

AN ELASTOPLASTIC ELBOW ELEMENT: THEORY AND APPLICATIONS

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= S U M M A R Y =

An original elastoplastic elbow element has been formulated and developed for inclusion in the library of a finite elements code. The library includes typical piping items such as the straight pipe (already presented, see paper F3/2 of 4th SMIRT), the elbow (herein presented) and the TJoint (under development) the program gives a complete 3D simulation for the piping under dynamic non linear conditions for the analysis of extreme accidental conditions (pipewhip earthquake etc).

The main features of the element are discussed herein :

Theory :

- a) Vlasov's thin shell theory is used to correlate generalized strains and midthickness displacements
- b) The displacement field is represented by means of a Fourier expansion in the circumferential direction, while a finite difference method is used to compute the derivatives of Fourier coefficients along the azimuthal direction
- c) Flow rule is given in terms of stress resultants and generalized strains ; in formulating the stress strain flow rule Iliouchine yield locus theory is used together with normality rule. The relative merits of this theory as compared to the more accurate involving Gauss points through the thickness is discussed with particular reference to the problem one is dealing with .
- d) Local equilibrium equations are waived and a virtual work procedure is used to compute the stiffness matrix of the elbow element.
- e) A static condensation procedure is used to reduce the overall stiffness matrix to a (12x12) matrix, where external generalized forces and displacements are represented so that compatibility with adjacent elements is possible.
- f) In order to account for large displacement the geometry (radius of curvature and central angle) is reevaluated at each step so that the element is compatible with a "large displacements" (updated Lagrangian approach).

Confirmations : comparisons have been made with published results both in the elastic and plastic field obtained both experimentally and by means of other computer codes (MARC, ADINA).

Applications : a code incorporating this special elbow element has been run for some pipe whip cases, the results have been compared with ones obtained by means of ADINA and FRUSTA codes.

1. Introduction

Pipes are generally idealized by means of straight or curved beams in order to keep the number of degrees of freedom within reasonable limits. The increase in stresses and flexibility as compared with the ones of a conventional beam are taken into account by means of stresses and flexibility factors. As it is well known, ASME 3 code / 1 / provides these factors based on a theory originally developed by Von Karman / 2 /. Further analyses have been performed e.g. by Jones / 3 /, Thompson / 4 /, Purdue and Vigness / 5 /, Ohtsubo and Watanabe / 6 /; however all these analyses assume elastic behaviour. In the plastic field data for in plane bending have been presented by Spence / 7 /. General finite element codes such as Marc / 9 / and Adina / 10 / have been used and data have been reported by Shimizu et al / 11 /, Sobel / 19 /.

The aim of this paper is the formulation, the testing and use of a finite element: an elbow in the elastic and plastic field. This element represent a further contribution to a finite element library for pipe items after the straight pipe / 12 /.

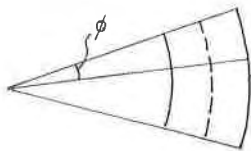
2. Formulation of the element

2.1 Strain-displacements relationships

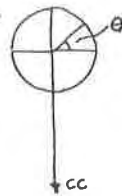
Vlasov's / 13 / thin shell theory is used, the equations are somewhat different than the classical ones, based on Love's / 8 / theory. The reasons for this difference are explained by Vlasov, however no large difference should arise by the use of one theory or the other one. Let us use the following notations

- ϕ the longitudinal angle
- θ the section angle
- r the pipe radius
- t the pipe thickness
- R the elbow radius
- u, v, w midthickness displacements in the axial, tangential and radial direction

With reference to fig.1 the generalized strain components elongations $\epsilon_\phi, \epsilon_\theta, \epsilon_{\theta\phi}$



Elbow Geometry



curvatures $\chi_\phi, \chi_\theta, \chi_{\theta\phi}$ are expressed in terms of the midthickness displacements by means of the following differential equations :

