

A CRITERION FOR ANALYSING FATIGUE CRACK INITIATION IN GEOMETRICAL SINGULARITIES

J. C. DEVAUX

*Framatome, Division des Fabrications, Centre de Calcul,
B.P. no. 13, F-71380 Saint Marcel, France*

Y. D'ESCATHA

*Service de l'Industrie et des Mines, Bureau de Contrôle de la Construction Nucléaire,
3, rue Devosges, F-2100 Dijon, France*

P. RABBE

Creusot-Loire, Centre de Recherche d'Unieux, F-42240 Unieux, France

A. PELLISSIER-TANON

Framatome, Tour Fiat, 1, place de la Coupole, F-92084 Paris-la-Défense, France

SUMMARY

It can be shown experimentally that the initiation of low cycle fatigue cracking at the tip of a notch tends to become independent of the radius of the notch when it becomes lower than a characteristic value which depends on the properties of the steel [1]. It is thus possible to conceive a method for verifying the resistance to initiation of low cycle fatigue cracking in components bearing acute built in notches, which does not require to consider the details of the geometric construction of the tip of the notch.

A theoretical justification of this result is that the damaging process of fatigue necessarily affects a zone of finite size, the length of which should be a characteristic quantity of the material. In the case of small scale yielding at the notch tip, where the behaviour of the plastic zone is governed by the elastic surrounding field, the fatigue cracking initiation criteria ought to be derived from the crack tip stress field calculated by an elastic analysis. It is thus proposed, following the work by ERDOGAN and SIH [2] and WILLIAMS and EWING [3] on brittle fracture, to determine the number of cycles to initiation N_a from the maximum variation of the normal stress $\Delta\sigma_{\theta\theta}$ at a characteristic distance d from the crack tip and to determine the initial direction of the crack propagation as the one corresponding to the maximum of $\Delta\sigma_{\theta\theta}(d)$.

To verify this theory, the number of cycles to initiate cracking has been measured on a set of compact tension specimens of a 316 (Z3 CND 17 12) steel presenting a large variety of notch depths and root radii. Several features of the observed behaviour are analysed to obtain independent estimates of the characteristic distance d , which all give a value close to 0.05 mm. The fatigue initiation curve relating N_a to $\Delta\sigma_{\theta\theta}(d)$ is drawn for this value of d . It is shown that this criterion allows to predict with a good accuracy the behaviour of test pieces of complex geometry presented in the next paper G.8.2. of this session.

1 - EXPERIMENTAL FACTS TO BE EXPLAINED AND MODELED

The experiments performed use notched CT specimens with various widths W , notch depths a , root radii ρ , and load levels ΔP or ΔK_I defined by :

$$\Delta K_I = \frac{\Delta P}{B\sqrt{W}} Y\left(\frac{a}{W}\right)$$

We call N_a the number of cycles to initiate a fatigue crack at the notch root, determined from the variation of the crack notch opening range with the number of cycles, as shown at figure 1. The material is a 316 type austenitic stainless steel and the tests are made at room temperature.

First observation

For given values of W , a and ΔP , N_a decreases when ρ decreases, except when ρ gets lower than some characteristic value ρ_1 : for $\rho < \rho_1$, N_a seems to remain constant. ρ_1 is found to be about 0.15 mm (figure 2).

Second observation

For large values of ρ , N_a is found to be well correlated with $\frac{\Delta K_I}{\sqrt{\rho}}$ for all geometries and loads. But this good correlation disappears for small values of ρ (figure 3). However it can be found again if ρ is made equal to a fixed value ρ_2 found to be about 0.35 mm in the expression $\Delta K_I/\sqrt{\rho_2}$ (which is then $\Delta K_I/\sqrt{\rho_2}$) when $\rho < \rho_2$ (figure 4).

2 - MODE I FATIGUE CRACK INITIATION IN SINGULAR ZONES

We find in real structures some singular geometries having a deep notch with a very small root radius (order of magnitude : 0.1 mm). Two examples of these "singular zones" are shown on figure 9.

Classical methods of fatigue evaluation are inadequate for such singular geometries where stresses and strains are very high at the notch tip and have very steep gradients (what value of stress intensity is to be taken ? In what angular direction at the notch tip ? If using a theoretical elastic stress concentration factor K_t , to what "nominal stress" must it be applied ?).

