CREEP BUCKLING AND INSTABILITY OF CIRCULAR CYLINDRICAL SHELLS IN AXIAL COMPRESSION

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Abstract

The process of creep buckling in circular cylindrical shells under a constant axial load starts from the initial elastic-plastic deformation at the instant of loading, proceeds through quasi-static deformation due to creep and terminates by almost instantaneous collapse. The final collapse occurs either through a very rapid quasi-static deformation due to creep, or through an instantaneous elastic-plastic bifurcation. In the cases of thin shells or shells under large axial compressions, in particular, the mode of the final collapse has been observed to be non-axisymmetric even if the initial elastic-plastic deformation is apparently axisymmetric. As the causes of the non-axisymmetric final deformation, there may be three possibilities: 1) unstable non-axisymmetric elastic-plastic bifurcation may occur in the process of axisymmetric quasi-static creep deformation, 2) stable non-axisymmetric bifurcation may occur in quasi-static creep deformation and develop thereafter quasi-statically, 3) non-axisymmetric deformation may occur at the instant of loading because of a certain initial imperfection and develop quasi-statically up to the final collapse.

The present paper is concerned with the theoretical and the experimental investigation into the instability in creep buckling of real circular cylindrical shells with special emphasis on the initiation of non-axisymmetric mode of deformation. Clamped aluminium circular cylindrical shells of radius to thickness ratio α = 50 and 100 under constant axial compression at 200°C are chosen.

Axisymmetric quasi-static deformations in creep buckling of clamped shells of a orthotropic elastic-plastic-creep material is first analyzed. It was found that the effects of plastic deformation and material anisotropy are not significant.

Then the corresponding experiments are performed. In order to examine the process of unstable deformations, besides measuring end-shortening, variation of the axial and the circumferential component of strain at several places of the shells are detected. It was found that the shells of α = 50 buckled axisymmetrically and the limit times and the strain distribution agreed well with the analytical results. In the shells of α = 100, however, non-axisymmetric deformations were observed already in early stage of the buckling process and the limit times were considerably shorter than those predicted by the analysis. In view of these observations, together with the previous results of the bifurcation analysis, it was concluded that the cause of the non-axisymmetric collapse in the shells of α = 100 was attributable to the quasi-static magnification of the initial imperfections due to creep.

1. Introduction

The creep buckling of circular cylindrical shells subject to an axial compression shows various processes of collapse. When an axially compressed load is sufficiently low in comparison with the classical elastic buckling load determined by Young's modulus and the radius to thickness ratio of the shells, for example, an almost axisymmetric quasi-static deformation proceeds until the final collapse. In other cases, the non-axisymmetric waves may develop gradually or suddenly in the course of the axisymmetric deformation. These phenomena have been analyzed from the theoretical point of view and elucidated considerably by using idealized and specific assumptions convenient for the analyses [1-9]. However, creep buckling processes in real circular cylindrical shells are far more complicated, and such idealized assumptions will never be realized. Indeed real shells always involve certain kinds of unknown irregularities, such as the imperfection of shell geometry, the eccentricity of the load, non-uniformity of temperature and so on.

The present paper is concerned with the theoretical and the experimental investigation of the process of creep buckling in real circular cylindrical shells with special emphasis on the mechanism of the initiation of non-axisymmetric deformations. Creep buckling of clamped aluminium circular cylindrical shells of radius to thickness ratio $\alpha = 50$ and 100 under constant axial compressions at 200°C are discussed. The axisymmetric quasi-static buckling of the shells of orthotropic elastic-plastic-creep materials are first analyzed by the finite-difference method combined with the method of numerical integration on the basis of Donnell's shell theory. Then the corresponding experiments are performed, and besides measuring the end-shortening, variation of the axial and the circumferential component of strain at several places of the shells are detected by use of resistance-strain-gages. Finally, the difference of the experimental results from the numerical predictions is discussed on the basis of the earlier theoretical works.

2. Analysis of the Axisymmetric Creep Buckling of Clamped Circular Cylindrical Shells Subject to Axial Compression

The creep buckling process in a circular cylindrical shell under an axial compression can be elucidated by analysing the axisymmetric quasi-static deformation together with the processes of the stable or unstable bifurcation. In order to clarify the ideal process accurately, we first analyze the axisymmetric quasi-static creep process of the relevant shells by incorporating the effect of the plastic deformation as well as that of the material anisotropy.

