A 2- AND 3-DIMENSIONAL ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF AN SENB FRACTURE SPECIMEN

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Abstract
Much of the basic materials data on which the application of fracture mechanics rests, is derived from the theoretical analysis of two dimensional models. Recent three dimensional elastic finite element analyses have revealed crack tip singularity behaviour different from that previously postulated. This has permitted a rational analysis of fracture test results that were thought questionable.

It was considered that because many nuclear components may have failure modes in the elasto-plastic regime, that a three dimensional analysis of a standard test specimen could be of value in quantifying the fracture initiation process. A finite element model of a single edge notch bend specimen, with a relatively coarse mesh has been used to carry out the analysis employing the MARC finite element package. Exactly the same element configuration has been analysed for the two dimensional plane stress and plane strain cases to provide a comparison.

It is possible to show from the results differences in behaviour of the specimens in terms of crack tip opening and the J integral (energy difference) evaluation. The crack opening displacement, strains and J integral all vary through the thickness, but not in a totally consistent manner. Plastic collapse, defined as failure in numerical convergence, also varies considerably. The results provide a useful insight, but there are limitations resulting from the coarse nature of the element mesh.
1. **Introduction**

In practical situations, where fracture mechanics methods are applied to the assessment of structural components, it is inevitable that three-dimensional configurations must be dealt with. In some situations this may not be too difficult, particularly if linear elastic conditions prevail. However, in situations when large scale yielding proceeds fracture, an approach involving considerable conservatism must be adopted, especially with nuclear components, to account for any uncertainty.

Since recent three-dimensional elastic finite element analyses [1, 2], have revealed crack tip singularity behaviour different from that previously postulated, which subsequently allowed a rational analysis of fracture test results, it was felt that some benefit may be derived from a similar analysis continued into the plastic regime.

In what follows, the results of both two and three-dimensional elasto-plastic finite element calculations are presented, using a relatively coarse mesh which is identical for both cases. The results have been examined in terms of J evaluations (incremental and total energy), crack tip opening displacement and local strain distributions. They provide some useful insight into what can be accomplished but, are somewhat restricted by the coarse nature of the mesh and a method of describing the elasto-plastic crack tip character of any selected section normal to the crack plane.

2. **Finite Element Computations**

The specimen which has been modelled was a single edge notch beam in bending, with a span to thickness ratio of four. The essential dimensions were depth and thickness 110mm and crack depth 45mm. The material properties assumed were those for the ferritic steel A533B given by Robinson [3], that is

\[
\sigma = \begin{cases} 
1260 \varepsilon^{0.23} & \text{for } \varepsilon \leq 0.08 \\
1050 \varepsilon^{0.158} & \text{for } \varepsilon > 0.08
\end{cases}
\]

The finite element representation for both two and three dimensional calculations is shown in figure 1. Both plane stress and plane strain conditions were assumed in the two dimensional analysis. In the three dimensional analysis, a quarter specimen was modelled using two brick elements to represent the half thickness. Nine and twenty seven Gauss points were used in the two and three dimensional analyses respectively. The mid-side node was moved to the quarter position, for all elements surrounding the crack tip.

The calculations were carried out using the MARC package on a CDC computer. The evaluation of the J integral was achieved using Parks [4] differential stiffness procedure. The whole crack front was moved, in the two dimensional analyses but for the three dimensional case, only the surface or mid-plane node was moved. This must cast some doubt on the value of the result because the increment only occurs between the surface and the first mid-side node along the crack front. However, it will be assumed that the results can have either,

a) some real meaning
or 

b) are at least relative to each other for all load increments.

The crack tip opening displacement was determined by extrapolation of the flank angle of the opening crack to the plane normal to the crack tip.

