LOCAL FAILURE OF REINFORCED CONCRETE UNDER MISSILE IMPACT LOADING

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SUMMARY

In reactor safety analyses it is frequently necessary to consider hazards caused by the impact of missiles. Such missiles may be external (for example: aircraft) or internal (for example: valves or fittings). In either case likely impact velocities are below the ordnance velocity region. ($<300 ms^{-1}$). Data here are lacking and there are no fully validated calculational methods. In the UK a co-ordinated programme exists for acquiring data and developing methods. The work covers a wide range of missile types and includes both concrete and metal targets. Several UK organisationscontribute to this programme: the UKAEA at the Safety and Reliability Directorate and at Winfrith; the MOD at AWRE Foulness; and the Nuclear Power Company with Taylor Woodrow Construction Ltd. There are also collaborative research programmes with other European nuclear safety organisations. Some of this work is reported in other contributions to these proceedings.

This paper deals with the theoretical modelling of the local response of reinforced concrete to missile impact loading. A computer code, SARCASTIC, has been written to serve as a vehicle for the assessment of specific constitutive models for concrete. The code is described: it is axisymmetric (or two dimensional) and uses an explicit integration, Lagrangian finite difference formulation of the equations of motion. Non linear strain tensors and a mesh rotation correction allow large deflection calculations to be performed.

One particular constitutive model for concrete is described. Elastic behaviour to failure is assumed. An analytic failure surface, defined in principal stress space, is used to distinguish failed regions. When failure is indicated, local cracking normal to the most tensile principal stress is assumed. A method for representing cracks in the finite difference scheme is described. Particular attention is paid to the representation of shear terms in the stress tensor when the local material is cracked. Spurious crack propagation from an initial failure is discussed. The inclusion of reinforcement with yielding and ultimate failure is outlined.

Code calculations are compared with experimental results from selected tests using deformable aluminium tube missiles striking concrete targets. The code predicts that the dominant failure mode is tensile cracking at stresses close to the uniaxial tensile stress. Crack orientations are predicted well but there are significant differences in the initial location of major failure. Possible reasons for this are discussed.

1. INTRODUCTION

Accidental missiles of many different sorts may give cause for concern in reactor safety studies. The reasons for concern may be diverse. For example, the only external missile hazards commonly identified are aircraft and tornado driven debris, and the integrity of the secondary containment is normally the point at issue. Whereas inside the reactor containment a wide variety of missiles may be generated from high pressure pipework or rotating machinery. Here rupture of the secondary containment is only one consideration; more important may be the possibility of damage to vital safety systems or coolant lines.

With the exception of a large aircraft, the crash of which may give a distributed load more akin to blast or seismic effects, missiles may fail a structure in two ways: locally, where a panel or wall is deformed or perforated without widespread damage; and structurally, where the load transmitted to the supporting structure is sufficient to cause collapse at or of the supports. Widespread structural failure may be important and should not be overlooked, but generally the more difficult design problem is local failure, and this paper deals with that only.

The size and nature of missiles vary widely. A military aircraft might typically be 25Te, crashing at around $200ms^{-1}$. Such a missile would be relatively deformable with the impact of the engines cushioned by airframe collapse. A fragment of a pump bowl from a coolant line might typically weigh a few kilograms and be ejected at $300ms^{-1}$. Such a missile would be relatively undeformable.

"Targets" hit by such missiles are on the whole less varied. Reactor containments themselves will be of reinforced (or perhaps pre-stressed) concrete. If missile barriers are incorporated into plant design to protect against identified missile hazards, they are likely to be of reinforced concrete or perhaps steel if space is at a premium. There may be some composite construction using scabbing plates.

Much military work has been done, and a good deal published, on relatively undeformable missiles striking metal and concrete at ordnance velocities ($>300ms^{-1}$). However, there are relatively few data in the velocity range of accidental missiles. The Safety and Reliability Directorate of the United Kingdom Atomic Energy Authority have undertaken a programme of research and development to provide data and validate theoretical methods for the treatment of missile impact. Several other UK organisations participate: AEE Winfrith; AWRE Foulness; and Taylor Woodrow Construction, acting for the Nuclear Power Company. Recently a common interest in missile impact problems has led to collaboration agreements with French and German nuclear safety interests.