Physically speaking, the fatigue damage of the material at the notch tip can only be produced by sufficient stresses and strains over a volume of metal involved in the micro mechanisms of fatigue.

The model we thus thought of, to explain and predict quantitatively the previous experimental results, uses as fatigue initiation parameter in small-scale yielding conditions, the quantity $\Delta\sigma_{\theta\theta}(d)$, which is the variation during the load cycle of the $\sigma_{\theta\theta}$ component of the stress tensor calculated on an elastic basis at the distance d characteristic of the fatigue processes (figure 8). This distance d is to be found in the study. In this model, initiation will then be predicted by entering a unique characteristic ($\Delta\sigma_{\theta\theta}(d)$ versus Na) curve of the material, which is to be found in the study. This model is thus an extension of the theory of ERDOGAN and SIH [2] and WILLIAMS and EWING [3] for brittle fracture to the fatigue crack initiation phenomenon at notch and crack tips, in small-scale yielding conditions.

As we consider in the present paragraph mode I (symmetrical) situations, $\Delta\sigma_{\theta\theta}(d)$ is for the moment taken on the axis of symmetry x .

Considering deep notches, we use the formula given by CREAGER [4] (figure 8), reducing here to :

$$\Delta\sigma_{\theta\theta}(d) = \frac{\Delta K_I}{\sqrt{2\pi x'}} \left(1 + \frac{\rho}{2x'}\right) \quad (x' = d + \frac{\rho}{2})$$

$$\Delta\sigma_{\theta\theta}(d) = \frac{2}{\sqrt{\pi}} \frac{\Delta K_I}{\sqrt{2d + \rho}} \cdot \frac{d + \rho}{2d + \rho}$$

For large $\frac{\rho}{2d}$ ratios,
$$\Delta\sigma_{\theta\theta}(d) \approx \frac{2}{\sqrt{\pi}} \frac{\Delta K_I}{\sqrt{\rho}}$$

And for very small values of $\frac{\rho}{2d}$,
$$\Delta\sigma_{\theta\theta}(d) \approx \frac{\Delta K_I}{\sqrt{2\pi d}} = \frac{2}{\sqrt{\pi}} \cdot \frac{\Delta K_I}{\sqrt{8d}}$$

We thus plot on figure 5 :

$$Z = \frac{\Delta\sigma_{\theta\theta}(d)}{\frac{2}{\sqrt{\pi}} \cdot \frac{\Delta K_I}{\sqrt{\rho}}} \quad \text{and} \quad Y = \frac{\Delta\sigma_{\theta\theta}(d)}{\frac{2}{\sqrt{\pi}} \cdot \frac{\Delta K_I}{\sqrt{8d}}}$$

(with $k = 8$ and also 6.5 as will appear in the interpretation).

Interpretation of the first observation

This is immediately done on curve n° 1 of figure 5. We see that Y , that is to say $\Delta\sigma_{\theta\theta}(d)$, increases when ρ decreases at constant ΔK_I and thus Na decreases ; but we also see that Y and thus $\Delta\sigma_{\theta\theta}(d)$ is constant within 10 % when $\frac{\rho}{2d} < 2$ or $\rho < 4d$. Consequently the first observation is thus interpreted and this gives that $4d$ is about 0.15 mm, whence d is about 0.04 mm.

Interpretation of the second observation

This is immediately done on curve n° 2 of figure 5. We see that for large values of $\frac{\rho}{2d}$, Z is approximately constant, meaning that the fatigue initiation parameter $\Delta\sigma_{\theta\theta}(d)$ can be taken as $\Delta K_I/\sqrt{\rho}$; and we see that this is no more true for small values of $\frac{\rho}{2d}$ where, initiation being given by $\Delta\sigma_{\theta\theta}(d)$ and Z decreasing rapidly, the corresponding values of $\Delta K_I/\sqrt{\rho}$ are much too high at initiation (second observation, figure 3). But we see also that the good correlation between $\Delta K_I/\sqrt{\rho}$ and Na obtained for large values of $\frac{\rho}{2d}$, is maintained down to low values of $\frac{\rho}{2d}$ provided that ρ is replaced by 6.5 d in the expression $\Delta K_I/\sqrt{\rho}$: we see on figure 5 that 6.5 d is better than 8d because Y, for $k = 6.5$, is at the same level for small values of $\frac{\rho}{2d}$ that Z for large values of $\frac{\rho}{2d}$ (roughly speaking between 0.9 and 1.0). Then the fatigue initiation curve ($\Delta\sigma_{\theta\theta}(d)$ versus Na) can be expressed with a good approximation in terms of $\frac{\Delta K}{\sqrt{\rho}}$ versus Na, ρ being replaced by 6.5 d for small values of $\frac{\rho}{2d}$ (the transition between the two curves Y and Z, their intersection, is at $\frac{\rho}{2d} = 3.25$ or $\rho = 6.5 d$). Now, the second experimental observation gives that 6.5 d is about 0.35 mm, whence d is about 0.05 mm.

