \(F(t)\) for various initial stress values, these values may be plotted on log-log coordinates, simultaneously giving \(n\) and \(A\) (cf. Figure 2).

For example, a series of tubes from a single manufacturing batch gives the following values:

\[ n = 5.9 \pm 0.5 \quad A = 3.6 \times 10^{-34} \]

while another series of tubes gives:

\[ n = 5.9 \pm 0.5 \quad A = 5.4 \times 10^{-34} \]

The values given here correspond to zircaloy behavior at temperatures below 850°C.

If we now consider the conventional rupture temperature versus equivalent stress diagram, a series of curves may be plotted for different values of \(v\) : in this case \(v\) is assigned the values 5, 10, 25 and 50°C/sec. For each pair of values \(\sigma_0, v\) there exists a temperature giving the limit \(F(t)\) value. Figure 3 shows that the resulting set of curves is valid above a 50 MPa equivalent stress value, which corresponds to a pressure of 80 bar; the experimental points no longer coincide with the calculated curves at lower stress values. Other behavior laws must therefore be determined for the \(\sigma + \beta\) zone and the pure \(\beta\) zone. For purposes of comparison, Figure 3 also shows two curves obtained at the Argonne Laboratory \(^9\) for \(v\) values of 5 and 55°C/sec.
These curves show that when $T$ and $\sigma$ vary simultaneously there is no single relation between equivalent stress and rupture temperature.

A simple code, processing the expansion of consecutive axial zones of cladding as a function of an axial clad temperature distribution and its va-

riation in time for a specified axial pressure variation, has been used to sim-

ulate a postulated LOCA during which the first peak temperature reaches 900°C. Three configurations are considered:
- an empty closed tube,
- an open tube,
- a pellet-filled closed tube with a plenum space.

The initial conditions correspond to a 90 bar internal pressure at 450°C.

Figure 4 shows that the expansion curves and burst temperatures are very different in each case. In particular, a 25% circumferential expansion is obtained without instability for the empty sealed tube; instability appears only when the temperature rises.

4. TWO-DIMENSIONAL ANALYSIS

It is clear that a two-dimensional analysis is not required for "long ballooning" (i.e. tube swelling under uniform or virtually uniform temperature conditions) since the stress and deformation model described in paragraph 2 provides correct results.

In the case of "short ballooning", due generally to hot spots or - more rarely - to a tube defect, experimental data tend to indicate that for any given geometry the swelling length is always of the same order of magni-

tude for a set of tubes. Thus, for example, a fuel rod cladding tube for a 17x17 PWR assembly tended to balloon over 40 to 60 mm, both in the FDNAR experiments and in the Springfields tests.

Finite-element calculations using the PASTEL and INCA codes from the CASTEM system /10,7/ showed that when a "point" defect is introduced (i.e. a defect of small dimension compared to the ballooning length) the experimental length of the ballooning is obtained.

Moreover, the calculation also show that during the first deformation phase - up to 10% equivalent deformation - the relative size of an initial, geometrical defect tends to diminish. This corresponds to the inequality $\frac{\delta W}{\delta W_0} < 0$ mentioned at the beginning of this paper.

After this phase, the calculation may be pursued only for major hypo-

thetical deformations, allowing for local pressure variation. This generally gives rise to convergence problems such that it is not feasible to repeat this type of calculation each time for a code coupled with a thermohydraulic module.

For this reason, by virtue of the geometrical reproductibility of short ballooning phenomena, a single calculation with the desired accuracy gi-

ves the true instantaneous stress values at each point of the ballooning zone for a given diameter, and allows the actual internal volume to be calculated. The volume and the gas pressure may then be recalculated at each step.
The results of the two-dimensional calculation relating the maximum swelling diameter, stress and internal volume are used to evaluate the correction factors for the single-dimension calculation. For the latter, the zones are delimited so that the ballooning is limited to a single axial layer.

5. CONCLUSION

The type of analysis discussed here has proved to be profitable in interpreting zircaloy clad burst experiments. The instability criterion defined above makes it possible to determine a constant-parameter correlation law for temperatures below 850°C. In particular, the dependence of the temperature rise rate is clearly revealed on the relation between rupture temperature and stress.

The analysis must now be extended to temperature ranges corresponding to the β phase.

Finally, two-dimensional analysis methods may be used to refine the stress calculation in the final ballooning phase without modifying the instability criterion.

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Fig. 1 - F(t) functions at different heating rates. Burst temperatures.

Fig. 2 - F(t) limit variation versus initial equivalent stress.

Fig. 3 - Burst temperatures versus initial stress at different heating rates.

Fig. 4 - Swelling of Pressurized Tube in LOCA conditions.