The UKAEA research programme has three main aims: to validate replica scaling for missiles impacting model concrete (this enables model experiments to be undertaken with confidence); to validate empirical formulae for perforation (this enables design calculations to be readily performed); and to develop computer models capable of representing the dynamic behaviour of concrete (this enables problems as yet unforeseen to be tackled). The experimental programme involves both concrete and metal targets hit by missiles of different deformabilities. Some of this work has been reported previously [1], [2] and some is reported in other contributions to these proceedings [3], [4], [5].

This paper is concerned with the development of computer models for the behaviour of reinforced concrete. No well-developed and widely accepted constitutive model exists for concrete, unlike many materials. It was felt that several different models would probably need to be formulated and assessed. It may even be that no one constitutive model is capable of representing all conditions. We have chosen, then, to write a short, straightforward, finite difference code for solving the dynamic equations of motion in a continuum and to use this as a vehicle for different constitutive models. This enables different models to be assessed in identical contexts. We describe in this paper the basic code and the first constitutive model.

2. BASIC CODE

The code is called SARCASTIC (Safety And Reliability Code for the Analysis of Stress Transients In Concrete). The basic equations are those for non-linear elasticity, formulated in Lagrangian co-ordinates. Both two dimensional and axisymmetric calculations are possible, but the axisymmetric form is quoted here since this is appropriate for missile impact.

Explicit integration is used; the course of calculation in a single time step follows the procedure outlined below:

1) calculate strains from displacements
2) calculate elastic stresses from strains (concrete and reinforcement separately)
3) adjust stresses according to constitutive model (concrete & reinforcement separately)
4) combine stresses for concrete & reinforcement
5) calculate internal forces from stresses
6) calculate accelerations, velocities, and adjust displacements
7) return to 1).

The basic equations used in these steps follow:

the components of the Lagrangian strain tensor are, in terms of displacement:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} + \frac{1}{2}\left[\left(\frac{\partial u_r}{\partial r}\right)^2 + \left(\frac{\partial u_z}{\partial r}\right)^2\right]$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u_z}{\partial z}\right)^2 + \left(\frac{\partial u_r}{\partial z}\right)^2\right]$$

$$\varepsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{2}\left(\frac{u_r}{r}\right)^2$$

$$\varepsilon_{rz} = \frac{1}{2}\left[\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right] + \frac{1}{2}\left[\frac{\partial u_r}{\partial r}\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\frac{\partial u_z}{\partial z}\right]$$

where $\varepsilon_{rr}, \varepsilon_{zz}, \varepsilon_{\theta\theta}$ are orthogonal strains in local r, z, θ directions; ε_{rz} is shear strain ($\varepsilon_{r\theta} = \varepsilon_{\theta z} = 0$); u_r, u_z are displacements in global r, z, directions.

The first terms in each of these equations are the linear strain terms and give a good approximation to the strains for small displacements. The code as written contains an option for calculations using the linear strain tensor only. This is adequate for many problems. When the non-linear strain tensor is used it is necessary to correct the stresses for the local rotations. This is done in the code by calculating the angle of local rotation, ϕ, given by:

$$\phi = \sin^{-1}\left(\frac{1}{2}\left[\frac{\partial u_z}{\partial r} - \frac{\partial u_r}{\partial z}\right]\right)$$

and subsequently correcting the direction of action of the stresses to allow for this. This formulation accounts for large displacements, but is of course still only valid for small material strains. This is felt to be no serious limitation in the case of concrete which is not a ductile material.

The elastic stresses are calculated from the strains by a generalised Hooke's law. The accelerations are calculated from the body forces using the Lagrangian equations of motion:

$$\rho \frac{\partial^2 u_r}{\partial t^2} + \left[\rho \frac{K}{\Delta t} \frac{\partial u_r}{\partial t}\right] = \frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r}$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} + \left[\rho \frac{K}{\Delta t} \frac{\partial u_z}{\partial t}\right] = \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{rz}}{r}$$

Where t denotes time and ρ density; K is a constant and Δt a timestep.

