AN EXPERIMENTAL APPROACH TO THE DESIGN OF NETWORK REINFORCEMENT AGAINST IN-PLANE SHEAR IN REINFORCED CONCRETE CONTAINMENTS

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SUMMARY

This experimental study was conducted to rationalize the design of network reinforcement arrangements in reinforced concrete containments (R.C.C.V) against combined earthquake and internal pressure loadings. Taking into consideration the stress conditions expected to be encountered in the design of R.C.C.V walls, about 20 orthogonally reinforced concrete flat plate specimens with the test area of 150x150cm and the thickness of 10cm were made and loaded up to failure by bi-axial membrane forces. Two layers of 810mm deformed bars in both directions were anchored outside the test area. Non-cracked and pre-cracked specimens were employed. The ratios of the two principal forces \( k = N_x/N_y \) ranged from -1 to +1. The angles between the larger principal forces and reinforcing bars were changed in five steps from 0° to 45°. Also, several series of experiments were conducted on about 300 push-off specimens with the shear area of 15x30cm and 10x20cm in order to make clear the shear transfer mechanisms across cracks in reinforced concrete members. This test included the specimens subjected to combined shear and axial tensile forces.

Followings are the main results obtained within the limit of the experiments.

(1) The strength capacity of concrete plate elements reinforced by network bars, which are subjected to membrane axial forces combined with shear, may be conservatively designed on the assumptions that the predominant cracks form in the diagonal directions of bar networks and that shear as well as direct tensile stress components be resisted only by reinforcements. In the range of 0.5 \( \leq k \leq 1 \), however, design should be based on the two directions of cracks perpendicularly crossed.

(2) As was expected by strain compatibility conditions, the deformations of skewed reinforced concrete plates were found to be considerably larger compared with those with bars placed parallel to the applied forces.

(3) Shear transferability across cracks, which is an important factor in sharing radial shear capacity in R.C.C.V. can be relied on up to the crack width of 0.3-0.4mm, but in the shear plane with reinforcement ratio of more than 1% yield strength of bars are not fully developed.

(4) Although aggregate interlock plays an important role in the shear transfer capacity, contributions of aggregate interlock, dowel action and direct stress components in bars carrying shear are markedly influenced by the directions of bars crossing the shear plane.
1. Introduction

As the first Japanese prestressed concrete containment for nuclear power station has been decided to be constructed in the near future, the Technical Code for Concrete Containments was drafted in July, 1977 by the Ministry of International Trade and Industry. In the Draft Code it is stipulated that the containments shall resist all the combinations of loads conceivable to occur during the life of the structure without causing excessive deformations and hampering the leak-tightness. This includes the situation in which internal accident pressure is overlaid with earthquake loadings. In such a loading condition a plate element in the cylindrical wall of a containment is supposed to be subjected to combined action of biaxial membrane forces and in-plane shear, giving rise to the differences in the directions of main reinforcements and principal forces (see Fig.-1).

Although several design methods have been proposed to estimate the strength capacities and deformational behaviors in such a reinforced concrete shell element, none of them have necessarily been substantiated by appropriate experiments simulating the stress state in the cylindrical wall. In the present experimental study tests were conducted in which orthogonally reinforced concrete plates were loaded up to failure by biaxial principal membrane forces, the directions of which did not coincide with those of the reinforcements. The test results were compared with the analytical results, and discussions were made in the light of provisions in the proposed Draft Code.

In addition to the above experiments push-off tests were conducted on reinforced concrete blocks to provide fundamental data for the design of wall-base slab connections against radial shear.