Conclusion

These two independant experimental observations can thus be interpreted by the present model, and give independantly two values of d of the same order : about 0.04 to 0.05 mm.

We thus expressed all the experimental resultats as $\Delta\sigma_{\theta\theta}(d)$ versus Na for $d = 0.05$ mm, but also for varying values of d. With the present model, the scatter of the experimental results $\Delta\sigma_{\theta\theta}(d)$ versus Na must be reduced to its minimum with the "true" value of d : the scatter obtained with varying value of d was statistically analysed (figure 7) and was found minimum for d equal to 0.05 mm.

The determination of the mean initiation curve ($\Delta\sigma_{\theta\theta}(d)$ versus Na) for $d = 0.05$ mm is presented at figure 6.

3 - EXTENSION OF THE MODEL TO REAL ENGINEERING CONDITIONS AND EXPERIMENTAL VERIFICATION

This very simple model can be at once extended to the real situations of three dimensional geometries, and general loading conditions : thermal loadings and non-symmetrical cases.

For instance in the two dimensional case with superposition of mode I and mode II conditions :

- the direction in which the crack initiates is the one for which $\Delta\sigma_{\theta\theta}(d)$ is maximum,
- the number of cycles to initiation is obtained by entering the fatigue initiation curve ($\Delta\sigma_{\theta\theta}(d)$ versus Na) with this maximum values of $\Delta\sigma_{\theta\theta}(d)$.

Two types of specimens representing the singular geometries of penetrations in vessels (figure 8), in particular representative conditions of welding, were tested to fatigue crack initiation (see paper G.8.2., 5th SMIRT [5]). One of them is predominantly in mode I conditions and the other one in mode II conditions. The $\Delta\sigma_{\theta\theta}(d)$, $d = 0.05$ mm, were determined by very accurate finite element calculations, using a radial mesh with singular elements at the tip (see paper G.8.2.). These experimental results, $\Delta\sigma_{\theta\theta}(d)$ and corresponding Na , are reported on the $\Delta\sigma_{\theta\theta}(d)$ versus Na mode I results of figure 6 and show a very good agreement.

4 - CONCLUSION

This study has allowed to determine a criterion allowing to calculate fatigue crack initiation in components bearing geometrical singularities, usable for industrial applications as is shown in paper G.8.2.

A complete development and justification of this criterion still requires :

- a systematic experimental verification in mixed mode loading,
- a study of an appropriate expression for large scale yielding,
- an appreciation for the cases of non proportional and thermal loadings,
- a study of the damage summation rules for combination of different loading cycles.

REFERENCES

- [1] RABBE P., AMZALLAG C., 1974, Etude de l'amorçage des fissures et de la vitesse de fissuration par fatigue de quelques aciers inoxydables austénitiques. Rév. Met. Déc. 74.
- [2] ERDOGAN F. and SIH G.C., 1963. On the Crack Extension in Plate under plane Loading and transverse Shear. J. Basic Eng., Déc. 63, 513-527.
- [3] WILLIAMS J.G., and EWING P.D., 1971, Fracture under Complex Stress, the angled Crack Problem, Int J. Fr., Mech, 8, 4.
- [4] CREAGER M., 1966. The elastic Stress Field near the Tip of a Blunt Crack. Thesis, Lehigh University.
- [5] BOSSER M., MERCIER F., AMZALLAG C., PONSOT J., 1979. Verification of the Resistance of PWR Thermal Sleeves to Thermal Fatigue. This Conference, paper G.8.2.

