FAILURE STRENGTH AND ELASTIC LIMIT
FOR CONCRETE: A COMPARATIVE STUDY

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ABSTRACT

Due to increased demand for realistic analysis of structures such as prestressed concrete reactor vessels and reactor containments, the formulation of general constitutive equations for concrete is of considerable importance. In the field of constitutive equations the correct definition of the limit state represented by the concrete failure surface is a fundamental need.

In this paper carried out by a Danish-Italian cooperation, several failure criteria obtained by different authors are compared with failure experimental data obtained with triaxial tests on concrete specimens.

Such comparison allow to carry out conclusive considerations on the characteristics of the concrete failure surface and on the advantages and disadvantages of the different criteria. Considerations are also reported on the definition of a limit elastic surface, whose knowledge is of fundamental importance for designers of complex structures in concrete.
1. INTRODUCTION

The study of stress-strain behaviour of triaxially loaded concrete is of considerably complex nature and the set-up of relevant constitutive equations depends clearly on the boundary conditions, i.e. elastic limit and failure strength.

While extensive research has been directed towards determination of the failure conditions, the interest for the elastic limit, inside which linear elasticity can be assumed, is of newer date and only few results are available. Determination of the failure surface can be obtained by measuring in principle only stresses, and failure is a uniquely defined phenomenon, as it occurs at the maximum load the material can carry.

However, determination of the elastic surface depends both on stresses and strains, and in addition, this surface is not related to a uniquely defined phenomenon. In spite of these difficulties, the location of the elastic limit has much importance also for the designer, as linear elasticity and linear viscoelasticity can be assumed inside this limit in the short-term and long-term, respectively.

2. GEOMETRICAL PRELIMINARIES

For a given loading rate and considering proportional loading, a limit criterion for an initially isotropic material in a homogeneous stress state can be expressed in terms of the three stress invariants. Alternatively, the criterion can be given in the form

\[ g(\sigma_1, \sigma_2, \sigma_3) = 0 \]  \hspace{1cm} (1)

where \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are the principal stresses that occur symmetrically.

Eq. (1) describes a surface in a cartesian coordinate system with axes \( \sigma_1, \sigma_2, \) and \( \sigma_3 \).

This surface can also be described by the coordinates \( (x, y, \theta) \), as shown in Fig. 1.

The coordinate \( x \) gives the distance of the projection of the stress point on the unit vector \( \mathbf{e} = (1,1,1)/3 \) on the hydrostatic axis, and \((y, \theta)\) are polar coordinates in the deviatoric plane that is orthogonal to \((1,1,1)\), cf. fig. 1 a) and b).

The term \( \sigma_c (\sigma_c > 0) \) denotes the uniaxial compressive strength. It is easily proven, e.g. \( [1] \), that:

\[ X = |ON| = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{3} \sigma_c} \] \hspace{1cm} (2)

\[ Y = |NP| = \frac{\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}}{\sqrt{3} \sigma_c} \] \hspace{1cm} (3)

\[ \cos \theta = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{2 \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}} \] \hspace{1cm} (4)

As we assume that \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \) and that tensile stresses are positive, then \( 0 \leq \theta \leq 60 \) holds. The use of eqs. (2), (3) and (4) corresponds to the use of the stress invariants, as \( x \) and \( y \) are proportional to the first stress invariant and the square root of the second invariant of the stress deviator, respectively, while the angle \( \theta \) reflects the influence of the third stress invariant. Thus, eq. (1) is replaced by:

\[ f(x,y,\theta) = 0 \] \hspace{1cm} (5)
The superiority of this form compared to eq. (1) clearly appears when expressing mathematically the trace of the limit surface in the deviatoric plane.

The meridians of the limit surface are the curves on the surface, where $\theta = \text{constant}$ applies. For experimental reasons, two meridians are of particular importance, namely the compressive meridian, where $\sigma_1 = \sigma_2 > \sigma_3$ holds, and the tensile meridian, where $\sigma_1 > \sigma_2 = \sigma_3$ applies.

Finally, to achieve a convenient comparison of experimental data and proposed models, the coordinates $x$ and $y$ have been normalized by the uniaxial compressive strength.

3. FORM OF THE FAILURE SURFACE

Basing on experimental facts, Newman and Newman [2] and the writers [1, 3, 4] conclude the following:

1) the meridians are curved, smooth and convex with $y$ increasing for decreasing $x$ - values;
2) the ratio, $y_t / y_c$, in which indices $t$ and $c$ refer to the tensile and compressive meridians respectively, (cf. Fig. 1) increases from approx. 0.5 for decreasing $x$ - values, but remains less than unity;
3) the trace of the failure surface in the deviatoric plane is smooth and convex for compressive stresses;
4) in accordance with 1), the failure surface opens in the negative direction of the hydrostatic axis.