2. Test Methods

The main dimensions and the loading scheme of reinforced concrete plate specimens are illustrated in Fig. 2. Only the central part of the specimen, the area and the thickness of which were 150x150cm and 10cm, respectively, was designated as the test area, around which additional thickened concrete area with slits was attached to anchor the reinforcing bars and to introduce principal membrane forces. Tensile loads were applied from both sides of the specimen by what is called tournament scheme, using two 100 ton hydraulic jacks in each direction. In order to introduce compressive forces four 50 ton jacks were employed. The plates were reinforced by Ø10mm deformed bars with the spacing of 20cm in two layers in each direction. The cover of the reinforcements was kept at 2 cm. The angles between the main bars and principal tensile forces were changed from 0° to 45° in five steps, while the ratios of the two principal forces $k = N_2/N_1$ were varied from -1 (pure shear) to +1 (biaxial equal tension). Several of the specimens were initially cracked by artificial splitting actions. Test was conducted at the age of 7 ~ 10 days after concrete gained strengths from 200 kg/cm² to 240 kg/cm². Strains in reinforcements were measured by electric wire strain gauges, while the overall deformations as well as the width of openings and the shear slips of cracks by cantilever type deformation gauges. The geometrical and strength data of the 20 specimens tested were listed in Table-1 together with the pertinent data.

3. Yield Strength of Orthogonally Reinforced Concrete Plate Elements

When a reinforced concrete plate element in which steels are placed in two directions at right angle is subjected to arbitrary orthogonal principal forces, the stresses in re-
inforcements and concrete per unit width can be expressed as follows based on the conditions of equilibrium in the directions normal and parallel to a crack [1] (see Fig. 3).

\[ Z_x = N_1 \cos^2 \alpha (1 + \tan \alpha \cdot \cot \phi) + N_2 \cos^2 \alpha (1 - \cot \alpha \cdot \tan \phi) + H \tan \phi \]
\[ Z_y = N_1 \sin^2 \alpha (1 + \cot \alpha \cdot \cot \phi) + N_2 \cos^2 \alpha (1 - \tan \alpha \cdot \cot \phi) - H \cot \phi \]
\[ D_b = (N_1 - N_2) \sin 2\alpha \sin^2 \phi - 2H \cot^2 \phi \]

where

- \( Z_x, Z_y \): tensile stresses in bars in \( x \) and \( y \) directions
- \( D_b \): compressive stress in concrete
- \( H \): shear resistance along a crack due to aggregate interlocking and dowel action
- \( N_1, N_2 \): principal stresses (\( N_1 > N_2 \), tension-positive)
- \( \alpha \): angle between the directions of \( x \) bar and \( N_1 \)
- \( \phi \): angle between the directions of \( y \) bar and crack.

Since the above three equations of equilibrium contain 5 unknowns, that is \( Z_x, Z_y, D_b, H \) and \( \phi \), the problem is essentially of indeterminate nature. Baumann [1] obtained the solutions using the principle of least work, while Duchon [2] solved the equations by applying the conditions of strain compatibility. If we assume the direction of crack as \( \phi \) and neglect the contribution of shear resistance along crack \( H \), the problem reduces to a statically determinate one.

For example by putting \( \phi = \pi/4, H = 0 \) and \( \phi = \alpha, H = 0 \), design formulas proposed by Leitz [3] and Peter [4], respectively, are obtained.

Fig. 4 shows the ratios of principal forces \( N_{y_{meas}} \), measured at the time of yield in \( x \) reinforcement and \( N_{y_{calc}} \), calculated by Leitz's theory. In Fig. 5 similar comparisons are made between the measured values \( N_{y_{meas}} \) and calculated yield values \( N_{y_{calc}} \), based on Duchon's proposals. In these figures not only the results of plate specimens, but also those obtained by the tests of cylindrical models, the details of which are reported in the paper J4/4 presented at this session [5], are plotted against the values of \( k = N_2/N_1 \).

From the comparisons of measured and calculated principal forces causing yield in \( x \) bars, it may be found that both Leitz's and Duchon's method predict the values slightly higher than those obtained in the experiments within the range from uniaxial to biaxial tensions (\( k = 0 \sim 1 \)). This may be attributed to the fact that while in the calculation assumption is made that cracks appear only in one direction even in the biaxial tensile state of stresses, actual plate element often have cracks in two directions that are crossing each other almost perpendicularly. Considering the above mentioned experimental evidences it seems rational to limit the allowable stress in reinforcements to 0.9 \( f_y \) as is specified in ASME Code [6]. Contrary to the tensile stress state in the range of pure shear the ratios of the values obtained by experiments and by calculations are more markedly influenced by the angles \( \alpha \) for the case of Leitz's formula than for Duchon's theory. This may be explained by the fact that while in this state of stresses yielding capacities of orthogonally reinforced elements are highly dependent on the angles \( \alpha \) in Leitz's formula, experimental values do not exhibit such a wide range of variations as may be estimated by the theory.
Examining the results obtained in the experiments on cylindrical models it may be judged as safe to apply either of the two theories up to the reinforcement ratio of 2.4%, provided that resultant principal stress state is not much apart from the uniaxial state of stress \((k = 0)\). However, the model reinforced with 2.4% steel and subjected to horizontal force without internal pressure failed in much lower loading stage than was estimated by the theories. Taking account of such a premature failure it seems necessary to limit the compressive stress in concrete in order that the effectiveness of compressive chord of assumed truss may fully develop.