Now, let us consider a thin circular cylindrical shell of mean radius R, thickness H and length L subject to axial force F. We take a system of orthogonal co-ordinates (x, θ, z), with the origin at the middle surface of the center of the undeformed shell and the radial coordinate z directed outward. Furthermore, let us denote the axial and the radial component of displacement in the middle surface of the shell by u and w, the axial and circumferential components of the membrane force, bending moment and strain by N_x, N_θ, M_x, M_θ, e_x and e_θ, respectively. According to Donnel's shell theory [10], the equation of equilibrium and the kinematic relation in circular cylindrical shells are expressed as follows:

$$N_x = -F/(2\pi R) \equiv -N, \qquad M_{x,xx} - N_\theta/R - Nw_{,xx} = 0 \tag{1}$$

$$e_x = u_{,x} + (1/2)(w_{,x})^2 - zw_{,xx} \qquad e_\theta = w/R \tag{2}$$

where the subscripts following comma denote the differentiation with respect to them.

In order to express the rate of membrane force and that of bending moment in terms of the velocity $\dot{u}$ and $\dot{w}$, we need to specify the constitutive equation. Let us assume that the total strain rate $\dot{e}_{ij}$ is expressible as the sum of the elastic part $\dot{e}^e_{ij}$, plastic part $\dot{e}^p_{ij}$ and creep part $\dot{e}^p_{ij}$. We further assume the orthotropic inelastic response of the material which is described by the uniaxial relations of the plastic and creep deformation:

$$\varepsilon^p = [(\sigma - Y_{ii})/K_{ii}]^{\ell}, \quad \gamma^p = [(\tau - Y_{ij})/K_{ij}]^{\ell}, \quad \sigma > Y_{ii}, \quad \tau > Y_{ij},$$
$$\varepsilon^c = B_{ii}\sigma^n t^m, \quad \gamma^c = B_{ij}\tau^n t^m, \qquad (3)$$
$$(i,\ j:\ i \neq j,\ 1,\ 2,\ 3,\ \text{not summed})$$

where, ε, γ, σ, τ and t are the axial strain, shear strain, axial stress, shear stress and time, respectively. In eq. (3) ℓ, n, m, Y_{ij}, K_{ij} and B_{ij} are material constants. Then eq. (3) may be readily extended to the multiaxial state of stress as follows:

$$\dot{e}^e_{ij} = \frac{1+\nu}{E}\dot{s}_{ij} - \frac{\nu}{E}\dot{s}_{kk}\delta_{ij}$$
$$\dot{e}^p_{ij} = \frac{9\ell}{4K}\frac{1}{(s^p_e)^2}\left(\frac{s^p_e - Y}{K}\right)^{\ell-1} C_{ijk\ell}s_{k\ell}C_{mnrs}s_{rs}\dot{s}_{mn}, \quad s^p_e > Y, \quad \dot{s}^p_e > 0 \qquad (4)$$
$$\dot{e}^c_{ij} = \frac{3}{2}mB(s^c_e)^{n-1}t^{m-1}D_{ijk\ell}s_{k\ell}$$

where s_{ij}, s^p_e and s^c_e are stress tensor and equivalent stresses corresponding to the plastic and creep deformation, and E and ν are Young's modulus and Poisson's ratio. K, Y, $C_{ijk\ell}$ and B, $D_{ijk\ell}$ are material constants expressible in terms of the constants in eq. (3).

Now, if we solve eq. (4) with respect to the stress rate, integrate the resulting relations through the shell thickness and replace the strain rates by $\dot{u}$ and $\dot{w}$ by use of eq, (2), we obtain the expression of $\dot{N}_x$, $\dot{N}_\theta$, $\dot{M}_x$ and $\dot{M}_\theta$ expressed in terms of velocities.

Then the fundamental equations in the present analysis expressed in terms of $\dot{u}$ and $\dot{w}$ can be eventually obtained by differentiating eq. (1) with respect to time, and by substituting $\dot{N}_x$, $\dot{N}_\theta$ and $\dot{M}_x$ into the resulting equations.

Lastly, the end-shortening of the shell δ can be given as follows:

$$\delta = -2[u]^{x=L/2}_{x=0} \qquad (5)$$

By use of the above equations, axisymmetric quasi-static creep buckling can be analyzed by solving first the rate boundary value problem by the method of finite difference and then integrating the resulting solutions numerically with respect to load or time [8].

