DAMAGE EQUATIONS FOR CREEP RUPTURE IN STEELS

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Two laws are proposed for thermal creep damage in steels, one in finite terms, known as the total damage theory, and the other in differential terms, known as the incremental damage theory, useful for dealing with cases of stress which varies with time.

A method is described for calculating the parameters of the material for the law of incremental damage, in order to reduce to a minimum the mean square deviation of the rupture times for the tensile constant-load creep tests, calculated with the total damage law and the incremental law. The procedure is applied to two steels of interest for nuclear installations - AISI 310 stainless steel and ferritic steel 2.25Cr 1Mo.

The results obtained show that for materials whose rupture depends almost exclusively on strain, such as AISI 310, one may assume the same parameter values for both laws while for materials with damage which is stress-dependent to a considerable degree, such as the 2.25Cr 1Mo steel, and above all for materials with a damage, that is practically strain-independent, the values of the parameters of the incremental damage law differ from those obtained for the total damage law from the statistical analysis based on the experimental data.
1. Introduction

While plants are still in the design stage, a study of creep damage is necessary in order to evaluate the life time or the remaining life span of structures operating under conditions such that their component materials are subject to creep. The damage caused by creep is the main cause of brittle rupture. The importance for nuclear power stations, and the most modern procedures for calculating time-to-rupture, have been demonstrated in several recent scientific meetings, such as the Congress of Rome, 1973 [1], those of Gothenburg, 1976 [2], The Hague, 1977 [3], London 1977 [4, 5] and 1978 [6], the "Creep Modeling Panel of Ispra [7], and the Ispra Course "Creep of Engineering Materials and Structures [8]". At these meetings there was a great deal of discussion about the true nature of creep, and the values which can best represent it (see for example also the recent interesting paper by Bräthe [11]), because the final aim is the law of damage, i.e. the link between the values assumed in order to measure the damage and the mechanical and physical variables which characterise the creep tests; in the case of thermal creep, these are, namely, strain, stress, time and temperature. For example, in [9] and [10], with statistical analyses of the results of constant-load traction tests, damage laws have been determined in finite terms for austenitic stainless steel AISI 310 and ferritic steel 2.25Cr 1Mo, respectively, both of which are of great importance to nuclear power stations. The damage laws described in [9] and [10], and which we shall call total damage laws, because they are expressed in finite terms, are of no use when dealing with cases of variable stress. For this purpose a differential rupture law has been derived from them, known as the incremental damage law, which, in a case of constant stress, will assume a form similar to the damage law obtained from constant-load tests, but with different parameter values. In the total damage law, the stress is the same as in the constant-load test, whereas in the other the stress is the true one, which is for instance variable in a constant-load test.

In this work we show how to calculate the parameters of the differential damage law, once those of the law in finite terms are known. Using a procedure of successive approximations, the new parameters can be determined in such a way that the mean square deviation between the rupture times calculated with the two laws for constant-load tests is minimum. The calculation method is applied to the two steels mentioned above and to a hypothetical material, which lend themselves well to illustrating the two different results which can be obtained according to whether or not the damage is stress-dependent.

2. Total Damage Law

The constant-load data of the experiments have been subjected to accurate analyses in [9] and [10], for a damage law of the type

\[ \frac{dN}{da} = C \exp \left( -\frac{Q}{RT} \right) \]

For appropriate bibliographical references we refer the reader to documents [1] - [8] and to the bibliography of the papers [9] and [10].
\[
D = D(d) \left( \frac{\alpha}{\varepsilon(d)} \right)^{\alpha} \frac{\partial T(d)}{T(\varepsilon)} \left( \frac{\sigma_o}{\sigma_o(d)} \right)^{\gamma} \left( \frac{T(t)}{t(d)} \right)^{\delta} = \left( \frac{\sigma_o}{\sigma_o(d)} \right)^{\gamma} \left( \frac{T(t)}{t(d)} \right)^{\delta} \]

where \( D(d) \) is the damage produced, at the reference temperature \( T(d) \), by the strain \( \varepsilon(d) \) and the initial stress \( \sigma_o(c) \) in the time \( t(d) \).