4. Deformations and Crack Widths in Orthogonally Reinforced Concrete Plate Elements

It has been recognized that the more the direction of main reinforcements differs from that of the principal force, the larger the deformations and the crack widths become. For instance new CEB code [7] specifies that width of cracks formed at 45° with respect to bars should be doubled compared with the case in which crack crosses the main bars at 70° ~ 90°. If we neglect the compressive strains in concrete and contributions exerted by concrete between cracks, following equations can be derived from the geometrical relations shown in Fig. 6 [1].

\[
\begin{align*}
\varepsilon_\phi &= \frac{\varepsilon_x}{\cos^2 \theta} + \frac{\Delta}{a_m} \tan \phi - \frac{\Delta}{a_m} \cot \phi \\
\varepsilon_\theta &= \frac{\varepsilon_x}{x} + \frac{\varepsilon_y}{y} \\
\Delta / a_m &= \varepsilon_x \cos \phi - \varepsilon_y \tan \phi \\
\varepsilon_1 &= \varepsilon_x \cos^2 (\phi - \alpha) - \sin (\phi - \alpha) \cos (\phi - \alpha) \Delta / a_m \\
\varepsilon_2 &= \varepsilon_y \sin^2 (\phi - \alpha) + \sin (\phi - \alpha) \cos (\phi - \alpha) \Delta / a_m
\end{align*}
\]

\text{eq. (2)}

where \(\varepsilon_\phi\) : strain perpendicular to cracks

\(\varepsilon_x, \varepsilon_y\) : strains in \(x\) and \(y\) reinforcements

\(a_m\) : average crack spacing

\(\Delta\) : relative slip between crack edges

Considering tensile strains in concrete between adjacent cracks, average tensile strains in reinforcements and average crack widths may be expressed as follows similar to that employed in the CEB Code [7].

\[
\begin{align*}
\varepsilon_{xm} &= \varepsilon_x \left[1 - \left(\frac{\sigma_x}{\sigma_x^*}\right)^2\right], \quad \varepsilon_{ym} = \varepsilon_y \left[1 - \left(\frac{\sigma_y}{\sigma_y^*}\right)^2\right] \\
\varepsilon_{m} &= \varepsilon_{\phi m} \cdot \frac{a_m}{a_m}
\end{align*}
\]

\text{eq. (3)}

Substituting the values in the equations (2) by the corresponding average values, average strains and average crack widths may be obtained.

Fig. 7 and Fig. 8 show the average strain behaviors parallel to the principal force \(N_1\), together with the calculated data. In Fig. 9 crack widths measured on the precracked specimens are compared with computed results. Both experimental and calculated results clearly show the tendency of marked increase in deformations and crack widths with the
inclusion of angle $\alpha$.

5. Shear Transfer Capacities Across Cracks, with Particular Reference to the Design of Radial Shear

In the foregoing discussions for the design of orthogonal reinforcement network in the containment's shell walls, it was confirmed rational to assume that the contribution of aggregate interlock across cracks should be ignored. For the design of wall-slab connections against radial shear, however, it has been pointed out by the tests specifically conducted to make clear the radial shear strength [8], that the shear transfer across a cracked plane could positively be incorporated in order to estimate the radial shear capacity.

To provide fundamental data on the shear transfer capacities across cracks about 300 pieces of push-off specimens were tested, varying concrete strength, shear reinforcement ratio, inclination of reinforcement, coexisting tensile stress perpendicular to shear plane, etc.