The term in brackets on the left hand side of these equations is a linear damping term. This may be used to apply structural damping; a special case of this is the application of critical or near critical damping when seeking a steady state solution to a given loading. For the cases discussed here no damping terms were used. The velocities and displacements are calculated from the accelerations by integration of these equations.

In the finite difference formulation of the equations the quantities of interest are defined at points on an interlacing mesh similar to that used by Otter [6]. Stress and displacement points exist outside the boundary; these are used in the definition of boundary conditions. The quantities necessary for the solution of the basic equations are all obtained by centred differencing; this includes the integration of the equations of motion, which is based on centred differencing in time.

The basic code has been validated for elastic material behaviour by comparison with analytical solutions and other methods for a range of problems. One such is shown in figure 1; displacements are plotted as a function of time for several radii on a circular aluminium plate. The plate is subjected to a triangular wave form load at the centre. The solid lines are a modal solution from Goldsmith et al [7] and the dashed lines are the SARCASTIC solution.

3. CONSTITUTIVE MODEL FOR REINFORCED CONCRETE

Concrete can be considered a two phase material, with a wide range of sizes of coarse particles embedded in a cement paste matrix. Its behaviour under stress is complex and anisotropic, with both microscopic and macroscopic fissures forming as failure is reached. For the purposes of discussion here we use "elastic" to mean linear in stress/strain relations, "plastic" to mean non-linear without macroscopic cracking, and "failure" to mean the onset of macroscopic cracking.

For this constitutive model the concrete is assumed to be isotropic and elastic up to failure. This is certainly not so in practice (see e.g., [8], [9]), but plasticity prior to failure is most marked in regions of compressive stress. It is felt that in many missile impact problems failure in tension will dominate because of the low tensile strength of concrete. In these circumstances plasticity is less important; its neglect is to some extent justified by the indications from results (see section 4) that tensile failure does indeed dominate. In common with other models ([10], [11], [12]) concrete cracking is not modelled in detail. The onset of failure is identified from stress-based criteria, and in the post-failure region the stresses surrounding a failed region are adjusted to be consistent with the failure.

3.1 Onset of Failure

Failure of concrete is sensitive to all three principal stresses. Because of the ractical difficulties of testing specimens by applying 3 independent stresses, experimental data are limited. Some workers such as Kupfer et al [9] have used specialised equipment to provide data with two independently applied stresses (3rd stress zero). From this and other work both Newman and Newman [8] and Ottosen [13] have deduced failure surfaces for concrete. These surfaces are represented in principal stress space; they represent somewhat triangular pyramids centred on the hydrostatic axis. We choose to use the surface suggested by Ottosen since it is entirely analytic and very suitable for incorporation in a computer code. The surface has the following properties:

It is defined in principal stress space

It uses four independent stress states to locate the surface

The Meridians (sections including the hydrostatic axis) are convex and parabolic

It is convex in the deviatoric plane (sections normal to the hydrostatic axis)

It opens continuously along the compressive hydrostatic axis

The cross section goes from nearly triangular at low hydrostatic stress to nearly circular at high hydrostatic stress

It is continuous everywhere

It has 120° symmetry in the deviatoric plane

The criterion can be readily expressed in terms of the stress invariants as

$$f(\underline{\sigma}) = A\frac{S_2}{\sigma_c^2} + \lambda(\theta)\frac{\sqrt{S_2}}{\sigma_c} + \frac{B J_1}{\sigma_c} - 1$$

where

$$\lambda(\theta) = K_1 \cos\left[1/3\cos^{-1}\left(K_2\cos 3\theta\right)\right] \quad ; \ \cos 3\theta \geqslant 0$$

$$\lambda(\theta) = K_1 \cos\left[\pi/3 - 1/3\cos^{-1}\left(-K_2\cos 3\theta\right)\right] \quad ; \ \cos 3\theta < 0$$

and

$$\cos\theta = \left(2\sigma_1 - \sigma_2 - \sigma_3\right)/\left(2\sqrt{3S_2}\right)$$

$\sigma_1, \sigma_2, \sigma_3$ are the principal stresses, J_1 is the first invariant of the stress tensor, S_2 is the second invariant of the stress deviator tensor, σ_c is the uniaxial compressive strength of the concrete and A, B, K_1, K_2 are constants.