In addition to a relation with four independent variables laws of type (1) with only three, two or one independent variable, have also been analysed [9, 10]. The statistical analyses involve the use of Fisher's F test, which examines whether the variance removed from the regression is significant when compared with the residual variance of estimates.

Taking density variation as the measure of damage, for AISI 310 stainless austenitic steel [9], from these investigations the conclusion can be drawn that the strain is the only independent variable which is always significant. For creep tests in argon at constant temperature, this total damage law can be used

\[
D = -\frac{\Delta e}{\rho_o} = H_o \varepsilon^\alpha \sigma_o^\gamma t^\delta
\]

where for \( T_o = 600^\circ C \), with the measurement units of the list of symbols

\[
H_o = H e^{-\frac{\beta}{T_o}} = 10^{-5} 17, \alpha = 0.7, \gamma = 0.06, \delta = 0.012
\]

For 2.25Cr1Mo ferritic steel [10] density variation is strongly stress-dependent. For instance at 550^\circ C the eq. (2) is valid where, with the measurement units of the list of symbols

\[
H_o = 10^{-5} 0.28 10^{-2}, \alpha = 0.65, \gamma = 2.5, \delta = 0.25
\]

Eq. (2) can also be written as

\[
\frac{d}{dt} \left[ \frac{D}{H_o \varepsilon^\alpha} \right] ^{1/\delta} = \sigma_o^\gamma t^\delta
\]

Eqs. (2) and (5) are perfectly equivalent for \( \sigma_o \) constant in time.

3. Incremental Damage Law

In generalising eq. (5) to the case of variable stress \( \sigma \), we postulate that the damage rate be a function of axial strain rate, axial strain, damage and stress, i.e.

\[
\dot{D} = f [\dot{\varepsilon}, \varepsilon, D, \sigma]
\]

where the dot above a variable indicates \( d/dt \).

An expression consistent with this function is obtained directly if we simply replace \( \sigma_o \) in eq. (5) by \( \sigma \), which upon integration yields the desired result

\[
D = H_o \varepsilon^\alpha \left[ \int_0^t \left( \frac{\varepsilon}{\sigma} \right)^{\delta} dt \right]^\delta
\]
If we again consider the constant-load test but now with the lateral contraction included, improved values of the material damage constants \( H_0, \alpha, \gamma, \delta \) may be computed by a least squares technique, using eq. (7) rather than eq. (2) in order to determine rupture times. For this purpose we need the constitutive law of creeping materials, i.e. the relationships between time, stress, strain and their derivatives.

4. Constitutive Law

To construct the constitutive law for a constant-load situation, we assume for the total strain

\[
\dot{\varepsilon} = \dot{\varepsilon}_i + \dot{\varepsilon}_c = \dot{\varepsilon}_i + \dot{\varepsilon}_c^T
\]

in which, following Hult's approach [13]/

\[
\dot{\varepsilon}_i = \dot{\varepsilon} i(d) = \frac{\sigma_o}{\sigma_o(d)} \frac{a(d)T(d)}{T} \frac{e^{-a(d)}}{e} = \sigma e^{-a(d)} \frac{e}{E^*}
\]

\[
\dot{\varepsilon}_c = \dot{\varepsilon}_c(d) \left( \frac{\sigma_o}{\sigma_o(d)} \right)^n e^{-b(d)T(d)} = K \sigma_o^n e^{-b/T}
\]

On the contrary, for Odqvist's approach [14]/ the initial strain we have

\[
\dot{\varepsilon}_i = \dot{\varepsilon} i(d) = \left( \frac{\sigma_o}{\sigma_o(d)} \right)^{n_1} e^{-c(d)T(d)} = K_1 \sigma_o^{n_1} e^{-c/T}
\]

\( \dot{\varepsilon}_i(d) \) and \( \dot{\varepsilon}_c(d) \) are the initial strain and the secondary strain rate, respectively, produced by the initial stress \( \sigma_o(d) \) at the reference temperature \( T(d) \). \( E^* \) is a fictitious Young modulus at the reference temperature \( T(d) \).