Fig.1

$$(16, P) \left\{ \begin{aligned} \epsilon_\phi &= \frac{[u\phi + v\cos\theta + w\sin\theta]}{R+r\sin\theta} ; \quad \epsilon_\theta = \frac{[v\phi + w]}{r} ; \\ \epsilon_{\theta\phi} &= \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{1}{R+r\sin\theta} \left[\frac{\partial u}{\partial \phi} - u\cos\theta \right] ; \quad \chi_\phi = \frac{R\cos\theta}{r(R+r\sin\theta)^2} - \frac{\sin^2\theta}{(R+r\sin\theta)^2} - \frac{1}{(R+r\sin\theta)^2} \frac{\partial^2 w}{\partial \phi^2} \\ &- \frac{\cos\theta}{R+r\sin\theta} \frac{\partial w}{\partial \theta} ; \quad \chi_\theta = -\frac{1}{r^2} \left[w + \frac{\partial^2 w}{\partial \theta^2} \right] ; \quad \chi_{\theta\phi} = \frac{-R}{r^2(R+r\sin\theta)^2} \left[(R+r\sin\theta) \frac{\partial u}{\partial \theta} \right. \\ &\left. - u\cos\theta - r \frac{\partial v}{\partial \phi} \right] - \frac{2}{r(R+r\sin\theta)} \left[\frac{\partial^2 w}{\partial \theta \partial \phi} - \frac{r\cos\theta}{R+r\sin\theta} \frac{\partial w}{\partial \phi} \right] \end{aligned} \right.$$

Let us consider then a particular section (i.e. a value of θ) and let us make a Fourier expansion for u, v, w :

$$(2a,c) \quad \begin{aligned} U &= U_0 + \sum_n (U_{1n} \cos n\theta + U_{2n} \sin n\theta) \\ V &= V_0 + \sum_n (V_{1n} \cos n\theta + V_{2n} \sin n\theta) \\ W &= W_0 + \sum_n (W_{1n} \cos n\theta + W_{2n} \sin n\theta) \end{aligned}$$

The terms U_0, V_0 etc. will be functions of θ only, then let $|\mathcal{E}|$ denote the vector of the generalized strains and $|\mathcal{S}(\theta)|$ the vector of the terms U_0, V_0 etc. at a particular θ location.

By taking both (1) and (2) into account one ends up with an equation of the type

$$(3a) \quad |\mathcal{E}(\theta, \theta)| = [A(\theta)] \cdot |\mathcal{S}(\theta)| + [B(\theta)] \left| \frac{\partial \mathcal{S}}{\partial \theta} \right| + [C(\theta)] \left| \frac{\partial^2 \mathcal{S}}{\partial \theta^2} \right|$$

The terms of the matrices $[A], [B], [C]$ are rather complicated trigonometric expressions and are here omitted for simplicity sake ; they are reported in ref / 24 /. Let us then assume that a finite differences procedure is used for the evaluation of the partial derivatives in (3), this is similar to the procedure used by Bushnell / 25 /, while the same problem has been solved by Ohtsubo et al / 6 / by the use of Hermitian polynomials. However let $|\mathcal{S}_i|$ represent the value of $|\mathcal{S}|$ at the i th longitudinal location, then

$$(3b) \quad \frac{\partial^n |\mathcal{S}_i|}{\partial \theta^n} = \sum_k \alpha_i^n |\mathcal{S}_k|$$

In this case a very simple Lagrangian expression has been used to formulate the various α_i^n terms. If (3a) and (3b) are considered one has a correlation of the type

$$(4) \quad |\mathcal{E}_{i,J}^n| = [A_{i,J,k,m}^n] \cdot |\mathcal{S}_{k,m}|$$

where the term $\mathcal{E}_{i,J}^n$ represents the n th component of the strain vector at the i th longitudinal and J th circumferential location and $\mathcal{S}_{k,m}$ represents the K_{th} component of the Fourier expansion at the m th longitudinal location. It should be mentioned that the computation of the kinematic matrix $[A_{i,J,k,m}^n]$ is comparatively time consuming, however the matrix is a function of geometry only and one can take advantage of this circumstance while formulating the appropriate program by suitable programming techniques (registration on files etc.).