4. COMPARISON OF SOME FAILURE CRITERIA WITH EXPERIMENTAL DATA.

Several important failure criteria have been proposed in the past, and some of them are evaluated in refs.[1, 2, 3, 4]. In addition, Newman and Newman [2], Hannant [5] and Hobbs, Pomeroy and Newman [6] contain a collection of different experimental failure data. In this paper, we concentrate on some of the several criteria proposed recently, namely those of Reimann - Janda [7, 8], Willam and Warnke [9], Chen and Chen [10], Cedolin, Crutzen and Dei Polli [11] and Ottosen [4]. These criteria will be compared mutually and with some representative experimental results.

The coefficients involved in the considered criteria are calibrated by some distinct strength values, for instance, uniaxial compressive strength $\sigma_c$, uniaxial tensile strength $\sigma_t$ ($\sigma_t > 0$), etc. In some criteria only a few strength values are necessary, while other need more strength values. Noting that all coordinate systems considered here are normalized by $\sigma_c$, the applied parameters are given in the table 1.

The Reimann-Janda criteria was originally proposed by Reimann [7], but it is here evaluated by using the coefficients proposed by Janda [8].

Fig. 2 shows the comparison of the considered criteria with the experimental data of Richart, Brandtzæg and Brown [12], Balmer [13], Hobbs [14, 15] and Ferrara et al. [16]. The figure shows the compressive and tensile meridians. Except for the proposal of Chen and Chen [10], a good agreement is obtained for all criteria. The greater discrepancy for the Chen and Chen model is due to the fact that it was used in a strain hardening plasticity model, and therefore it only includes the first two stress invariants, so that a circular shape of the failure surface is predicted in the deviatoric plane.

While the failure surface proposed by Willam and Warnke [9] intersects the hydrostatic axis for large negative $x$-values (in this case $x = -14$), the other surfaces open in the negative direction of the hydrostatic axis. The predicted shape in the deviatoric plane, where $x = -2$, is shown in Fig. 3 for the criteria considered. The proposals of Reimann-Janda and Cedolin, Crutzen and Dei Polli [11] both involve singular points (corners).
In addition, the trace of the latter proposal is concave along the tensile meridian.


Great importance is related to biaxial stress states and Fig. 4a shows a comparison for all criteria, except that of Cedolin, Cruzen and Dei Poli [11], with the experimental results of Kupfer, Rüsch and Hilsdorf [17]. All criteria in Fig. 4a show a good agreement and especially those of Willam and Warnke [9] and of Ottosen [4] are in very good agreement with the experimental data, also when tensile stresses occur.

Comparison of Figs. 2 and 4a shows that the model of Chen and Chen [16] is much better suited for prediction of biaxial failures than of triaxial ones. For biaxial loading, the proposal of Cedolin, Cruzen and Dei Poli [11] is compared with the other criteria in Fig. 4b. It appears that the concavity along the tensile meridian has a worse effect on the obtained curve.

Comparison in general of Figs. 3 and 4 reveals that even small changes in the form of the trace in the deviatoric plane have considerable effects on the biaxial failure curve. Indeed, the latter curve is the intersection of the failure surface with a plane, having a rather small angle to the hydrostatic axis.

This emphasizes the need for a very accurate description of the trace in the deviatoric plane. In general, it may be concluded that fitness of a failure criteria can only be estimated, when comparison with experimental data are performed in at least three planes of different type.

5. ELASTIC LIMIT

While in principle the behaviour of concrete is nonlinear from the beginning of the loading, it is generally considered as an acceptable approximation to adopt linear elasticity, when loading is well inside the failure surface. This assumption is supported by the behaviour observed on microscopic level. Investigations by e.g., Newman and Newman [2, 18] and by Kotsovos and Newman [19] identify three levels of behaviour defined by:

1. a limit where bond cracking is initiated; these microcracks are stable under sustained loading. (2) Onset of continuous cracking, where cracking primarily occurs within the mortar matrix. These cracks may be visible on the specimen surface and are unstable under sustained loading. Therefore this limit corresponds to the long-term strength of the concrete and the volume of the material achieves its minimum here. (3) The ultimate strength as described in the previous sections.

Overcoming of the elastic limit is not related to a uniquely defined phenomenon, and its location is therefore subject to considerable scatter. Only relatively few experiments have been concerned with determination of the elastic limit. However, based on the test results in the compressive meridian plane of Ferrara et al. [15] and defining the elastic limit in this plane as the stress state where the tangential shear modulus equals 70% of the initial modulus, we obtain the result shown in Fig. 5.