To determine the constants which appear in eqs. (10) and (11), statistical analyses were carried out analogous to those described in § 2, in ref. [9] for the AISI 316 austenitic stainless steel and in ref. [15] for the 2.25Cr 1Mo ferritic steel. For instance, with the measurement units of the list of symbols, for the AISI 310 steel at 600°C in argon, we have

\[
K_1 = K_1 e^{-873} = 5.10^{-12} \quad n_1 = 5
\]

\[
K = K e^{-873} = 0.74 \cdot 10^{-11} \quad n = 7
\]

and for the 2.25Cr 1Mo steel at 550°C we have

\[
K_1 = K_1 e^{-823} = 1.10^{-12} \quad n_1 = 5.5
\]

\[
K = K e^{-823} = 0.45 \cdot 10^{-8} \quad n = 5.6
\]
For the sake of simplicity, in the following paragraphs we shall use Hult's approach with a fictitious Young modulus $E^* = 10,000$ MPa.

Eq. (8) can be written as

$$
\dot{\varepsilon} = \dot{\varepsilon}_i + \dot{\varepsilon}_c 
$$

(16)

and if we now replace $\sigma_o$ by $\sigma(t)$ in eqs. (16), (9), (10) and (11), we have an incremental constitutive law valid for variable stress.

5. **Constant-Load Test**

5.1 **Time-Dependent Strains and Stresses**

From ref. (11) we consider the case of large-scale deformation with transient creep excluded in a constant-load test at temperature $T(d)$. Consider a bar of instantaneous cross-sectional area $A(t)$ under a constant load $P_o$ suddenly applied at $t = 0$. The bar experiences a continuously increasing stress

$$
\sigma(t) = \frac{P_o}{A(t)}
$$

(17)

due to the effect of lateral contraction, and the existence of a finite rupture time is assured. Employing the logarithmic definition for large-scale strain, we write the lateral strain rate as

$$
\dot{\varepsilon}_T(t) = \frac{\dot{R}(t)}{R(t)} = \frac{1}{2} \frac{\dot{A}(t)}{A(t)}
$$

(18)

where $R(t)$ is the instantaneous cross-sectional radius.

We use constitutive law (16) in which $\varepsilon$ has been put equal to the elastic strain, and also assume that the steady creep is incompressible ($v_s = \frac{1}{2}$), thus

$$
\dot{\varepsilon}_T(t) = -v_e \left[ \frac{P_o}{A(t)E^*} \right]^{\frac{n}{2}} \frac{1}{2} K - \frac{P_o}{A(t)}
$$

(19)

where $v_e$ is the elastic Poisson ratio.

This last result in combination with eq. (18) leads to the differential equation in $A(t)

$$
K \dot{P}_o^n dt = \frac{3v_e P_o}{E^*} A^{-2} dA - A^{-1} dA, \quad t > 0
$$

(20)

The initial condition for eq. (20) is prescribed at $t = 0^+$ (i.e. immediately after the load is applied), i.e.

$$
A(0^+) = A_0
$$

(21)

Differential eq. (20) with initial condition (21) may be integrated to yield for $n > 1$

$$
\left( \frac{A}{A_o} \right)^n \left[ 1 - \frac{2n v_e \sigma_0}{(n-1)E^*} \left( \frac{A}{A_o} \right)^{-1} \right] = 1 - \frac{2n v_e \sigma_0}{(n-1)E^*} K \sigma_o^n t, \quad t > 0
$$

(22)
where here \( \sigma_0 = P_o/A_o \), the initial stress. Eq. (22) gives the function \( A_o/A(t) = \mathcal{F}(t) \) in implicit form, but explicit expressions may be obtained for approximate cases.

For example, we can retain the elastic term but approximate \( A_o/A \) within the brackets by unity, and accordingly obtain

\[
\frac{A_o}{A(t)} = \left[ 1 - \frac{n(n-1)E^*}{(n-1)E^* \varepsilon_0} K \sigma_o^n t \right]^{-1/n}, \quad t > 0. \tag{23}
\]

The stress now follows immediately from eq. (23) as

\[
\sigma(t) = \sigma_o \frac{A_o}{A(t)} = 1 - \frac{n(n-1)E^*}{(n-1)E^* \varepsilon_0} K \sigma_o^n t \right]^{-1/n}, \quad t > 0. \tag{24}
\]