2.2 Stress strain relationships

It is convenient to formulate the stress-strain relationships in terms of generalized stresses (stress resultants) and generalized strains (elongation and curvatures). The stress resultants vector $|L(\theta, \theta)|$ can be computed in terms of the local stress vector by an integration through the thickness. It should be mentioned that for stress resultants the duality principle does not hold, as an example the two shears $S_{\theta\theta}$ and $S_{\theta\phi}$ are not equal, however if the shell is thin (i.e. the ratio t/r is small) one can assume that the duality rule holds, then the components of the stress resultants are related to the local stresses by the approximate relationships :

$$(5a, f) \left\{ \begin{array}{lll} N\phi = \int_{-t/2}^{t/2} \sigma_{\phi} dz & N\theta = \int_{-t/2}^{t/2} \sigma_{\theta} dz & S = S\phi\theta = S\theta\phi = \int_{-t/2}^{t/2} \tau_{\phi\theta} dz \\ M\phi = \int_{-t/2}^{t/2} z \sigma_{\phi} dz & M\theta = \int_{-t/2}^{t/2} z \sigma_{\theta} dz & T = M\phi\theta = M\theta\phi = \int_{-t/2}^{t/2} z \tau_{\phi\theta} dz \end{array} \right.$$

Further the theory will be formulated under the hypotheses of negligible transverse shear. Obviously enough σ_{ϕ} , σ_{θ} , $\tau_{\phi\theta}$ are the non zero components of the stress tensor.

If an incremental approach is used and Prandtl-Reuss equations are modified in order to have relationships of the type / 21,22 / :

$$(6) \left| \dot{\sigma}_{i,J}^n \right| = \left[C_{i,J}^{n,m} \right] \cdot \left| \dot{\epsilon}_{i,J}^m + Z \dot{\epsilon}_{i,J}^{m+3} \right| \quad \begin{array}{l} n = 1,3 \\ m = 1,3 \end{array} \quad Z = \text{elevation}$$

where $\dot{\sigma}_{i,J}^n$ represents the increment in the nth component of stress. Obviously enough the matrix $\left[C_{i,J}^{n,m} \right]$ is a function of the local stress state and is consequently variable through the thickness. If (6) equations are substituted into (5) and if $|L|$ denotes the stress resultants vector one yields :

$$(7) \left| \dot{L}_{i,J}^k \right| = \left[b_{i,J}^{k,l} \right] \cdot \left| \dot{\epsilon}_{i,J}^l \right| \quad \begin{array}{l} k = 1,6 \\ l = 1,6 \end{array}$$

(the dot denotes differentiation with respect to time). However the computation of the $[b]$ matrix involves an integration through the thickness, as matrices $[b]$ and $[c]$ are related by an integral relationships. This may be done by the assumption of Gauss points through the thickness.

However a more simple even if more approximate relationships can be used; as already suggested by Hoffman et al / 17 /; it involves the use of Iliouchine's / 14 / theory and the use of normality principle and associated flow rules / 15 /. Iliouchine gives collapse relationships for a shell in terms of the three following functions

$$(8a, c) \left\{ \begin{array}{l} Q_m = m_{\phi}^2 + m_e^2 - m_{\phi} m_e + 3 m_{\phi e}^2 \\ Q_n = n_{\phi}^2 + n_e^2 - n_{\phi} n_e + 3 n_{\phi e}^2 \\ Q_{mn} = m_{\phi} n_{\phi} + m_e n_e - 0.5 m_{\phi} n_e - 0.5 m_{\phi e} n_{\phi e} + 3 m_{\phi e} n_{\phi e} \end{array} \right.$$

where

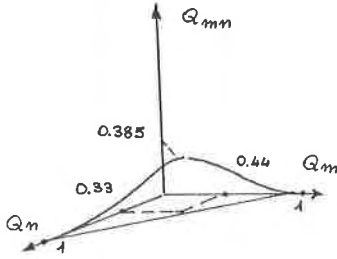
$$\begin{array}{lll} m_{\phi} = M_{\phi} / M_0 & m_e = M_e / M_0 & m_{\phi e} = M_{\phi e} / M_0 \\ n_{\phi} = N_{\phi} / N_0 & n_e = N_e / N_0 & n_{\phi e} = N_{\phi e} / N_0 \\ M_0 = \sigma_s t^2 / 4 & N_0 = \sigma_s t & \beta = M_0 / N_0 \end{array}$$

and where σ_s is the current yield stress. The collapse curve is given in terms of the three parameters Q_m , Q_n , Q_{mn}

$$(9) \quad \Gamma(Q_m, Q_n, Q_{mn}) = \text{zero}$$

A visualization of Iliouchine's curve is shown in fig.2

The locus given by Iliouchine is very difficult to be handled mathematically however the tangent planes could be used instead of (9) with reasonable accuracy, so that the yield



