A FRACTURE MECHANICS APPROACH TO PREDICTING THE EFFECTS OF WARM PRESTRESSING AND ITS APPLICATIONS TO PRESSURE VESSELS

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SUMMARY

A theory of warm prestressing based on the J-integral is described. The theory is validated using experimental warm pre-stressing data obtained on a carbon-manganese steel, two pressure vessel steels and mild steel. The theory is applied to the pressurised water reactor and the effects of warm prestressing evaluated after irradiation damage to the pressure vessel, and in the case of a loss of coolant accident. Warm prestressing increases the resistance to fracture of the vessel in the latter case, at least for deeply penetrating cracks, and inhibits the initiation and propagation of the cracks. The benefits of warm prestressing for shallow cracks is less certain and a more detailed analysis is required.

1. INTRODUCTION

Warm prestressing (WPS) consists of increasing the temperature of a cracked structure above its ductile-brittle toughness transition temperature, applying a load and then fully or partly unloading before cooling to the operating temperature below the transition. If the structure is re-loaded to failure at the operating temperature it fails at a stress intensity factor, K_f, above its fracture toughness value K_c. The phenomenon is illustrated in Fig. 1 where K_1 and K_2 results from WPS loads at temperatures T_1 and T_2, and T_3 is the operating temperature. The apparent increase in the resistance to failure after WPS is due to the increase in yield stress on cooling.

The general effects of overstressing have been reviewed in ref. [1], and a qualitative description of WPS in terms of failure mechanisms given in ref. [2]. Recently a theory of WPS based on fracture mechanics concepts has been proposed and experimentally validated [3]. This theory is formulated in terms of the J-integral, where J is interpreted in the Eshelby [4] sense as being the force on the crack tip and plastic deformation enclosed by the contour of integration. It involves only elastic components in its evaluation and will be henceforth referred to as J_e. This definition contrasts with the form of J commonly used in finite element calculations, which involves both elastic and plastic components and will be called J_{e-p}. Provided the path of integration is outside the plastic zone the two forms of J are identical. If the path goes through the plastic zone J_{e-p} appears numerically to be path independent [5], whereas consistent with its physical interpretation as a force on plasticity, J_e is path dependent [6].

In ref. [3] it was proposed that failure occurs when the total force J_e on all regions in the plastic zone where plastic flow can occur reaches a critical value, $J_c = K_c^2/E'$, where E' equals Young's modulus for plane stress and $E/(1-\nu^2)$, where ν is Poisson's ratio, for plane strain. WPS gives rise to residual zones (areas of residual plastic strain) at the operating temperature where, because the stress is below the yield stress, plastic flow cannot occur. These residual zones are a consequence of the increase in yield stress between the WPS and operating temperatures (Fig. 2). J_e must therefore be evaluated around a path which encloses only the plastic enclave where plastic flow is possible (Fig. 2a). Clearly this requirement means that after WPS J_e is zero at T_3 until extra load is added (Fig. 2b,c).

2. THEORY

The theory developed in ref. [3] has been solved in the small scale yielding regime in ref. [7]. The resulting equations governing failure are,

$$K_c^2 = \bar{\sigma}_3 \left[\frac{(K_f-K_2)^2}{(\bar{\sigma}_2+\bar{\sigma}_3)} - \frac{(K_1-K_2)^2}{(\bar{\sigma}_1+\bar{\sigma}_2)} \left(1-F\left(\left[\frac{(K_f-K_2)(\bar{\sigma}_1+\bar{\sigma}_2)}{(K_1-K_2)(\bar{\sigma}_2+\bar{\sigma}_3)}\right]^2\right)\right) + \frac{K_1^2}{\bar{\sigma}_1}\left(1-F\left(\left[\frac{(K_f-K_2)\bar{\sigma}_1}{K_1(\bar{\sigma}_2+\bar{\sigma}_3)}\right]^2\right)\right)\right] \quad \text{for case 1} \qquad (1a)$$