3. Creep Buckling Experiments of Clamped Circular Cylindrical Shells Subject to Axial Compression

In order to discuss the process and the mechanism of buckling, we need not only the global quantities, but also local ones. Hence we measured experimentally the strain distribution of the shells as well as the end-shortening and the limit time of creep buckling.

Aluminium alloy (JIS A-5052 BE-O) at 200°C is

R	H	α=R/H
40.0	0.8	50
39.8	0.4	100

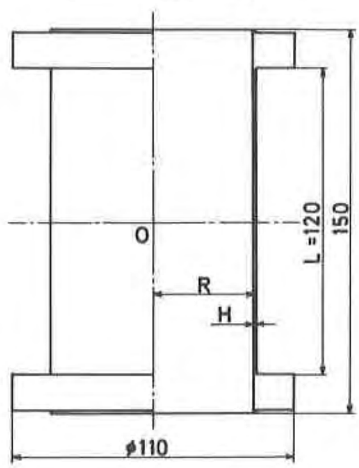


Fig. 1 Cylindrical Specimen

chosen for the specimen to facilitate the strain measurement by means of the resistance-strain-gages. The material was provided in the form of extruded bars of the diameter 120 mm in order to cut out the calibration specimens not only in axial but also in circumferential direction of the shells. The material constants in eqs. (3) and (4) were determined as follows:

$$E = 6200\ \mathrm{kgf/mm^2},\quad \nu = 0.32,\quad Y_{11} = 7.3\ \mathrm{kgf/mm^2},\quad Y_{22} = Y_{33} = 6.2\ \mathrm{kgf/mm^2},$$
$$\ell = 2.857,\quad K_{11} = 21.4\ \mathrm{kgf/mm^2},\quad K_{22} = K_{33} = 17.2\ \mathrm{kgf/mm^2},\quad n = 6,\quad m = 1, \qquad (6)$$
$$B_{11} = 6.34 \times 10^{-9}\ (\mathrm{kgf/mm^2})^{-6}h^{-1},\quad B_{22} = B_{33} = 1.41 \times 10^{-8}\ (\mathrm{kgf/mm^2})^{-6}h^{-1}$$

The circular cylindrical shells were machined from the material into the geometry shown in Fig. 1. Two kinds of radius to thickness ratio $\alpha = 50$ and 100 were taken.

Axially compressed mean stresses were chosen to be $\sigma_m = 5$, 6 and 7 $\mathrm{kgf/mm^2}$.

4. Results and Discussion

4.1 Analytical and Experimental Results

Fig. 2 shows the results of the end-shortening obtained from the analyses for various combination of α and σ_m. Solid, dashed and chain lines in the figure are, respectively, the result of analysis incorporating plastic deformation and material anisotropy (A), that incorporating only the material anisotropy (B) and that of the plastic deformation only (C). As regards the case of (C), the material constants in the axial direction are adopted.

As shown in Fig. 2, the difference in the limit times between the results of A, B and C is no more than 10%. In view of the fact that the limit times of creep buckling predicted by the time-hardening hypothesis of the constitutive equation of creep are twice as large as those by strain-hardening hypothesis [8], the effects of the plastic deformation and that of the material anisotropy in this case does not have an essential influence on creep buckling.

On the other hand, the limit times depend remarkably on the values of σ_m and α. As regards the stress dependence in the case of A and $\alpha = 50$, in particular, the limit times for $\sigma_m = 5$, 6 and 7 $\mathrm{kgf/mm^2}$ correspond to 138.6 h, 44.0 h and 16.3 h, respectively.

According to the previous paper [11], it was elucidated that the limit time in the quasi-static instability of a circular shell obeying creep law $\varepsilon^c = B\sigma^n t^m$ with axisymmetric initial imperfection may be approximated sufficiently by the following equation:

$$T_\ell = [1.78(1-\rho)^{0.966+0.034n}/(n-1)^{0.413}] \times [(1.50 - 0.26n + 0.015n^2) - 10^{\rho/(4.35-1.61\rho^2)} - \log_{10}\overline{W}_0] \qquad (7)$$

where the non-dimensional time T and load parameter ρ are defined as $T = B\sigma_m^n t^m/\varepsilon_{c\ell}$ and $\rho = \sigma_m/\sigma_{c\ell}$, and $\sigma_{c\ell}$ and $\varepsilon_{c\ell}$ are the classical elastic buckling stress $\sigma_{c\ell} = [E/\sqrt{3(1-\nu^2)}](H/R)$ and the corresponding strain $\varepsilon_{c\ell} = \sigma_{c\ell}/E$, respectively. Moreover, $\overline{W}_0 = \overline{w}_0/H$ is the amplitude of the initial imperfection reduced with respect to the thickness of the shell. If the magnitude of the elastic-plastic bulged deformation at the instant of loading is identified