Iliouchine Yield Locus Fig.2

locus could be given as

$$(9') \quad a Q_n + b Q_m + c Q_{mn} = 1$$

where the coefficients a, b, c are functions of the actual Q_m, Q_n, Q_{mn} values. This formulation is in progress, however if Q_{mn} is small (what may be a reasonable hypothesis in this case), it is known that a very good approximation is

$$(9'') \quad Q_n + Q_m = 1$$

This formulation has been used for simplicity sake in this particular work. Then if (9'') is used together with the usual normality rule, i.e.

$$(10a) \quad \dot{\epsilon}^k = \dot{\lambda} (\partial \Gamma / \partial L_k)$$

if one assumes that a generalized strain hardening rule is postulated in terms of

$$(10b) \quad \epsilon_p = f(\sigma_p)$$

if one assumes that σ_p is a constant throughout the whole thickness, so that the plastic energy dissipation rate is given by

$$(10c) \quad \dot{\epsilon}_p = (\partial f / \partial \sigma_p) \sigma_p \dot{\sigma}_p t$$

by equating the internal dissipation rate to the external one comes out with the following equation

$$(11a) \quad \dot{\lambda} = \frac{\partial f}{\partial \sigma_p} \sigma_p \dot{\sigma}_p t / \left\{ \sum_k L_k (\partial \Gamma / \partial L_k) \right\}$$

and by the use of equation (9'') to compute the derivatives $(\partial \Gamma / \partial L_n)$

$$(11'a) \quad \dot{\lambda} = \frac{\partial f}{\partial \sigma_p} \sigma_p \dot{\sigma}_p \frac{t}{2}$$

Then the generalized Prandtl-Reuss equations are obtained

$$(12a, f) \quad \begin{aligned} \dot{\epsilon}_\phi &= \left(\frac{\partial f}{\partial \sigma_p} \right) \sigma_p \dot{\sigma}_p t \left(\frac{N\phi - 0.5 N_e}{No^2} \right) & \dot{\chi}_\phi &= \left(\frac{\partial f}{\partial \sigma_p} \right) \sigma_p \dot{\sigma}_p t \left(\frac{M\phi - 0.5 M_e}{Mo^2} \right) \\ \dot{\epsilon}_e &= \left(\frac{\partial f}{\partial \sigma_p} \right) \sigma_p \dot{\sigma}_p t \left(\frac{Ne - 0.5 N\phi}{No^2} \right) & \dot{\chi}_e &= \left(\frac{\partial f}{\partial \sigma_p} \right) \sigma_p \dot{\sigma}_p t \left(\frac{Me - 0.5 M\phi}{Mo^2} \right) \\ \dot{\epsilon}_{e\phi} &= 3 \left(\frac{\partial f}{\partial \sigma_p} \right) \sigma_p \dot{\sigma}_p t \frac{N\phi e}{No^2} & \dot{\chi}_{e\phi} &= 3 \left(\frac{\partial f}{\partial \sigma_p} \right) \sigma_p \dot{\sigma}_p t \frac{M\phi e}{Mo^2} \end{aligned}$$

On the other hand again by considering (9'') the actual value of σ_p is given by

$$(11b) \quad \sigma_p = \frac{1}{t} \left\{ (N\phi^2 + Ne^2 - N\phi Ne + 3 N\phi e^2) + \frac{1}{\beta^2} (M\phi^2 + Me^2 - M\phi Me + 3 M\phi e^2) \right\}^{1/2}$$

By differentiation and by the use of equations (11b) and (12a, f) one yields a relationship of the type :

$$(12) \quad \dot{\epsilon}^k = \frac{1}{t} \left(\frac{\partial f}{\partial \sigma_p} \right) [\gamma_{kj}^p] \cdot | \dot{L}_j |$$

The derivation of the $[\gamma_{kj}^p]$ matrix is here omitted for simplicity sake, however it is quite straightforward. In the same way by adding the elastic contributions, i.e. a matrix

$[\gamma_{KJ}^E]$, which is again been here omitted one has :

$$(12') \quad \left| \dot{\epsilon}^K \right| = \frac{1}{t} \left\{ \left(\frac{\partial \sigma}{\partial \sigma_p} \right) [\gamma_{KJ}^P] + [\gamma_{KJ}^E] \right\} \left| \dot{L}_J \right|$$

Then by inversion one yields equation (7) again.