$$K_c^2 = \bar{\sigma}_3 \left[\frac{(K_f-K_1)^2}{(\bar{\sigma}_3-\bar{\sigma}_1)} + \frac{K_1^2}{\bar{\sigma}_1}\left(1-F\left(\left[\frac{(K_f-K_1)\bar{\sigma}_1}{K_1(\bar{\sigma}_3-\bar{\sigma}_1)}\right]^2\right)\right)\right] \quad \text{for case 2} \qquad (1b)$$

and

$$K_c = K_f , \qquad \text{for case 3} \qquad (1c)$$

where the various cases are defined by,

$$\text{Case 1,} \quad K_1/\bar{\sigma}_1 > (K_1-K_2)/(\bar{\sigma}_1+\bar{\sigma}_2) > (K_f-K_2)/(\bar{\sigma}_2+\bar{\sigma}_3) \qquad (2a)$$

$$\text{Case 2,} \quad K_1/\bar{\sigma}_1 > (K_f-K_1)/(\bar{\sigma}_3-\bar{\sigma}_1) > (K_f-K_2)/(\bar{\sigma}_2+\bar{\sigma}_3) > (K_1-K_2)/(\bar{\sigma}_1+\bar{\sigma}_2) \qquad (2b)$$

$$\text{Case 3,} \quad (K_f-K_1)/(\bar{\sigma}_3-\bar{\sigma}_1) > K_1/\bar{\sigma}_1 > (K_f-K_2)/(\bar{\sigma}_2+\bar{\sigma}_3) > (K_1-K_2)/(\bar{\sigma}_1+\bar{\sigma}_2) \qquad (2c)$$

and $\bar{\sigma}_i$ are the flow stresses at temperatures T_i, $i = 1,2,3$.

The function F(z) is given by [7]

$$F(z) = (1-z)^{\frac{1}{2}} - \frac{z}{2} \ln \left| \frac{1 + (1-z)^{\frac{1}{2}}}{1 - (1-z)^{\frac{1}{2}}} \right|$$

For a given value of K_c, equation (1) can be solved for K_f. Typical values of K_f for various WPS loads K_1 and K_2, are shown in Fig. 3, where $\bar{\sigma}_3 = 1.25\bar{\sigma}_2 = 1.25\bar{\sigma}_1$. In general K_f is not a sensitive function of flow stress. Failure curves such as those shown in Fig. 3 can be used to assess the increased resistance to fracture arising from WPS.

3. VALIDATION OF THEORY

The WPS theory and its concepts have been validated in ref. [3] and [7] using experimental data obtained on a carbon-manganese steel, a carbon-manganese pressure vessel steel (experimental data from ref [8]) and A533B pressure vessel steel (experimental data from ref [9]). Recent data on mild steel [10] has also been analysed here using equations (1) and (2). The results of these analyses are shown in Fig. 4 where the experimentally determined stress intensity factors at failure, K_f, are shown plotted against the values predicted using theory. This data covers a wide range of WPS loadings from $K_2 = K_1$ (no unloading before cooling) to $K_2 = 0$ (total unloading before cooling). The good agreement between measured and predicted values of K_f gives confidence in the theory.

4. APPLICATIONS TO PRESSURE VESSELS

There are two main areas where WPS may influence the safety of cracked pressure vessels in a light water reactor. The first results from irradiation damage, and is applicable to the belt-line region. Irradiation increases the yield stress of A533B steel and decreases the toughness, in the sense that the ductile-brittle transition temperature is increased [11]. Typically the yield stress increases by about 40 per cent for a neutron dose of around 2×10^{19} n/cm^2. Assuming the pressure vessel contains an inherent defect prior to irradiation damage, and that this defect does not propagate sub-critically due to cyclic loading, then the normal pressure loading provides a WPS, because the irradiation causes the yield stress to increase from $\bar{\sigma}_1$ to $\bar{\sigma}_3$ while the pressure loading is maintained. This corresponds to the case when $K_1 = K_2$ and $T_1 = T_2$ i.e. $\bar{\sigma}_1 = \bar{\sigma}_2$. Assuming a 25 per cent increase in yield stress at the operating temperature T_1, and K_c is the irradiation embrittled toughness, then $\bar{\sigma}_3 = 1.25\bar{\sigma}_1 = 1.25\bar{\sigma}_2$ and from Fig. 3 the effective toughness of the vessel material is increased by about 20 per cent i.e. $K_f \simeq 1.2K_c$ when $K_1 = K_c$.