The corresponding interpolation curve is based on the results of Riccioli et al. [20].

The elastic limit for uniaxial compressive loading using the aforementioned definition equals 40% of the uniaxial compressive strength. Some earlier experimental results are also shown in this figure, namely the "onset of fracture propagation" of Newman and Newman [18], the "crack initiation" of Launay and Gachon [21, 22], the "onset of stable fracture propagation" of Kotsovos and Newman [19] and the "quasi-elastic limit" of Launay and Gachon [21, 22].

For comparison, the failure surface of Ottosen [4], and the limit for design stresses - according to the ASME-Code [23] - are also shown. Several interesting conclusions can be inferred from Fig. 5. It is obvious that the experimental data sets do not correspond to the same physical phenomenon as different definitions were adopted. In addition, differences may also be due to different concrete types and loading rates.
However, the results of "onset of fracture propagation" of Newman and Newman [18],
the "crack initiation" of Launay and Gachon [21, 22] and the "onset of stable fracture pro-
gagation" of Kotsovos and Newman [19] support each other.
In the original papers they were in fact all considered as limits within which no failure can
occur and they were therefore suggested as limits for design stresses or lower bounds for
any type of failure. On the contrary, the "elastic limit" of Ferrara et al. [16] and the
"quasi-elastic limit" of Launay and Gachon [21, 22] may be considered as linearity limits
and in fact they also support each other. It is interesting to note that the ASME-code [23]
seems to be in close agreement with the curves defining the elastic limit than with the ear-
lier mentioned curves. Whether a larger region than defined by the ASME-code could be
considered as allowable stress states seem still to be an open question.
Regarding the shape of all the surfaces in Fig. 5 - except the failure curve - it is im-
portant to note that they are closed in the negative direction of the hydrostatic axis.
A further difference compared to the failure surface is that for hydrostatic pressures not
too close to the origin these surfaces seem to be rotational symmetric around the hydro-
static axis, cf. the results of Kotsovos and Newman [18], and those of Launay and Ga-
chon [21, 22]. For small hydrostatic pressures, the experimental elastic limit leaves the hy-
drostatic axis. Accepting the usual normality principle of hardening plasticity, dilatation
then has to occur when crossing the elastic limit. This is against experimental evidence
showing that dilatation occurs just before failure and hardening plasticity theories for con-
crete may therefore to some extent be questioned.
6. CONCLUSIONS

The present study has concentrated on two aspects of concrete behaviour; (1) com-
parison of some failure criteria with representative experimental failure data; and (2) evalua-
tion of the appearance of the elastic surface.
On this basis, the following conclusions seem to be supported:
1. A considerable number of experimental failure data is available and in general good
agreement exist. These tests are sufficient to describe the features of the failure sur-
face.
2. It has been shown that fitness of a failure criterion can only be estimated when compa-
rison with experimental data are performed in at least three planes of different type.
3. Some recently proposed failure criteria are in very close agreement with experimental
data over a wide range of stress states. Especially the proposals of Williams and War-
4. The definition and location of the elastic limit is still open to discussion, but it appears
that it is located well within bounds where any type of failure can occur.
5. The ASME-code [23] and the elastic limit seem to be comparable.
6. The surfaces for elastic limit and also for what might be considered as lower bounds
of any type of failure are closed in the negative direction of the hydrostatic axis.
For hydrostatic pressures not too close to the origin, these surfaces seem to be
roughly rotational symmetric around the hydrostatic axis.
7. The location of the elastic limit in the compressive meridian plane seems to some
extent to question the use of the normality principle in hardening plasticity theories.
REFERENCES


TABLE 1:

Parameter values applied in the failure criteria considered.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_1/\sigma_2$</th>
<th>$\sigma_1/\sigma_3$</th>
<th>$X - \gamma_{cl}$</th>
<th>$X - \gamma_{tt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reihman - Janda [7,8]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>William and Warneke [9]</td>
<td>0.08</td>
<td>1.15</td>
<td>-2.20</td>
<td>2.87</td>
</tr>
<tr>
<td>Chen and Chen [10]</td>
<td>0.08</td>
<td>1.15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cedolini, Creuzen and Del Poli [11]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ottosen [4]</td>
<td>0.08</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
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$\sigma_1$, uniaxial tensile strength ($\sigma_1 > 0$), $\sigma_2$, uniaxial compressive strength ($\sigma_2 < 0$)

$\sigma_{ob}$, biaxial compressive strength; $X$, $\gamma_{cl}$ and $X$, $\gamma_{tt}$, additional parameters [9].