2.3 Strain hardening

The usual hypotheses about strain hardening rules are made, both the bilinear formulation and the Ramberg-Osgood theory are implemented ; elastic unloading is considered.

2.4 Stiffness formulation

Local equilibrium equations could be used to interrelate stress resultants and formulate the final equations in terms of forces and displacements. Vlasov gives equilibrium equations, while Sanders's equations / 16 / have been used quite often. However in this case, the usual virtual work procedure has been resorted to; the following formulation has been used (it represents a slight simplification with respect to the one used by Bathe for ADINA). Let $\{P\}$ be the vector of the external loads and $\{d\}$ the associated displacement vector, then one has letting t and $t+\Delta t$ sprints denote value at two different time steps

$$L_{i,J}^{K,t+\Delta t} = L_{i,J}^{K,t} + \dot{L}_{i,J}^K \cdot \Delta t$$

Then the rate of work dissipation is

$$\dot{W} = \sum_i \sum_J \sum_K \bar{A}_{i,J} L_{i,J}^{K,t+\Delta t} \cdot \dot{\epsilon}_{i,J}^K$$

where $\bar{A}_{i,J}$ is an obvious veighting factor (area) in the quadrature formula . Then the rate of external works

$$\dot{W} = \sum_i \sum_J P_{i,J}^{t+\Delta t} \cdot \dot{d}_{i,J}$$

This formula can be rewritten by a suitable manipulation as

$$\dot{W} = \sum_m P_m^{t+\Delta t} \dot{\delta}_m$$

Omitting the details of the calculation and letting

$$(13) \quad \begin{aligned} P_m^e &= P_m^{t+\Delta t} - \sum_K \sum_J L_{i,J}^{K,t} \bar{A}_{i,J} A_{i,J,m}^K \\ Z_{m,p} &= \sum_K \sum_i \sum_J \sum_n \bar{A}_{i,J} A_{i,J,m}^K b_{i,J}^{K,n} A_{i,J,p}^n \end{aligned}$$

one has the final equation :

$$(14) \quad \left| P_m^e \right| = \left[Z_{m,p} \right] \cdot \left| \dot{\delta}_p \right|$$

Besides eq (13) can be slightly modified as by equilibrium at time t the second term represents $\left| F_m^{*,t} \right|$ the forces equilibrated at time t (of course modified so that they are consistent with $\left| \delta \right|$ vector), finally one yields

$$(14') \quad \left| P_m^{*,t+\Delta t} - F_m^{*,t} \right| = \left[Z_{m,p} \right] \cdot \left| \delta p \right|$$

This formulation allows an equilibrium iteration procedure, such as the one used in ADINA / 10 /.

Besides it should be mentioned that the stiffness matrix $\left[Z_{m,p} \right]$ has to be condensed in order to make it compatible with other pipe element; this can be done quite easily as the loads are supposed to be applied only at the ends of the elbow and to represent overall section loads (i.e. the ones compatible with the usual beam assumptions). This circumstance allows a condensation procedure (see the text of Przemieniecki / 18 / for theory description) to the degrees of freedom corresponding to section deformations; with reference to formula (2a,c) they are :

$$U_o, U_{11}, U_{21}, V_o, V_{11}, V_{21} \quad \text{at both the ends}$$

Then two matrices are obtained, an overall stiffness matrix (12x12) and a connection matrix correlating all the other Fourier terms to the explicit ones. Besides it should be mentioned that it is assumed that end sections are to remain plane, while unrestrained radial displacements are supposed to take place. Different hypotheses could be made, however this one seems to be comparatively simple and reasonable, if larger stiffness matrices are not formulated (for a discussion of this point see also the paper by Ohtsubo et al / 6 /).

2.5 Orientation matrix

In paragraph 2.4 the stiffness matrix has been formulated in terms of local coordinates, however the usual techniques can be used, based on the formulation of the orientation matrix. Besides large displacements are taken into consideration in the formulation of the orientation matrix, which is then reformulated at each load step. The procedure is here omitted for brevity sake ; it is sufficient to say that it represents an extension of Bolourchi's et al theory / 20 , 24 /.

Besides changes in the elbow geometry due to large displacements can be considered and a reevaluation of the elbow radius and central angle may take place; however this procedure should not be used at each step, as it involves the reevaluation of the matrix $\left[A_{i,J,K,m}^n \right]$, what is very time consuming.