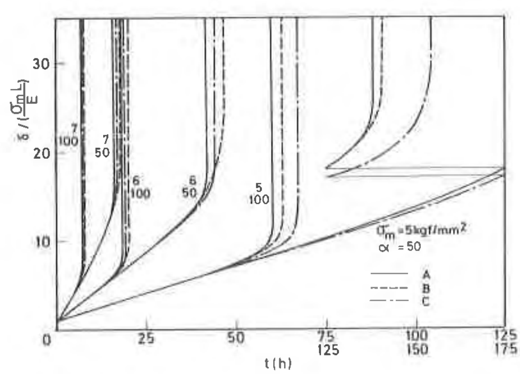
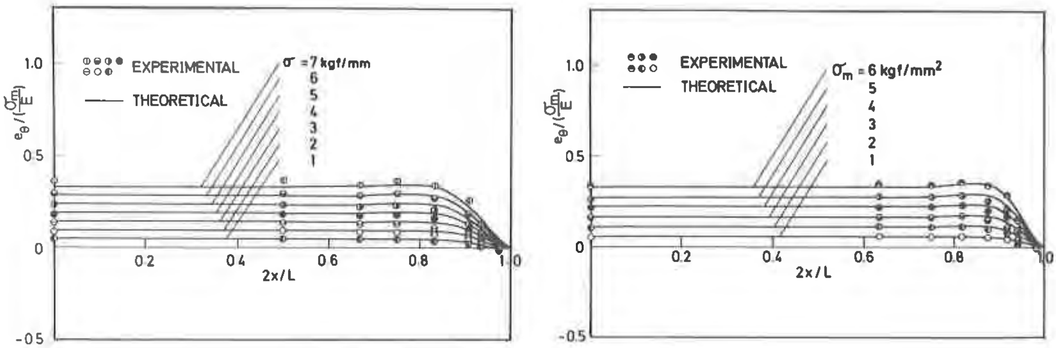


Fig. 2 Variation of End-Shortening



(a) α = 50 (b) α = 100

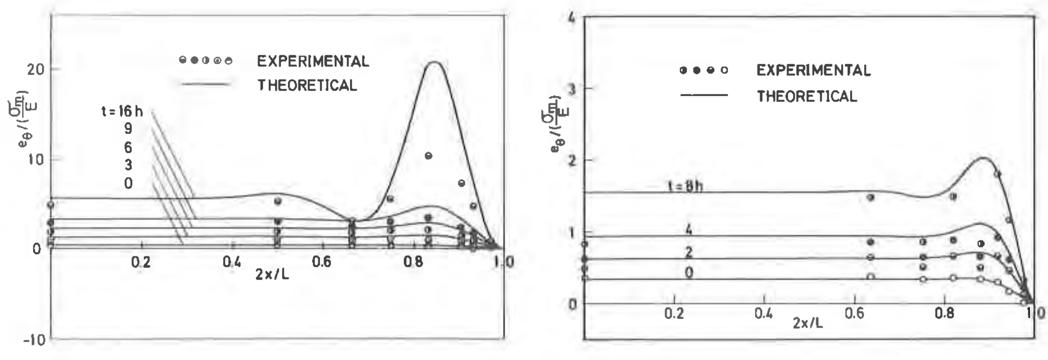
Fig. 3 Distribution of Circumferential Strain at the Process of Loading

as the magnitude of the initial imperfection, the corresponding values of t_ℓ calculated from eq. (7) for α = 50 are t_ℓ = 145.0 h (σ_m = 5 kgf/mm^2), 44.8 h (σ_m = 6 kgf/mm^2) and 16.4 h (σ_m = 7 kgf/mm^2), which agree quite well with the theoretical results shown in Fig. 2. Therefore eq. (7) may describe the stress-limit time relation also for the present clamped circular cylindrical shells. Though the pertinence of the recent revision of ASME Boiler and Pressure Vessel Code from the safety factor of 10 of limit time to the load factor of 1.5 has been discussed in some papers [12], eq. (7) may provide more general informations to estimate the validity of the revision.

Now the relation between the analytical and the experimental results are summarized in Fig. 3 through 5.

