FURTHER STUDIES ON STRESS INTENSITY FACTORS OF SEMI-ELLIPTICAL CRACKS IN PRESSURIZED CYLINDERS

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SUMMARY

The authors have used, in the past, the three-dimensional stress intensity magnification factor, $M_{KS}$, for a semi-elliptical surface crack in a flat plate with a curvature correction factor, $M_C$, to estimate the stress intensity magnification factor, $M_K = M_C \cdot M_{KS}$, for unpressurized and pressurized inner semi-elliptical cracks and unpressurized outer semi-elliptical cracks in pressurized and thermally shocked cylinders. Recent papers by Atluri/Kathiresan, Helliot/Labbens/Pellissier-Tanon and McGowan/Raymund, however, showed that while this plate analogy with curvature correction provided reasonable estimates of the stress intensity factors at the deepest crack penetration, it underestimated the stress intensity factors at the cylindrical surface. The source of this discrepancy was traced to the curvature correction factor $M_C$, which was re-evaluated for various crack configurations and cylindrical geometries studied.

Using the updated $M_C$ together with the previously derived $M_{KS}$, stress intensity factor magnification factor, $M_K$, was rederived for:

1. Pressurized and unpressurized inner semi-elliptical cracks of two crack aspect ratios of $b/a = 0.2$ and $0.98$ at crack depth of $b/(R_0-R_1) = 0.4$, $0.6$, and $0.8$ in pressurized cylinders with outside-to-inside radius ratios of $R_0/R_1 = 3/2$, $5/4$, $7/6$, and $10/9$.

2. Unpressurized outer semi-elliptical cracks of two crack aspect ratios of $b/a = 0.2$ and $0.98$ at crack depths of $b/(R_0-R_1) = 0.4$, $0.6$, and $0.8$ in pressurized cylinders with outside-to-inside radius ratio of $R_0/R_1 = 3/2$, $5/4$, $7/6$, and $10/9$.

Other than isolated local deviations of approximately 10%, these results were in good agreement with Atluri's results for outer semi-elliptical cracks of $b/a = 1.0$ and crack depths of $b/(R_0-R_1) = 0.6$ and $0.8$ in a pressurized cylinder of $R_0/R_1 = 3/2$ as well as an inner pressurized semi-elliptical crack of $b/a = 0.2$ at crack depth of $b/(R_0-R_1) = 0.8$ in a pressurized cylinder of $R_0/R_1 = 3/2$. Similar agreements between the interpolated results for an outer crack of $b/a = 0.6$ at crack depth of $b/(R_0-R_1) = 0.4$ in a pressurized cylinder of $R_0/R_1 = 3/2$ and those of Blackburn and Helen's inner as well as outer elliptical crack for $b/a = 0.6$, $b/(R_0-R_1) = 0.4$ and $R_0/R_1 = 1.46$ and Atluri's outer elliptical crack for $b/a = 0.6$, $b/(R_0-R_1) = 0.4$ and $R_0/R_1 = 3/2$ are noted.
1. Introduction

In previous papers [1, 2, 3], the authors proposed the use of a three-dimensional stress intensity magnification factor, $M_{KS}$, for a semi-elliptical surface crack in a flat plate together with a curvature correction factor, $M_C$, for estimating the resultant stress intensity magnification factor, $M_K = M_C \cdot M_{KS}$, for unpressurized and pressurized inner semi-elliptical cracks and unpressurized outer semi-elliptical cracks in pressurized and thermally shocked cylinders. Figure 1 shows schematically the logic behind this approximate procedure where uncertainty in the curvature correction factor, $M_C$, lead the authors to list separately $M_{KS}$ and $M_C$ in place of the resultant stress intensity magnification factor of $M_K$. These $M_{KS}$ and $M_C$ values were reported in graphical forms in reference [1-3].