3. Discussion of Results

A possible measure of the quality of finite element representation in modelling the specimen, is to compare the elastic stress intensity value when the yield stress is first reached. This is shown in Table 1; the theoretical value being calculated using the results presented in STP 401 [5]. At this point the first problem arises with evaluation of the three dimensional results. One can either assume the incremental area to be used with $\Delta U$ is a triangle or a distorted form reflected by the weighting given to the nodal points. This leads to an area of either one eighth or one twelfth that associated with crack advance through the complete thickness. These differences are shown in the table for both the surface and mid-plane of the specimen. In addition, the stress intensities were calculated from the total energy and area of remaining ligament using

$$J = \frac{2U}{E(W-a)}$$  \hfill (2)

<table>
<thead>
<tr>
<th></th>
<th>STP 410</th>
<th>FINITE ELEMENT</th>
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<tbody>
<tr>
<td>$\text{MN}^{-3/2}$</td>
<td>Triangle</td>
<td>Weighted</td>
</tr>
<tr>
<td>Mid Plane 3D Surface</td>
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<td>16.36</td>
</tr>
<tr>
<td>2D Plane Strain</td>
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</tr>
<tr>
<td>2D Plane Stress</td>
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<td>14.53</td>
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</table>

Table 1 Comparison of elastic stress intensities.

Figure 2 shows the load plotted against load point displacement and in addition the theoretical plastic collapse load is indicated as determined from the solution due to Green and Hundy [6]. For the finite element results, collapse is taken as the load at which numerical convergence fails to be achieved. If instead of using the tensile yield strength to calculate collapse, the maximum Mises stress at the first yield position is substituted, then the predicted value is 45% greater than indicated by finite element calculations. In comparison, a real A533B specimen with the following dimensions, width 108.9mm, depth 113.8mm and crack length approximately 46mm, gave a load maximum of 730 x $10^3$ MN. However this value was preceded by a small amount of stable tearing. The open triangles on figure 2 represent experimental data for load point displacement as a function of applied load.

Figure 3 is a graph showing the variation in crack mouth opening, while figure 4 shows the displacement at the first quarter node point behind the crack tip. In all three
figures it can be seen that reasonable agreement exists between the load and displacement for the two dimensional plane strain case and three dimensional analysis.

As pointed out earlier, there is a problem in choosing the area of crack increment to be used in deriving a J value, from the incremental energy difference in the three dimensional analysis. If one assumes that, the incremental energy over the whole loading range has a meaningful variation and can be related to J, then the two and three dimensional cases can be compared in terms of the ratio $\Delta W/\Delta U_0$ where the latter is the initial value associated with elastic loading. Figure 5 shows just such a comparison and indicates that the two dimensional plane strain results fall between the mid-plane and outer surface values of the three dimensional solution.

An alternative means of evaluating J, is to use equation 2, but such an approach only provides an overall value for the three dimensional case. This approach was followed but not using the constant value 2.0. A constant was determined for elastic loading, from the relationship between $K$, $J$ and $U$ using the theoretical value of $K$ from STP 41C as given in Table 1. The constants were 1.83 for the three dimensional and plane stress cases and 1.87 for the plane strain case. Up to the point of collapse, predicted by Green and Hundy [6], the J versus load curves are very similar, but diverge beyond this point and the three dimensional results increase more quickly with applied load, in a way similar to that shown in figure 5 for the mid-plane and plane strain case.

An area of considerable interest in non-linear fracture mechanics, is the relationship between J and crack opening displacement (COD), which has the form

$$ J = MxY $$  \hspace{1cm} (3)$$

The crack tip opening displacement was determined by projecting the flank opening angle to a line at the crack tip and normal to the plane of the crack. It should be noted that such projections are completely insensitive to variation in crack tip profile between the surface and mid plane. The coefficient $M$ was determined for $\sigma_Y$ constant e.g. 470 MPa$^{-2}$ and $\sigma_Y$ equal to the Mises stress. The value of $J$ was that determined from the use of equation 2 with the coefficients 1.83 and 1.87 as appropriate. The value of $M$ in equation 3 increases to a maximum for large scale yielding and these are tabulated for the different conditions and presented in Table 2.