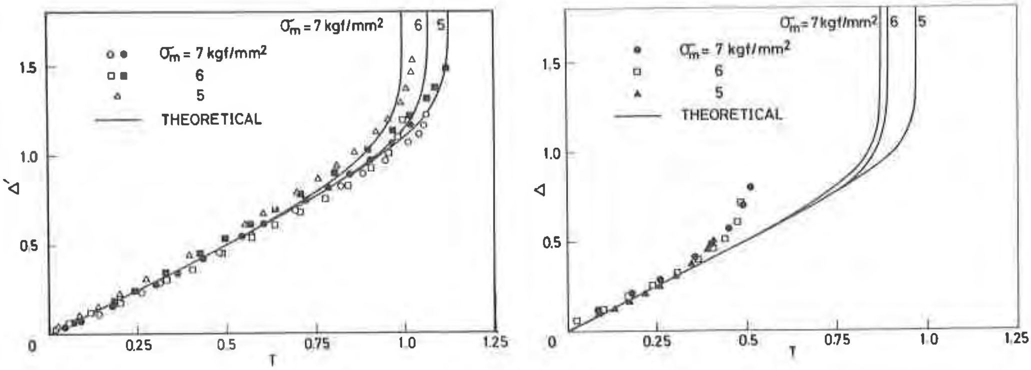
Fig. 3 shows the elastic and plastic distribution of the circumferential strain at the instant of loading for the cases of α = 50, σ_m = 7 kgf/mm^2 and α = 100, σ_m = 6 kgf/mm^2. Each circle in the figure is the average value of the experimental ones measured by two strain gages attached diametrically, while the solid lines show the theoretical results in the case of A. It is found that, in both cases of α = 50 and 100, the experimental and the theoretical results agree almost completely.

The corresponding strain distributions in the succeeding process of creep under constant



(a) α = 50, σ_m = 7 kgf/mm^2 (b) α = 100, σ_m = 6 kgf/mm^2

Fig. 4 Distribution of Circumferential Strain at Constant Load



(a) α = 50 (b) α = 100

Fig. 5 Variation of Non-Dimensional End-Shortening

load are shown in Fig. 4. In the case of α = 50 shown in Fig. 4(a), except close vicinity of the final collapse (t = 16 h), the theoretical and the experimental results coincide with each other within the accuracy of 10%. However, in the case of α = 100, though both results coincide at the stage of t = 0, the experimental results starts to show remarkable irregularities in the shape of waves afterward, and the discrepancy between the theoretical and the experimental values in the bulged part is magnified remarkably as the creep deformation proceeds.

Fig. 5(a), (b) show the relation between non-dimensional end-shortening and non-dimensional time $T = (\dot{e}^c/e_{c\ell})t$ reduced by the axial component $\dot{e}^c$ of creep strain rate in perfect shells and the classical elastic buckling strain $e_{c\ell}$, while the ordinate is the creep component of the non-dimensional end-shortening $\Delta' = \delta'(e_{c\ell}L)$. In the case of α = 50 shown in Fig. 5(a), the experimental results coincide fairly well with the theoretical predictions. However in the case of α = 100, though the theoretical predictions for buckling times are about T = 0.9, the corresponding results of experiment are about T = 0.5, which is a remarkably different tendency from the case of α = 50. Moreover, the pattern of the final collapse of the shells of α = 100 were found to be always heptagonal contrarily to the results of the analysis.

As observed in the above discussion, the theoretical and the experimental results for the cylindrical shells of α = 50 agreed well not only with respect to the strain distribution and the end-shortening, but also with respect to the patterns of the final collapse. Hence, as far as the shells of α = 50 are concerned, the adequacy of the theory was confirmed. However in the shells of α = 100, the experimental results had a remarkably different tendency from that of theory. In order to investigate the cause of these discrepancies, the distribution of the circumferential strain in the circumferential direction was measured in the process of creep buckling. The ordinate of Fig. 6 shows the circumferential component of strain, while the abscissa is the circumferential coordinate θ. As shown in the figure, though an almost uniform deformation is observed at the instant of loading, remarkable waves appear quite rapidly and grow to the non-axisymmetric waves observed in the final stage of buckling. Accordingly, the irregularity of the strain distribution in Fig. 4(b) and the reduction of the limit time shown in Fig. 5(b) in the shells of α = 100 are attributable to the development of non-axisymmetric waves and the resulting accerelation in the rate of deformation.