Recent papers by Blackburn and Helen [4], Atluri et al. [5, 6], Heliot et al. [7], and McGowan and Raymund [8], however, showed that while the authors plate analogy with curvature correction provided reasonable estimates of the stress intensity factors at the deepest crack penetration, it underestimated the stress intensity factors at the cylindrical surface. The source of this discrepancy was traced to the curvature correction factor, $M_C$, which did not adequately model the bending moment incurred in the surfaced flawed, curved beam. As a result, the basic modeling of the curvature correction factor was changed to account for this overturning moment. The updated $M_C$ together with the previously derived $M_{KS}$, was then used to redefine the stress intensity factor magnification factor, $M_K$. In the following, the redefined $M_C$ and some of the resultant $M_K$ are discussed.

2. Curvature Correction Factor $M_C$

The curvature correction factor derived in the previous study consisted of comparing two-dimensional solutions of internal or external edge-cracked pressurized cylinders with those of single-edged crack plates. Care was taken to model the side restraints inherent in the solution procedure using three-dimensional alternating technique [9]. In addition, a variable curvature correction, $M_C(\theta)$, which varied with the local depth, was used to account for the change in rigidity along the periphery of the semi-elliptical crack.

In comparing this curvature correction with the previously mentioned solutions [4-8] as well as a recent surface flaw solution factor by Raju and Newman [10] and Atluri [11], one error and one oversight were uncovered. The avoidable numerical errors induced by comparing the stress intensity factors of an "edge-cracked, pressurized cylinder" with the stress intensity factors of an "edge-cracked, flat plate with crack-face loading" were found to be significant enough such that when the former was replaced by an "edge-cracked, unpressurized cylinder with crack-face loading," the resultant curvature correction factor, differed significantly from those shown in reference [1,3]. Notable difference between the previous and newly derived curvature correction factors is the dependence of the latter on the cylinder geometry, $R_o/R_i$ where $R_o$ and $R_i$ are the external and internal radii, respectively.

In applying the newly derived curvature correction factors, which were computed for a two-dimensional, edge-cracked pressurized cylinder, this factor was further corrected to account for the increased bending rigidity in a pressurized cylinder with a semi-elliptical crack. This second adjustment was made by comparing the two-dimensional stress intensity magnification factor of Gross et al. [12] with the surface flawed solutions of reference [13].

The oversight mentioned above was in the use of variable $M_C(\theta)$ which by definition is unity at the cylindrical surface. In reality, the inherent overturning movement, which is severely restrained in the solution procedure of the alternating technique, should increase
the stress intensity factor at the cylindrical surface. The previously used $M_K(8)$ which decreased towards the cylindrical surface was thus replaced with the variation in stress intensity magnification factor derived for a surface flawed plate subjected to uniaxial tension. The new resultant curvature correction factor, $M_C$ thus consisted of the once adjusted curvature correction factor, times the variation in stress intensity magnification factor due to the overturning moment.

3. Results

Using the new curvature correction factor, the resultant stress intensity magnification factor, $M_K = M_C - M_{KS}$, are presented in graphical forms for the three cases in reference [13]. In the following some of these results are discussed.

3.1 Unpressurized Inner Cracks

Unpressurized inner semi-elliptical cracks of two crack aspect ratios of $b/a=0.2$ and 0.98 at crack depth of $b/(R_o - R_i) = 0.4, 0.6$, and 0.8 in pressurized cylinders with outside-to-inside radius ratios of $R_o/R_i = 3/2, 5/4, 7/6, $ and 10/9 were obtained [13]. Figure 2 shows that for a semi-elliptical crack with a crack aspect ratio of $b/a = 0.2$ and crack depth of $b/(R_o - R_i) = 0.8$ in a cylinder of $R_o/R_i = 3/2$, the $M_K$ value is in reasonable agreements with that of Atluri et al. [5] other than the 10% difference at two locations.

3.2 Pressurized Inner Cracks

Pressurized inner semi-elliptical cracks of two crack aspect ratio of $b/a=0.2$ and 0.98 at crack depth of $b/(R_o - R_i) = 0.4, 0.6$, and 0.8 in pressurized cylinders of $R_o/R_i = 3/2, 5/4, 7/6$ and 10/9 were also obtained [13]. Figure 3 shows that for the same semi-elliptical crack discussed in Figure 2 the $M_K$ value is in reasonable agreement with that of Atluri et al. [5] other than the same 10% differences at two locations.