Stress states for which $f(\underline{\sigma}) \geqslant 0$ represent failures.

Uniaxial compressive and tensile strength measurements provide two of the independent constants. The other constants are derived from these in a manner suggested by Ottosen, based on a mean of independent measurements from a number of workers.

3.2 Mode of Failure

Macroscopic fracture patterns for concrete failure indicate in general cracking normal to the maximum principal strain. In uniaxial tension this is a single cleavage crack; in uniaxial or biaxial compression multiple cracks occur in parallel. For this model we have chosen to introduce cracking normal to the maximum principal stress in all cases where the failure surface is exceeded. The hoop direction is always principal, so radial cracks are straightforward. For circumferential cracks the code stores the angle at which the first crack occurs and this is subsequently treated as defining the direction of anisotropic behaviour. A second orthogonal circumferential crack is permitted. There is an inconsistency here in that if cracks close and carry shear forces, the normal to an existing crack need not be a principal direction. This is likely to happen only in the later stages of impact and in an already failed area. In this model we approximate non-orthogonal cracking by permitting an orthogonal crack if the maximum tensile stress at failure is at an angle less than $\pi/4$ to an existing crack.

3.3 Post-failure Model

The modelling of failed concrete has not been definitively resolved. A treatment by Carlton & Bedi [11] which has had some success in modelling impact represents failure in tension by cracking with a reduction of stress normal to the crack to zero, and a reduction of shear stress across the crack to zero when a specified strain is exceeded. Marchetas et al [10] however, in a prestressed concrete reactor vessel analysis, have chosen to use an arbitrary factor of 0.5 to reduce the shear stress carried by a crack. In compressive failure, Carlton & Bedi use an elastic/perfectly plastic model, correcting stresses back to their original failure surface. Marchetas et al employ a similar procedure, but base it on a uniaxial compressive stress limit only.

We have chosen to investigate a somewhat different model. Cracking is initiated as outlined above. If the crack initiated is in tension, the tensile stress normal to the crack is reduced to zero over a short, pre-set, relaxation time. This relaxation of stress follows Marchetas and Carlton & Bedi; it improves the stability of the calculations. Some physical justification for this may be argued also; the stress, normal to a crack, just outside a given element of concrete will only reach zero when the crack has propagated across the element. From crack initiation to full cracking will take a finite time. Because of the problems of assigning any real crack width to what is in the model a failed area of concrete, we choose to represent cracks in tension as having no shear carrying capacity. In compression we treat shear carrying capacity as a frictional effect and allow the crack to carry shear stress up to a maximum proportional to the compressive stress normal to the crack. For the calculations reported here an arbitrary friction factor of 0.5 was used.

When one stress is adjusted for the presence of a crack, the other principal stresses must be adjusted also to make the stress field consistent. The correction used here is:

for one crack where $\sigma_1 \longrightarrow \sigma_1'$

$$\sigma_2' = \sigma_2 + \nu'(\sigma_1' - \sigma_1) \qquad ; \nu' = \nu/(1-\nu)$$

$$\sigma_3' = \sigma_3 + \nu'(\sigma_1' - \sigma_1)$$

for two cracks where $\sigma_1' \rightarrow \sigma_1$ and $\sigma_2 \rightarrow \sigma_2'$

$$\sigma_3' = \sigma_3 + \nu(\sigma_1' - \sigma_1) + \nu(\sigma_2' - \sigma_2)$$

Similar corrections are used by Adamik [12] and Marchetas [10].