<table>
<thead>
<tr>
<th>$M = \frac{J}{6\sigma_Y}$</th>
<th>$\sigma_Y = 470 \text{ MPa}^{-2}$</th>
<th>$\sigma_Y &gt; 470 \text{ MPa}^{-2}$</th>
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<td>3D</td>
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<td>2D Plane Strain</td>
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Table 2 Variation in the Coefficient $M$ relating J and COD.

Though somewhat remote, i.e. 3.75mm behind the crack tip, the displacement of the quarter node point is plotted in figure 6. It can be seen from this that the surface opening is less than that at the mid-plane. If the assumption is made that the surface
layer can be said to conform to plane stress deformation in the three dimensional case, then using J and the coefficients in Table 2, surface and mid-plane COD values can be calculated.

The surface and mid-plane J values were calculated from the incremental energy values at maximum load, assuming an area of crack release defined by a triangular wedge, as discussed earlier. Then, using the value of $M = 1.7$ for the mid-plane and $1.05$ for the surface (plane stress value), the mid-plane COD is $77\%$ larger than the surface value. It is of interest to note that, Robinson [7] using the infiltration method on Charpy pre-crooked specimens observed that the surface COD was some $20$ to $25\%$ smaller than the mid-plane value. The calculated crack tip COD was $48 \times 10^{-5}$ m and compares with $33.4 \times 10^{-5}$ m for the projected flank angle value.

There are a number of observations that can be made on the numerical results in general. After some initial trials it was found that the plastic load increments could be as large as the initial load to first yielding, without detriment in convergence to the stress – strain curve being followed. Large steps could be taken up to a point at which the solution would not converge, and then restarted at a lower load to approach collapse with smaller load increments. At some Gauss points around the crack tip, not necessarily the first to yield, the Mises stress approached three times the initial yield value.

Initial yield occurred for the three dimensional solution, close to the quarter thickness plane and it was observed that the maximum strain normal to the crack plane also peaked at the quarter thickness for all load increments. This is shown in figure 7 for the elastic and maximum load, where the strains at the line of the Gauss points closest to the crack plane and tip have been plotted. In both cases, the strains reduce towards the mid-plane, which was taken as a plane of symmetry in the calculations. One is forced to question, how much this behaviour is a consequence of finite element modelling and assumptions regarding the response of nodal displacements defined to provide symmetry.

Finally, the collapse load, i.e. the point at which the numerical solution would not converge, varied considerably for the three cases, being $1100$ KN, $835$ KN and $600$ KN for the two dimensional plane strain, three dimensional and two dimensional plane stress solutions respectively.

4. Concluding Remarks

As with the elastic three dimensional stress and strain fields around a crack tip, differences also occur when large scale yielding is present and this is really not surprising. The finite element method of analysis does appear capable of providing information on what is happening. A much finer element mesh than used in the present calculations is necessary. Using a very much finer element arrangement close to the crack tip may make it practical to use changes in crack profile to examine differences on various planes taken on sections through the thickness and normal to the crack.

In three dimensional elastic solutions for cracked bodies, the stresses and strains can be used on planes normal to the crack for the evaluation of $K$, the stress intensity. Thus a $K$ profile can be constructed along the crack tip to describe through thickness variation. An equivalent for elasto-plastic crack tip conditions does not appear to be available at the present time, although the $HRR$ [8] equations may be applicable if $J$ can be shown to have a meaning on any selected plane through the crack.
5. **Acknowledgements**

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**References**


Figure 2  Plot of Load Against Load Point Displacement for Each Specimen

Figure 3  Variation in Crack Mouth Opening With Applied Load

Figure 4  Variation in Crack Tip Opening for the Quarter Node Point Behind the Crack Tip
Figure 5  Variation of Energy Release with Applied Load

Figure 6  Through Thickness Variation in Crack Opening for the Quarter Node Point Behind the Crack Tip (Distance 3.75mm)

Figure 7  Strain Variation Through the Thickness at the First Line of Gauss Points. Ahead of the Crack Tip.