Figure 4 shows a comparison between the interpolated $M_K$ obtained for a crack with aspect ratio of $b/a=0.6$, crack depth of $b/(R_o - R_i) = 0.4$ in a pressurized cylinder of $R_o/R_i = 1.5$ with those of Blackburn and Helen [4] for a pressurized cylinder of $R_o/R_i = 1.46$. Other than the 20% difference at the cylindrical surface, the two results are in excellent agreements with each other.

3.3 Unpressurized Outer Cracks

Unpressurized outer semi-elliptical cracks of two crack aspect ratios of $b/a=0.2$ and 0.98 at crack depths of $b/(R_o - R_i) = 0.4, 0.6$, and 0.8 in pressurized cylinders radius ratios of $R_o/R_i = 3/2, 5/4, 7/6$, and 10/9 were obtained [13]. Figures 5 and 6 show the excellent agreements with the results of Atluri and Kathiresan [6] for an outer semi-circular crack at two crack depths of $b/(R_o - R_i) = 0.6$ and 0.8 in a pressurized cylinder of $R_o/R_i = 3/2$.

Figure 7 shows comparison between interpolated $M_K$ for an external crack with aspect ratio of $b/a=0.6$ and crack depth of $b/(R_o - R_i) = 0.4$ in a pressurized cylinder of $R_o/R_i = 1.5$ with those of Atluri and Kathiresan [6] and those of Blackburn and Helen [8] for a pressurized cylinder of $R_o/R_i = 1.46$. While the agreements between all three results are reasonable, the Blackburn $M_K$ is lower at the cylindrical surface and is consistent with their result in Figure 4.

4. Conclusions

The updated stress intensity magnification factors for inner unpressurized and pressurized semi-elliptical cracks and outer semi-elliptical cracks in pressurized cylinders are in reasonable agreements with the isolated results generated by the others. Limited comparisons
of the interpolated stress intensity magnification factors are also in reasonable agreements with others. The results in reference [13] should thus provide reasonable estimates of the stress intensity factors for a large variety of semi-elliptical cracks in pressurized cylinders.

5. Added Note

Since the corrected curvature correction factor, $M_c$, for a thick-walled cylinder of $R_0/R_1 = 2/1$ is nearly unity up to crack depth in excess of $b/(R_0-R_1) = 0.8$, the stress intensity magnification for unpressurized and pressurized semi-circular cracks at the inner surface of this pressurized cylinder remains unchanged. The excellent agreements between the results obtained by the authors procedure and those obtained by Atluri et al. [5] using the three-dimensional hybrid displacement finite element method are noted.

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References


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FIGURE 1. PROCEDURE FOR ESTIMATING STRESS INTENSITY FACTOR OF AN INTERNAL SEMI-ELLIPTICAL CRACK IN A PRESSURIZED CYLINDER.

FIGURE 2. STRESS INTENSITY MAGNIFICATION FACTOR OF AN UNPRESSURIZED INNER SEMI-ELLIPTICAL CRACK IN A PRESSURIZED CYLINDER.

FIGURE 3. STRESS INTENSITY MAGNIFICATION FACTOR OF A PRESSURIZED INNER SEMI-ELLIPTICAL CRACK IN A PRESSURIZED CYLINDER.
FIGURE 4. STRESS INTENSITY MAGNIFICATION FACTOR OF A PRESSURIZED INNER SEMI-ELLIPtical CRACK IN A PRESSURIZED CYLINDER.

FIGURE 5. STRESS INTENSITY MAGNIFICATION FACTOR OF AN OUTER SEMI-CIRCULAR CRACK IN A PRESSURIZED CYLINDER.

FIGURE 6. STRESS INTENSITY MAGNIFICATION FACTOR OF AN OUTER SEMI-CIRCULAR CRACK IN A PRESSURIZED CYLINDER.

FIGURE 7. STRESS INTENSITY MAGNIFICATION FACTOR OF AN OUTER SEMI-ELLIPtical CRACK IN A PRESSURIZED CYLINDER.