Of course the important proviso in the foregoing example was that sub-critical crack growth did not occur. Fatigue crack growth is expected during the lifetime of the pressure vessel and it is highly probable that the crack tip will advance through the residual zone formed by the increase in yield stress (compare Fig. 2) and so wipe out any benefits accrueing from WPS. However, the above example illustrates how WPS may produce benefits, and also a possible mechanism by which these benefits may be removed.

A second example where WPS may occur is in a loss of coolant accident (LOCA) |9|. Here the situation is complicated by simultaneous changes in load and temperature resulting from thermal transients. Fig. 5 (after ref. |12|) shows the type of stress intensity variation with time for various longitudinal defects in the belt-line region during a LOCA. Here a is the crack length and w is the wall thickness (216mm). Also shown are the locus of points for $K_1 = K_c$, corresponding to initiation of crack growth, assuming pre-irradiated toughness values. No account of WPS was allowed for in the analysis.

In a LOCA the inside of the pressure vessel would cool to below the ductile-brittle transition temperature relatively quickly. Hence small defects would propagate at relatively small stress intensities (Fig. 5). In this case the material at the crack tip is subject to a simultaneous increase in yield stress and load, and WPS effects are likely to be small, if not totally absent. However, deeper cracks propagate after many minutes, and after the load on them has peaked (compare the $^a/w = 0.5$ curve in Fig. 5). Here a WPS effect is to be expected and may result in non-propagation of the crack although the stress intensity exceeds the toughness. For example, assume that the peak loading, $K_1 = 230$ MPa $m^{\frac{1}{2}}$ when $^a/w = 0.5$, occurs while the temperature T_1 at the crack tip exceeds the transition temperature, and that $\bar{\sigma}_1 = 375$ MPa. Without WPS effects initiation of crack growth occurs at a temperature T_3, 35 minutes after the commencement of the LOCA. At this temperature let $\bar{\sigma}_3 = 470$ MPa. From Fig. 5 $K_2 = K_c = 143$ MPa $m^{\frac{1}{2}}$ and $1.25\bar{\sigma}_1 = 1.25\bar{\sigma}_2 = \bar{\sigma}_3$, assuming the unloading to K_2 occurs at T_1. Hence $K_1/K_c = 1.6$, $K_2/K_1 = 0.62$, and from Fig. 3 initiation of crack propagation is predicted when $K_f = 1.74K_c = 243$ MPa $m^{\frac{1}{2}}$. Thus WPS effects in this case inhibit the onset of crack growth by raising the materials resistance to fracture, by 70 per cent from 143 MPs $m^{\frac{1}{2}}$ to 243 MPa $m^{\frac{1}{2}}$.

Similar arguments apply to all crack lengths where the peak loading occurs before the predicted initiation of crack propagation determined ignoring WPS. Furthermore the same trends would occur if irradiated toughness values were used in the analysis. These would change the values of $^a/w$ at which initiation occurs, but not the predicted behaviour.

5. DISCUSSION

The beneficial effects of WPS may occur whenever the yield stress of a loaded cracked material increases. As seen from the foregoing section this may arise due to irradiation damage or to temperature changes. In some circumstances the effects of WPS may be wiped out, this is so when crack growth by fatigue, or some other mechanism causes the crack tip to extend beyond the residual zone of plastic strain caused by the loading K_1 at temperature T_1.

Another case where WPS may influence safety assessments is when pressure vessels are proof tested at temperatures below the operating temperature. There the actual operating load provides the WPS and this may be sufficient, depending on the size of pre-existing defect, to effectively increase the toughness of the material at the proof test temperature, and thus invalidate the calculation of tolerable defect sizes based on this test.

These effects are predictable and quantifiable using the present theory.