Next, if we ignore the initial strain \( \varepsilon_0 \), the creep strain as obtained by substituting eq. (24) by eqs. (1c) and (10), is given by

\[
\mathcal{E} \approx \dot{\varepsilon}_c = \frac{-\ln \left[ 1 - \frac{n(n-1)E^*}{(n-1)E^* \varepsilon_0} K \sigma_o^n t \right]}{n(n-1)E^* \varepsilon_0} - \frac{n(n-1)E^*}{(n-1)E^* \varepsilon_0} \sigma_o^n t \right]^{-1/n}, \quad t > 0. \tag{25}
\]

5.2 Calculation of Rupture Time by Total and Incremental Damage Laws

The experimental results suggest the hypothesis that in creep tests rupture occurs when damage reaches a critical value \( D_{Rc} < 1/2 \). It should be observed that this is a mean value obtained from a fairly long part of the useful length of ruptured samples: ref. \( \text{14}/ \) demonstrates that the damage accumulates near the rupture zone where damage grows rapidly and tends toward unity. If we know, for a given temperature, the value of \( D_{Rc} \), the time at which rupture occurs for a given initial stress \( \sigma_0 \) can be calculated both by the total and the incremental damage law.

For total damage law, using eqs. (2) and (25) we obtain the following relationship between initial stress \( \sigma_0 \) and rupture time \( t_{Ri} \)

\[
D_{Rc} = H \left[ \frac{-\ln \left[ 1 - AK \sigma_0^n t \right]}{A} \sigma_o^n t \right] \left[ \sigma_o^n t \right]^{-1}, \quad t > 0 \tag{26}
\]

with \( A = \frac{n(n-1)E^*}{(n-1)E^* \varepsilon_0} \)

\[
\text{For incremental damage law from eqs. (5), (24) and (25) by integration we have}
\]

\[
D_{Rc} = H \left[ \frac{1}{A} \right] \left[ -\ln \left[ 1 - AK \sigma_0^n t \right] \right] \sigma_o^n t \left[ \sigma_o^n t \right]^{-1} \left[ \sigma_o^n t \right]^{-1}, \quad t > 0 \tag{28}
\]
We have computed rupture times for different values of the initial stress $\sigma_0$ by eq. (27) and eq. (28) using Newton's method. Table I(a) gives rupture times for AISI 310 stainless steel using the same values (3) for both damage laws and $D_{Rc} = 250 \cdot 10^{-5}$. We have calculated the square difference of rupture times

$$S.D. = \left( \frac{\log t_{Rt} - \log t_{Ri}}{\log t_{Rt} + \log t_{Ri}} \right)^2$$

and for data of Table I(a) S.D.=0.4 $\cdot$ 10^{-2}, Table I(b) is valid for 2.25Cr 1Mo ferritic steel with values (4) and $D_{Rc} = 150 \cdot 10^{-5}$, and in this case S.D. = 0.128.

For a hypothetical material with the material parameters of 2.25Cr 1Mo steel except $H_0 = 0.175 \cdot 10^{-8}$, $\alpha = 0$, $\gamma = 6.5$, $\delta = 1$, S.D. reaches the value 0.515 (Table I(c)).

5.3 Computation of Material Parameters for Incremental Damage Law

We have computed the rupture times many times by eq. (28) changing the values of material parameters. We have reached low S.D., i.e. a good agreement between predictions of the two damage laws, using the same parameters for both laws for AISI 310, $\alpha = 0.67$, $\gamma = 2.4$, $\delta = 0.24$ for 2.25Cr 1Mo steel and $\gamma = 6.6$, $\delta = 0.8$ for the hypothetical material.

6. Conclusions

We have recalled two damage laws: one in finite terms, called total damage law and another in differential terms, called incremental damage law. The material damage parameters of the first law can be computed by statistical analyses of experimental measurements in constant-load tests. In this paper we have proposed a method for computing material damage parameters for incremental damage law, the only one suitable to deal with time-dependent stress. We have shown that for materials with damage which does not depend strongly on stress, such as AISI 310 austenitic stainless steel, we can assume the same values of material parameters for total and incremental damage laws. On the contrary, the more stress-dependent and strain-independent damage is, the greater is the necessity to compute new values of the parameters passing from total to incremental damage law, as we have realised for 2.25Cr 1Mo steel and for a hypothetical limit material.