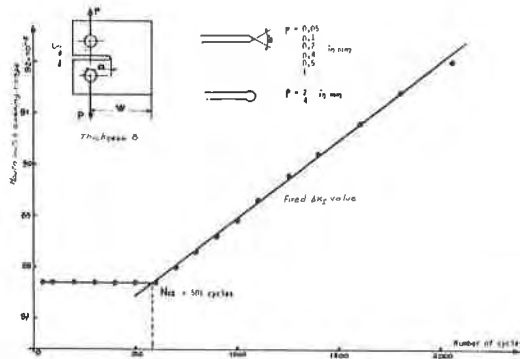


FIGURE 1 Measurement of fatigue crack initiation on CT specimens : mouth notch opening range versus number of cycles, for given specimen and ΔK_I value.

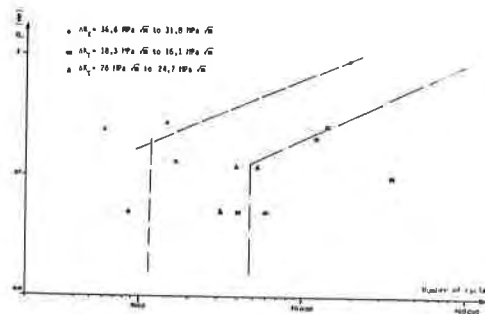


FIGURE 2 Variation of N_a with root radius for fixed values of ΔK_I .

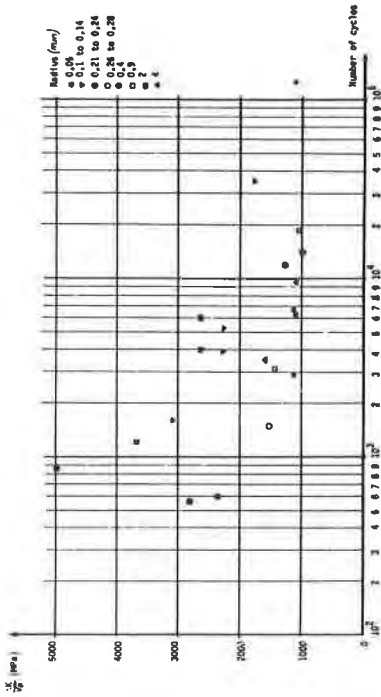


FIGURE 3 Variation of Na with ratio $\Delta K_I/\sqrt{\rho}$.

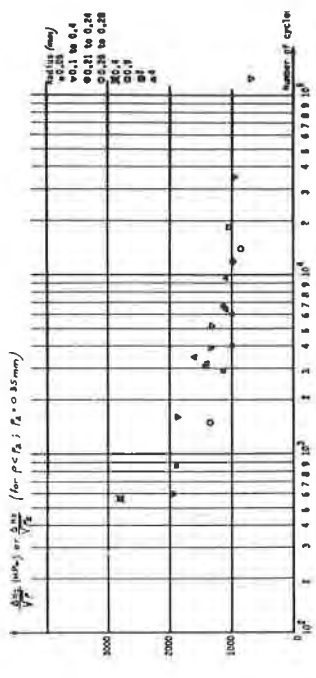


FIGURE 4 Variation of Na with $\Delta K_I/\sqrt{\rho}$, for $\rho > \rho_2$ ($\rho_2 = 0.35$ mm) and with $\Delta K_I/\sqrt{\rho_2}$, for $\rho < \rho_2$.

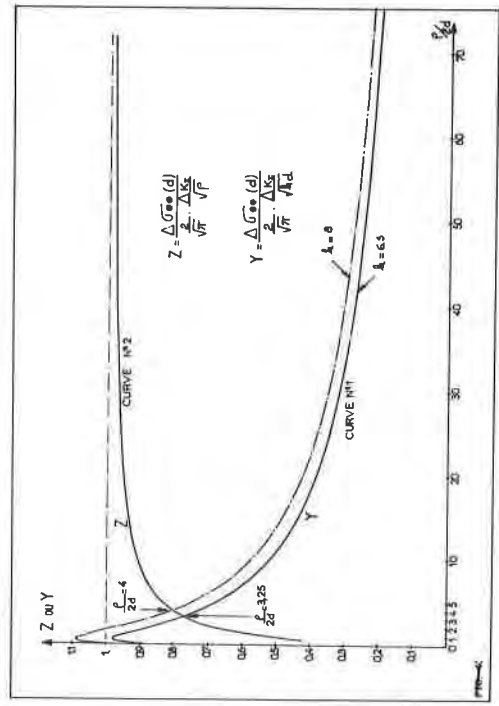


FIGURE 5 Expression of the reduced values $Z = \Delta\sigma_{00}(d)/(\frac{2}{\sqrt{\pi}} \cdot \frac{K_I}{\sqrt{\rho}})$ and $Y = \Delta\sigma_{00}(d)/(\frac{2}{\sqrt{\pi}} \cdot \frac{K_I}{\sqrt{\rho_2}})$ ($k = 8$ and 6.5), as a function of $\frac{\rho}{2d}$.