2.6 Mass matrix

Work is in progress for the formulation of a consistent mass matrix, however for many applications a lumped mass approach is supposed to be sufficient.

3. Confirmations

3.1 Elastic analyses

Experimental and theoretical data have been presented by Ohtsubo and Watanabe /6/ ; the same cases have been rerun by means of PAMEL code, which incorporates the finite element just described. In all the cases Fourier expansion has been limited to the fourth order, while four sections are assumed along the azimuthal angle, fourteen Gauss points are assumed at each section. The results are shown in figs 3; the elbow is a 90° elbow where λ is the pipe factor and \bar{b} the radius ratio :

$$\lambda = Rt/r^2 \quad \bar{b} = R/r$$

Very good agreement has been found between the data obtained by the use of PAMEL and Ohtsubo et al's /6/ experimental and theoretical data.

A comparison has been made with data obtained by Sobel /19/ by the use of Marc /9/; the results have been reported in fig.4; again good agreement has been obtained.

3.2 Plastic analyses

Very few data have been published on the plastic analysis of elbows and the experimental data are very few. Data have been published by Shimizu et al /11/ by the use of ADINA and Marc codes and some experiments on the same geometry are available through the work of Sumikawa et al /23/. Analyses for the case of in plane moment have been reported also by Spence /7/. PAMEL program has been used for the analysis of some of these cases, the results are reported in figs 5,6,7. Some differences obtained for the peak deformation in the section are due to the use of Iliouchine model which tends to smooth the deformation pattern in the section; however the comparison has been considered satisfactory enough and some differences are tolerated due to the large gain in computing time.

4. Implementation

The finite element just described and named PAMEL (Plastic Analysis Membrane Elbows) has been implemented on a CDC 7600 machine, to which AMN is permanently connected. The peculiarities of this machine have been considered ideal for a computation, which requires high velocities and large storage areas.

The PAMEL element is part of the F.E. library of a computer code named PAULA /26/, which is currently under continuous development; the finite element library presently contains straight pipe /12/, the present elbow element, a linear pipe-elbow element derived from SAP4 /27/ and restraint elements. A T Joint element will be added very soon. The finite element library has been incorporated in ADINA to form PAULA (Plastic Analysis Using Library and Adina)

5. Application examples

A typical pipe run is shown in fig.8. It has been analyzed by means of PAULA (incorporated)

rating PAMEL) and by means of the simplified pipe whip code FRUSTA B /2B/, which uses partial models of the whole pipe run, the results are compared in fig.9.

6. Conclusions

An elastoplastic elbow element has been formulated and described; some applications obtained by the use of a code incorporating such elements have been described and compared with the ones obtained by the use of a simplified code. Some difference exists between the results, however due to the huge increase in computing costs obtained by the use of such sophisticated codes (PAULA) as compared to the more simplified ones (FRUSTA), the use of the more complicated ones should be limited as much as possible.