Acknowledgement

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References


October, 1976.


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Table I - Rupture times in constant-load tests for AISI 310 stainless steel (at 873 K), 2.25Cr 1Mo steel (at 823 K) and a hypothetical material

<table>
<thead>
<tr>
<th>MPa (kg/mm²)</th>
<th>(a) AISI 310 stainless steel</th>
<th>(b) 2.25Cr 1Mo steel</th>
<th>(c) Hypothetical material</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>τ_Rt</td>
<td>τ_Ri</td>
<td>τ_Rt</td>
</tr>
<tr>
<td>49 (5)</td>
<td>22·10⁶</td>
<td>22·10⁶</td>
<td>51·10⁶</td>
</tr>
<tr>
<td>98 (10)</td>
<td>17·10⁴</td>
<td>17·10⁴</td>
<td>9730</td>
</tr>
<tr>
<td>147 (15)</td>
<td>9900</td>
<td>9900</td>
<td>913</td>
</tr>
<tr>
<td>196 (20)</td>
<td>1320</td>
<td>1320</td>
<td>157</td>
</tr>
<tr>
<td>245 (25)</td>
<td>277</td>
<td>276</td>
<td>38</td>
</tr>
<tr>
<td>294 (30)</td>
<td>77</td>
<td>76</td>
<td>12</td>
</tr>
</tbody>
</table>
List of Symbols

\( \alpha \) exponent of \( \dot{\varepsilon}_c \) in damage law, eq. (1)

\( \beta \) exponent in temperature term in damage law, eq. (1)

\( \gamma \) exponent of \( \sigma^o \) in damage law, eq. (1)

\( \delta \) exponent of \( t \) in damage law, eq. (1)

\( \dot{\varepsilon}(t) \) total axial strain \( [\%] \)

\( \dot{\varepsilon}_l(t) \) total lateral strain \( [\%] \)

\( \dot{\varepsilon}_c(t) \) axial creep strain \( [\%] \)

\( \dot{\varepsilon}_i \) initial strain \( [\%] \)

\( \nu_e \) elastic Poiisson’s ratio

\( \rho(t) \) density

\( \rho_o \) initial density

\( \sigma \) one-dimensional tensile stress \( [\text{MPa}] \)

\( \sigma^o \) constant stress \( [\text{MPa}] \)

\( \Delta \rho \) density variation

\( a \) exponent in temperature term in constitutive law for \( \dot{\varepsilon}_t \), eq. (9)

\( b \) exponent in temperature term in constitutive law for \( \dot{\varepsilon}_s \), eq. (10)

\( c \) exponent in temperature term in constitutive law for \( \dot{\varepsilon}_l \), eq. (11)

\( n \) exponent of \( \sigma^o \) in constitutive law for \( \dot{\varepsilon}_c \), eq. (10)

\( n_i \) exponent of \( \sigma^o \) in constitutive law for \( \dot{\varepsilon}_i \), eq. (11)

\( t \) time \( [\text{h}] \)

\( t_R \) rupture time \( [\text{h}] \)

\( A(t) \) instantaneous circular cross-sectional area

\( A_o \) initial cross-sectional area immediately after load application

\( D \) damage, eq. (1)

\( E^* \) fictitious elastic modulus, eq. (9)

\( H \) constant in damage law, eq. (1)

\( H_o \) equal to \( H e^{-B/T_o} \), eq. (2)

\( K \) constant in constitutive law for \( \dot{\varepsilon}_c \), eq. (10)

\( K_i \) constant in constitutive law for \( \dot{\varepsilon}_i \), eq. (11)

\( K' \) equal to \( K e^{-B/T_o} \), eq. (13)

\( P_o \) constant load suddenly applied at \( t = 0 \), eq. (17)

\( R(t) \) instantaneous cross-sectional radius, eq. (18)

\( S.D. \) square difference, eq. (29)

\( T \) absolute temperature

\( T_o \) constant temperature

\( \dot{\cdot} \) indicates \( \frac{d}{dt} \).