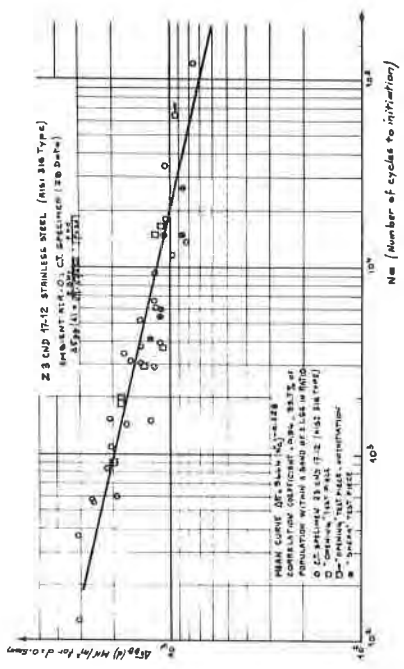


FIGURE 6 Experimental initiation values Na as a function CT specimens of 316 SS at room temperature for $d = 0.05$ mm, with least square best fit linear expression on log scale.

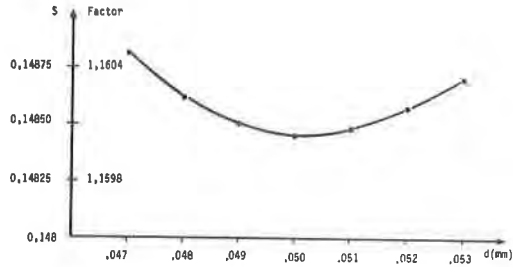


FIGURE 7 Mean square deviation S from the least square best fit initiation curve as a function of d for the same experiment as in figure 6.

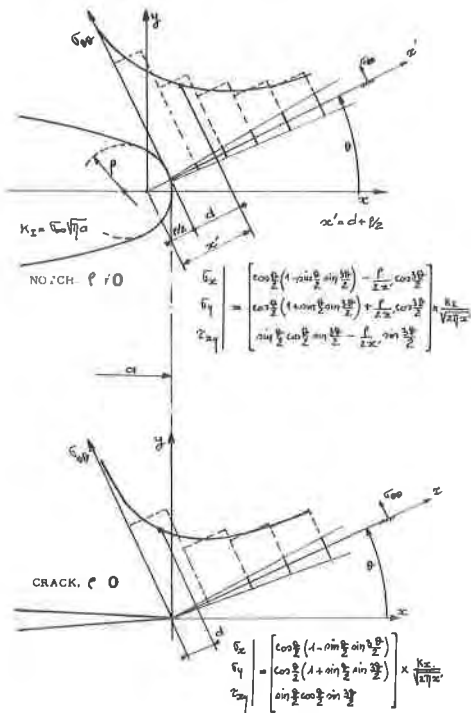


FIGURE 8 Expression of $\Delta \sigma_{\theta\theta}(d)$ as a function of the factors ΔK_I , ρ and d.

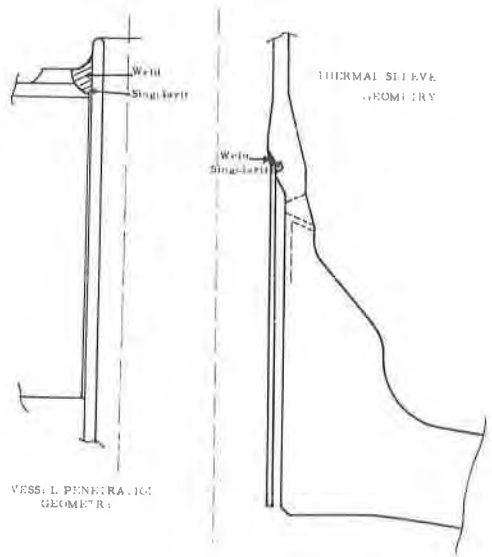


FIGURE 9 Examples of built in geometrical singularities (test pieces used in [4]).