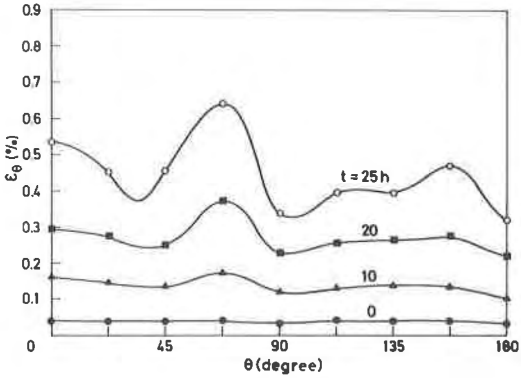


Fig. 6 Distribution of Circumferential Strain in the Circumferential Direction (α = 100, σ_m = 6 kgf/mm^2)

4.2 Cause of the Initiation of Non-Axisymmetric Waves

The process of creep buckling in circular cylindrical shells with axisymmetric initial imperfection may be divided into three cases [3]: 1) the case in which the axisymmetric deformation proceeds quasi-statically until the final stage of buckling, 2) the case in which a stable bifurcation occurs in the course of the axisymmetric quasi-static deformation, and the resulting non-axisymmetric deformation are magnified quasi-statically thereafter, 3) the case in which an unstable bifurcation occurs in the course of axisymmetric quasi-static deformation and the instantaneous non-axisymmetric buckling occurs. According to the analysis of the paper [3], the boundaries of the above cases are mainly determined by $\bar{W}_0$ and ρ mentioned previously. In the case of the shells of α = 100 adopted in the present experiment, if we take the elastic-plastic bulged deformation at the instant of loading as the initial imperfection, we have $\bar{W}_0 = 2.5\sim3.5 \times 10^{-2}$ for the cases of σ_m = 5(ρ = 0.132), 6(0.159) and 7 (0.185) kgf/mm^2. Then the condition of the initiation of the bifurcation is about $\rho > 0.25$ in the case of n = 6 and m = 1, and that of the instability is $\rho \geqq 0.3$ irrespectively of n, m, $\bar{W}_0$. Hence, neither stable nor unstable bifurcation will occur in the present shells.

As another possibility of the occurrence of the non-axisymmetric waves in the shells of α = 100, there may be a mechanism in which a small non-axisymmetric initial imperfection in the shell is magnified in the process of creep. However, in view of the fact that the pattern of the final collapse of the shells of α = 100 was always heptagonal in spite of that the form of the initial imperfection is far from definite, some mechanisms inducing definite modes of deformation should be operative. According to the previous paper on the effect of the axisymmetric and non-axisymmetric initial imperfections on the quasi-static creep buckling of the cylindrical shells [4, 13], it was ascertained that there is a specific combination of the wave length of the axisymmetric and the non-axisymmetric initial imperfections which accelerate the quasi-static deformation most easily. Though the non-axisymmetric initial imperfections may consist of a combination of various waves, only a specific non-axisymmetric pattern develops preferentially. If the initial axisymmetric elastic-plastic bulged deformation produced by the clamped ends is regarded as the initial imperfection, the number of the non-axisymmetric waves for α = 100 predicted by the literature [13] is 8 or 9. Hence though this is a rather approximate analysis, it predicts well the qualitative tendency of the non-axisymmetric buckling of the real circular cylindrical shells of α = 100.

4.3 Reduction of the Limit Time due to the Existence of Non-Axisymmetric Waves

As observed in Fig. 5(b), the shells of α = 100 always collapsed considerably earlier than predicted by the analysis. However, as mentioned above, a bifurcation does not occur in this case. Hence the discrepancies between the experimental and the analytical results for limit times may be again attributable to the development of the non-axisymmetric waves due to the initial imperfection as dicussed in the paper [13].

5. Conclusion

The creep buckling process of axially compressed clamped circular cylindrical shells of radius to thickness ratio of α = 50 and 100 was investigated from theoretical and experimental points of view with special emphasis on the mechanism of the initiation of the non-axisymmetric waves. The shell specimens of α = 50 buckled axisymmetrically, and the corresponding limit times agreed very well with the theoretical predictions. The shell specimens of α = 100 collapsed always in the mode of heptagonal contrarily to the results of the analysis, and the limit times were considerably shorter than the theoretical results. In order to investigate the cause of the discrepancies, the possibilities of the bifurcation buckling and the development of the non-axisymmetric initial imperfections in the shells were discussed by use of the results of the previous analyses. It was found that the discrepancies observed in the shells of α = 100 are to be attributable to the latter cause.

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