Using the test results obtained by the specimens with bars at right angle to shear plane, average shear stresses divided by the square root of $F_c$ (compressive strength of concrete kg/cm²) were plotted against the difference of $p_fy$ and $\sigma_v$ in Fig. 10 ($p_fy$ - product of reinforcement ratio and yield stress, $\sigma_v$ - axial stress tension-positive). Considering also the case in which the structure had been precracked design shear stress to be applied to the estimation of ultimate strength may safely be given as follows.

$$\tau_u = (0.05 (p_fy - \sigma_v) + 0.5) \sqrt{F_c} \quad \text{..........................................................eq. (4)}$$

Since the shear transfer capacities are highly dependent on the inclination of reinforcements relative to the shear plane (see Fig. 11), a reduction factor should be applied to equation (4), which was found to be approximately expressed as $\sin^2(70^\circ - \theta)$.

Supposing that at the region of junction between wall and slab exists a shear crack with arbitrary angle $\theta$ with respect to horizontal plane, ultimate radial shear stress may be calculated as below, taking due account for the effect of simultaneous action of vertical axial stresses (see Fig. 12).

$$\tau_u = \{0.05[(p_fy - \sigma_v \cos^2 \theta) \sin^2 (70^\circ - \theta)] + 0.5\} \sqrt{F_c} + \sigma_v \sin \theta \cos \theta \quad \text{....eq. (5)}$$

where

- $\tau_u$ : ultimate nominal shear stress (kg/cm²)
- $\sigma_v$ : normal stress acting on horizontal plane (kg/cm²)
- $p_fy$ : yield stress of vertical bars (kg/cm²)
- $p$ : vertical reinforcement ratio
- $\theta$ : inclination of shear crack with respect to horizontal plane (degree)
  - $0^\circ \leq \theta \leq 45^\circ$
- $F_c$ : compressive strength of concrete (kg/cm²)

The applicability of the proposed formula was verified against the test results obtained by the models reported in the previous paper J4/6 [8].

6. Conclusions

Within the limit of experiments following conclusions could be arrived at;

1. In the range of stress combinations occurring in cylindrical walls of reinforced...
concrete containments with orthogonal bar arrangement, membrane yield strength of shell elements could be predicted with an appropriate margin of safety by Leitz's formula, provided limitations be attached to compressive stress in concrete and design yield stress in reinforcements.

(2) When designing the cylindrical walls reinforced only by vertical and circumferential bars, due consideration should be paid to deformations and cracking behaviors of the shell wall.

(3) The design formula proposed by the authors, which were based on the test results of what is called push-off test, seems to be applicable to the estimation of ultimate shear transfer capacities at the junction of a shell and a basement slab.

References


Fig. 1: Stress state in reinforced concrete containment under the combined action of internal pressure and earthquake
Fig.-2 Sketch of the reinforced concrete plate specimen and the loading scheme.

Fig.-3 Equilibriums of forces in cracked reinforced concrete plate elements subjected to membrane principal forces.

Fig.-4 Comparison of measured and calculated principal forces $N_1$ causing yield in X bar (Leitz's theory).

Fig.-5 Comparison of measured and calculated principal forces $N_1$ causing yield in X bar (Duchon's theory).
Fig. 6 Geometrical relationship of deformations in cracked orthogonally reinforced concrete plate element.

Fig. 8 Comparison of observed and calculated average strains in the direction of principal force $N_1$ ($\alpha=45^\circ$, $k=0$).

Fig. 7 Comparison of observed and calculated average strains in the direction of principal force $N_1$ ($\alpha=30^\circ$, $k=0$).

Fig. 9 Comparison of observed and calculated crack widths in precracked specimens (initially non-cracked).
Fig.-10 Shear transfer strengths obtained by push-off specimens with reinforcements perpendicular to shear plane (initially non-cracked).

Fig.-11 Shear transfer strengths obtained by push-off specimens with reinforcements inclined to shear plane (initially cracked).

Fig.-12 Mechanism of resisting radial shear near cylindrical wall-base slab connections.
Table I Geometrical properties and strength data of plate specimens.

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ϕ: angle between y-bar and crack

N1 ≥ N2 (tension-positive, compression negative)
angle between principal force N1 and x bar