3.4 Reinforcement

The reinforcement is modelled as discrete bar elements passing through the u_r, u_z points in the finite difference grid. We take an idealised tri-linear stress-strain curve with failure at a specified ultimate strain. The reinforcement is assumed capable of supporting tensile stresses along the bar only. Whilst this is an assumption, it is borne out by experimental observations [14] that for both rigid and deformable missiles striking concrete targets, virtually all the reinforcement wires that fail show necking and characteristic tension failures.

In the uncracked state, concrete and reinforcement are assumed to be perfectly bonded and in identical states of strain. When a crack is formed the modelling is altogether more difficult. As a crack opens the reinforcement de-bonds near the crack to an extent which must depend on the reinforcement surface and crack width. No experimental work appears to have been published on this, so for inclusion in a code of this type it is necessary to make some bond length assumption. For simplicity we have chosen to assume the "gauge length" for reinforcement straining as constant and equal to the distance between points in the finite difference grid.

Reinforcement and concrete stresses are calculated separately in the code, and combined on a proportional cross sectional area basis in the calculation of internal forces on an element of concrete.

4. RESULTS

Some preliminary results only are reported here. In an earlier publication Alderson et al [1] reported tests using a deformable aluminium missile impacting a circular reinforced concrete target with a massive ring beam support. We compare our results here with one test in that series. The target geometry is sketched in figure 2; an idealised load/time curve for the missile is shown in figure 3. Figure 4 shows the target deflections at an 80mm radius measured experimentally. The error bars are derived from displacement transducers located on opposite sides of a target diameter.

The calculational comparison used concrete compressive and tensile strengths close to those measured experimentally (35 MPa compressive strength 5 MPa tensile strength) and reinforcement yield stress and ultimate strain measured experimentally. In the experiments it was noted that the massive ring beam showed some small movement during the impact. Two sets of boundary conditions were used because of this. The first is a rigid boundary at the inner target diameter, the second a rigid boundary at the outer diameter.

Figure 5 shows displacement time curves for the smaller diameter, and figure 6 shows the corresponding damage pattern at peak deflection. Figures 7 and 8 show equivalent data for the larger target diameter.

The model predicts a rather stiff response for the smaller diameter; the large diameter predictions are somewhat closer to those experimentally measured. Boundary conditions clearly exercise a major influence.

The damage pattern in both cases is similar in form of the overall failed area, and cracks have a similar orientation to those observed in experimental sections of targets [1]. However, the code predicts initiation of failure at the centre rear face, and the whole central zone is failed. The experimental sections show only slight cracking in the central zone, and one major circumferential crack corresponding approximately to the boundary of the cracked area in the code predictions. There are a number of radial cracks on the target as in the code model, but predictions here cannot be exact because of the axisymmetry of the model.

The reason for the difference in development of crack pattern is not clear; it may be due to the lack of plasticity prior to failure in the model. In the real target plasticity will allow a certain redistribution of stress prior to failure.

An examination of the stresses at failure for cracked regions in the calculation indicates that virtually all first cracks are initiated when the major principal stress is tensile. The use of a "correct" tensile stress limit is, then, important. Direct tensile measurements are desirable in theory but in practice difficult to perform adequately. Brazilian splitting tests are more straightforward, but platen or packing effects may make the stress state in the specimen more complex than the plane stress assumption normally made.

Modulus of rupture beam tests may be more appropriate; generally they will yield a higher apparent strength.

5. CONCLUSION

A constitutive model for the dynamic behaviour of concrete has been formulated and incorporated in a finite difference code. Preliminary results compare reasonably with experimental work but there are several areas in which a more detailed investigation of the effects of modelling parameters is needed. Following this it should be possible to assess the need for further refinements of modelling. Pre-failure plasticity in particular may be important. This work is proceeding at SRD, as part of a co-ordinated UK programme on missile impact effects.