6. CONCLUSIONS

A fracture mechanics theory of WPS has been briefly outlined and validated using experimental data. The theory is applicable to nuclear reactors and may be used to quantify the beneficial effects, if any, of WPS arising from increases in the yield stress due to irradiation damage, and the thermal transients occurring in a LOCA. In the latter case WPS improves the resistance to fracture of the pressure vessel to deep cracks, but it may have no effect on the initiation and propagation of shallow defects.

7. ACKNOWLEDGEMENTS

The author wishes to thank Dr. D.A. Curry for making his experimental results available before publication.

This work was performed at the Central Electricity Research Laboratories and is published by permission of the Central Electricity Generating Board.

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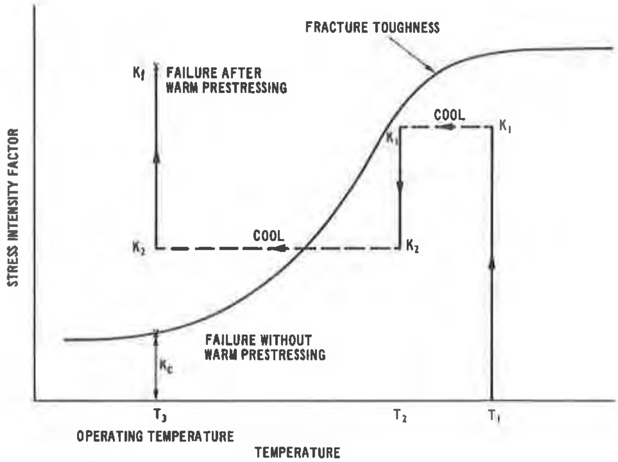