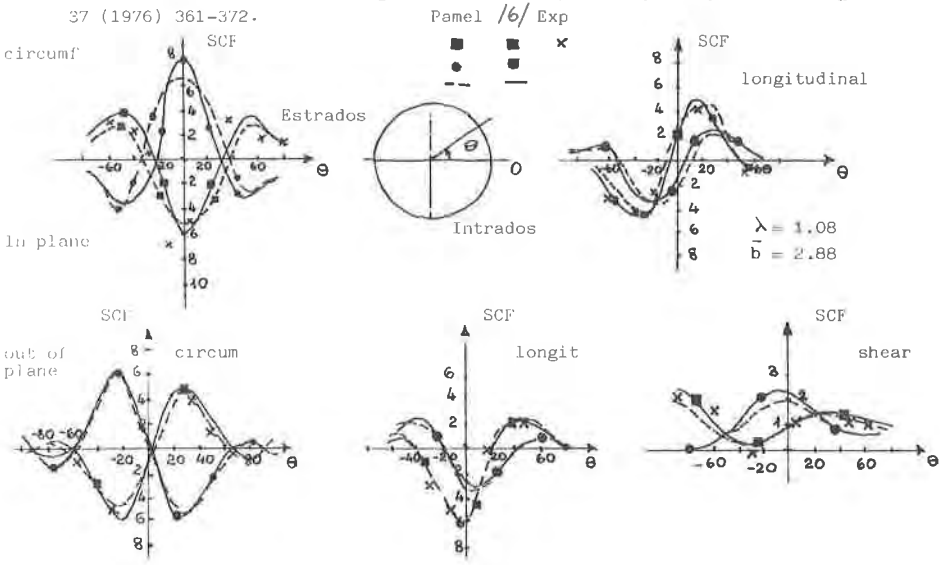
7. Acknowledgements

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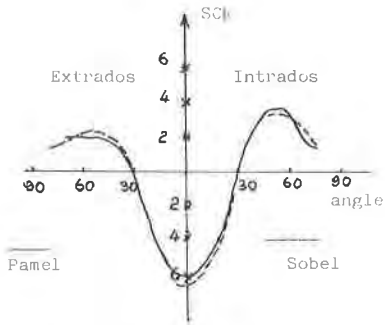
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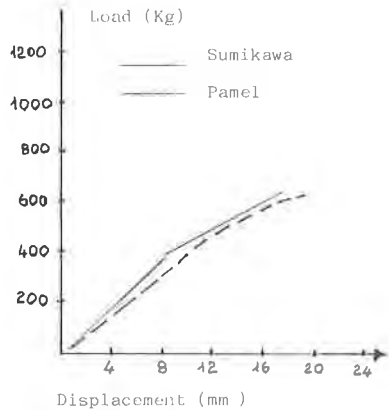
Stress Concentration Factors (SCF) for elbow from ref 6

Fig. 3



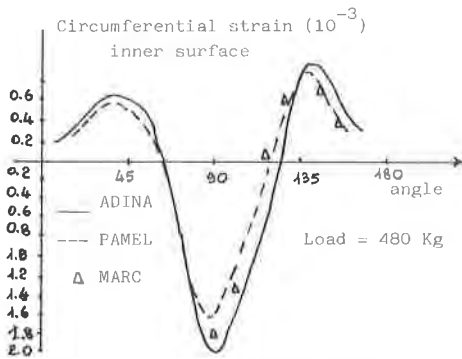
Stress concentration Factor for Sobel/19/Data

Fig.4



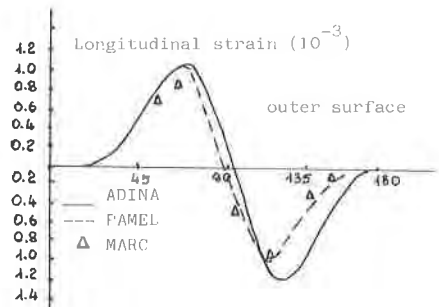
Displacement - Tests by Sumikawa et al

Fig. 5



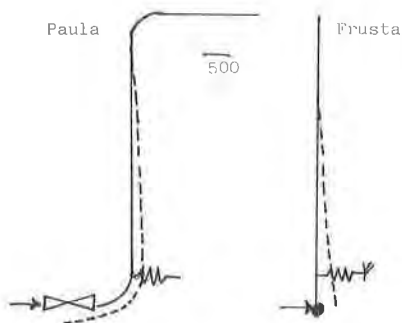
Comparison among ADINA, PAMEL, MARC Results by Shimizu et al /11/

Fig.6



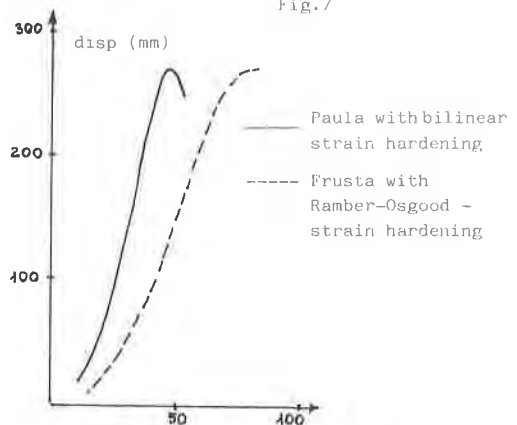
Comparison among ADINA, PAMEL, MARC Results by Shimizu et al /11/

Fig.7



Geometry and deformation Pattern for Paula and Frusta

Fig.8



Comparison between FRUSTA and PAULA Displacement at restraint

Fig.9