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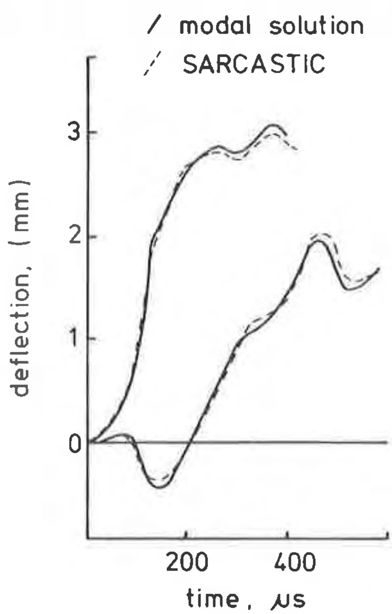

Figure 1

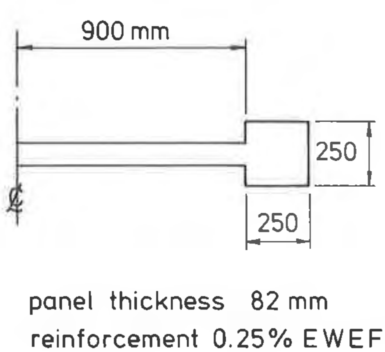

Figure 2

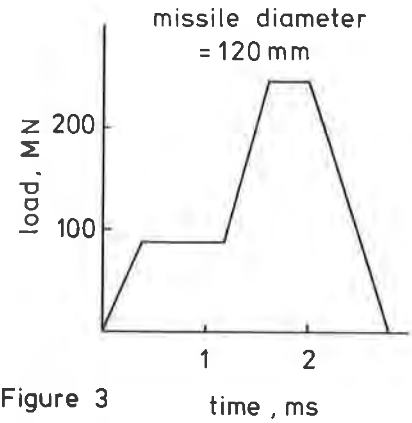

Figure 3

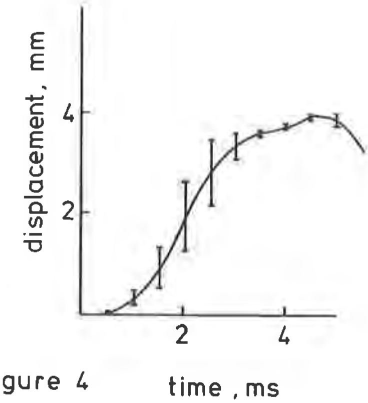

Figure 4

Target damage map
at 3.8 msec

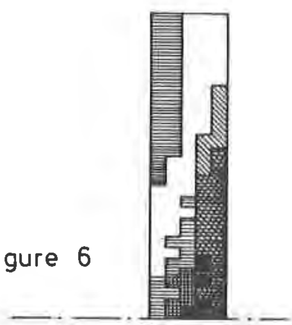

Figure 6

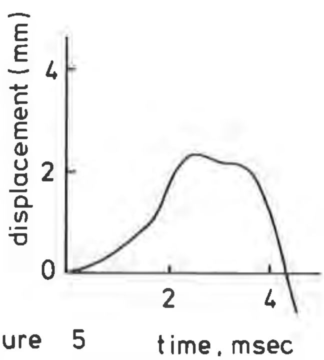

Figure 5

Target damage map
at 3.8 msec

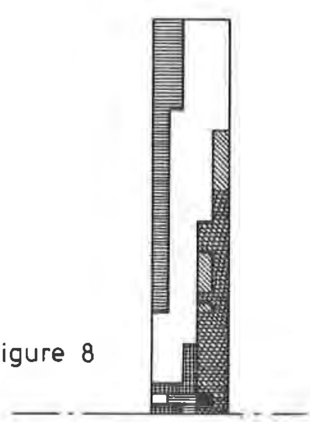

Figure 8

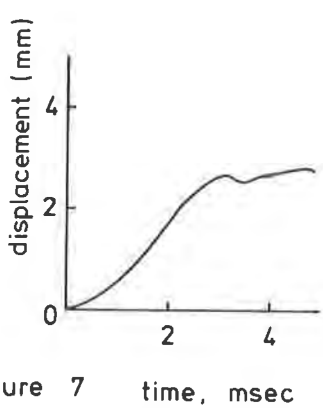

Figure 7

one r/z crack
one hoop crack
two r/z cracks
one r/z and one hoop crack
two r/z and one hoop crack