FIG. 1 SCHEMATIC REPRESENTATION OF WARM PRESTRESSING AND THE RESULTING INCREASE IN RESISTANCE TO FRACTURE

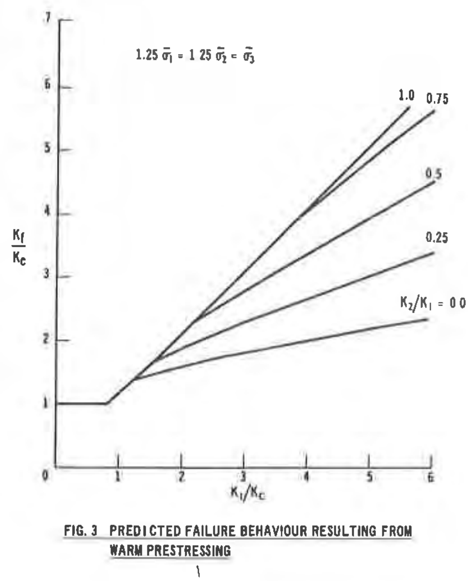

FIG. 3 PREDICTED FAILURE BEHAVIOUR RESULTING FROM WARM PRESTRESSING

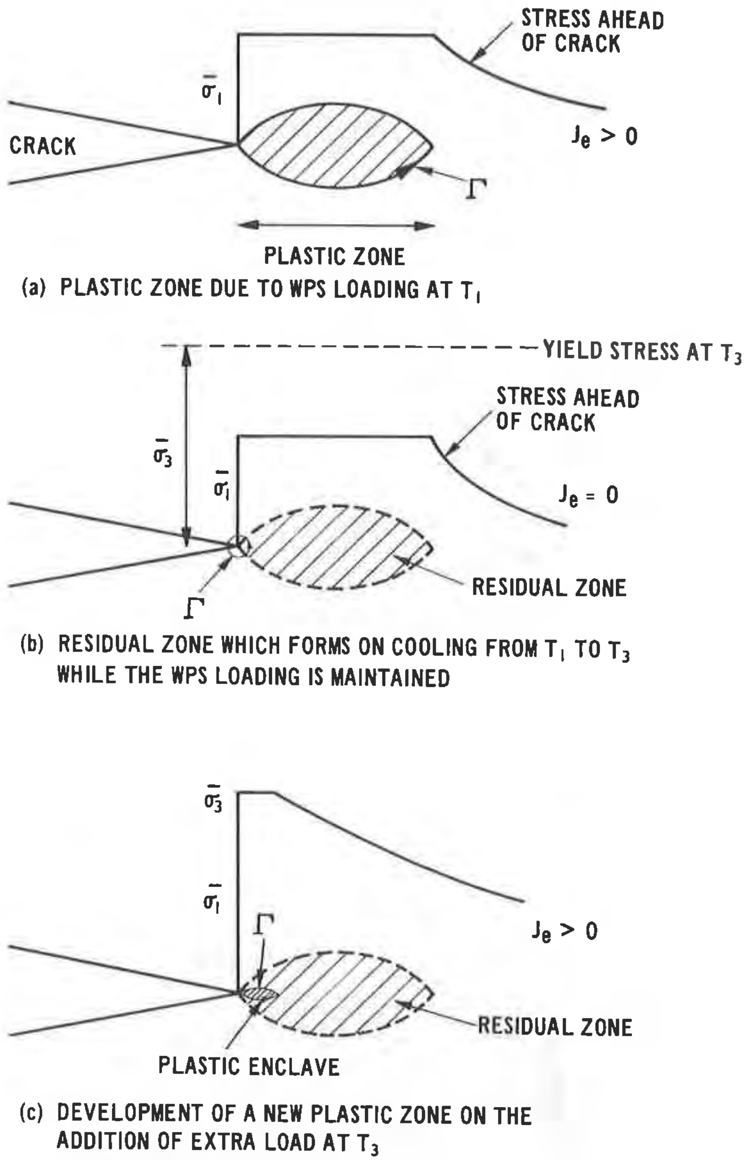

Fig. 2 The Formation of Residual Zones by WPS and Value of J Evaluated for Contour Γ which Includes only Areas Where Plastic Flow can Occur

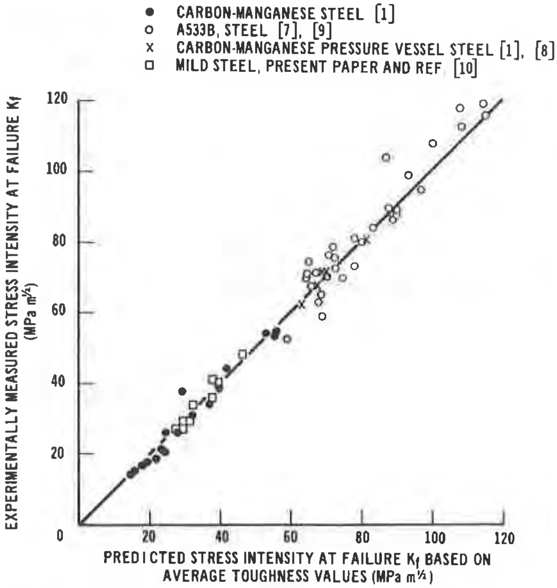

FIG. 4 VALIDATION OF WARM PRESTRESSING THEORY

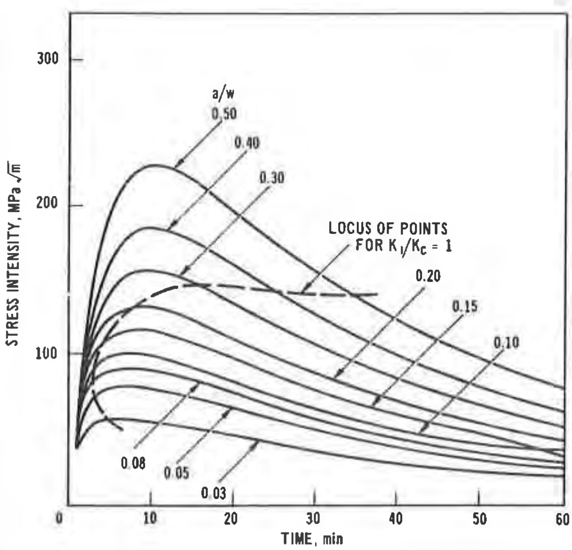

FIG. 5 STRESS INTENSITY FACTOR K_I vs TIME FOR DIFFERENT RELATIVE CRACK DEPTHS a/w, SHOWING THE LOCUS OF POINTS AT WHICH THE CRITICAL LEVEL FOR INITIATION K_c WOULD BE ATTAINED IN THE ABSENCE OF WARM PRESTRESS [12]