CONSTITUTIVE MODELLING IN PLASTICITY

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Abstract

Constitutive modelling of plastic behavior is treated as a sequence of a constitutive analysis, identification testing and a solution of the corresponding inverse problem. The constitutive analysis based on the internal variables approach generates a whole spectrum of plastic models of various degrees of flexibility. A higher flexibility enables more adequate modelling, however, the complexity of the corresponding identification testing program and the solution of the inverse problem grows with increasing flexibility very rapidly. Examples of the models of the spectrum suitable for monotoneous or cyclic loading are briefly analysed. An iteration procedure specifying a model adequate to an elastic-plastic stress analysis problem is suggested. A possibility to utilize inhomogeneous identification tests, each of which contains far more information than a homogeneous one, is mentioned.

1. Introduction

One of the most limiting factors of the present elasto-plastic stress analysis is the lack of analytical models of materials which are compatible in accuracy with currently developed methods of solutions of boundary-value problems. An existence of an universal model of plastic behavior with a range of validity comparable in scope to the linear elastic model cannot be expected [1]. Specialized models have to be seeked. Specification of such models can be based on an identification procedure, where an elastic-plastic constitutive analysis provides a format of the constitutive equations and data from suitably designed identification tests are used as input data for solution of the corresponding inverse problem, i.e. determination of the unknown material functions in the constitutive equations of the model.

As an appropriate constitutive framework we adopt the internal variables model of plasticity suggested in [2]. The constitutive equations are generated by two potentials ϕ and F. The internal energy $\phi(\underset{\sim}{e}, \underset{\sim}{\xi})$ which is a function of elastic strain $\underset{\sim}{e}$ and the internal parameter $\underset{\sim}{\xi}$ (here $\underset{\sim}{\xi}$ is assumed to be a second order symmetric tensor) determines the stress $\underset{\sim}{\sigma}$ and the generalized force $\underset{\sim}{A}$ through the relations

$$\underset{\sim}{\sigma} = \frac{\partial \phi}{\partial \underset{\sim}{e}} , \qquad \underset{\sim}{A} = \frac{\partial \phi}{\partial \underset{\sim}{\xi}} \tag{1}$$

The constitutive equations for the rates of plastic strain $\underset{\sim}{p}$ and $\underset{\sim}{\xi}$ are expressed through the plastic potential $F(\underset{\sim}{\sigma}, \underset{\sim}{A})$ (a superimposed dot means time differentiation)

$$\begin{aligned} \dot{\underset{\sim}{p}} &= \lambda \frac{\partial F}{\partial \underset{\sim}{\sigma}} , \\ \dot{\underset{\sim}{\xi}} &= -\lambda \frac{\partial F}{\partial \underset{\sim}{A}} , \end{aligned} \quad \left| \begin{array}{lll} \lambda \geq 0 , & \text{if} & F(\underset{\sim}{\sigma}, \underset{\sim}{A}) = 0 \\ \lambda = 0 , & \text{if} & F(\underset{\sim}{\sigma}, \underset{\sim}{A}) < 0 . \end{array} \right. \tag{2}$$

The scalar λ is determined by the consistency relation $F(\underset{\sim}{\sigma}, \underset{\sim}{A}) = 0$ which has to be satisfied during a plastic deformation process. The small strain approach is adopted, i.e. total strain $\underset{\sim}{\varepsilon} = \underset{\sim}{e} + \underset{\sim}{p}$, no temperature effects are considered. It is assumed that $F(\underset{\sim}{\sigma}, \underset{\sim}{A}) = 0$ determines a convex surface in the $\underset{\sim}{\sigma}, \underset{\sim}{A}$ space called a generalized yield surface; according to (2) $\dot{\underset{\sim}{p}}$ and $\dot{\underset{\sim}{\xi}}$ are normal to it.

To specify further the constitutive equations (1), (2) three additional assumptions (satisfied e.g. by many metals) are introduced: the validity of the linear Hooke's law for elastic strain, plastic incompressibility, and initial isotropy (more precisely the existence of an isotropic reference state). Hooke's law means that $(1)_1$ has the form $\underset{\sim}{\sigma} = \underset{\sim}{c}\,\underset{\sim}{e}$, $\underset{\sim}{c}$ is a constant tensor. This implies that $\phi = \psi(\underset{\sim}{e}) + \varphi(\underset{\sim}{\xi})$, hence $\underset{\sim}{A} = d\varphi / d\underset{\sim}{\xi}$, i.e. $\underset{\sim}{A}$ is independent of $\underset{\sim}{e}$. Due to plastic incompressibility only the deviatoric stress $\underset{\sim}{s} = \underset{\sim}{\sigma} - (\mathrm{tr}\,\underset{\sim}{\sigma}/3)\underset{\sim}{1}$ appears in F. The initial isotropy causes that the potentials φ and F depend on the arguments $\underset{\sim}{s}$, $\underset{\sim}{A}$ and $\underset{\sim}{\xi}$ only through their basic and mutual invariants (for convenience $\underset{\sim}{\xi}$ and $\underset{\sim}{A}$ are divided into spherical and deviatoric parts, i.e. $\underset{\sim}{\xi} = \alpha \underset{\sim}{1} + \underset{\sim}{a}$, $\underset{\sim}{A} = k\underset{\sim}{1} + \underset{\sim}{K}$; $II_a = \mathrm{tr}\,\underset{\sim}{a}^2$, $II_{sK} = \mathrm{tr}\,\underset{\sim}{s}\,\underset{\sim}{K}$, $III_{sK^2} = \mathrm{tr}\,\underset{\sim}{s}\,\underset{\sim}{K}^2$ etc.)

$$\begin{aligned} \varphi &= \varphi(\alpha, II_a, III_a) , \\ F &= F(k, II_s, II_K, II_{sK}, III_s, III_K, III_{sK^2}, III_{s^2K}) . \end{aligned} \tag{3}$$

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2. Spectrum of models

If only some of the invariants are retained in (3) the constitutive eqs. (1), (2) and the consistency relation yield a special approximative model of a plastic material. Retaining different groups of invariants in (3) a whole spectrum of plastic models of a various flexibility, and hence a potential accuracy, results. From practical point of view it is useful to consider two branches of the spectrum: A) the models suitable for description of plastic behavior under monotoneous loading; as then the stress state remains always on the generalized yield surface, only local information concerning F and its changes suffice for description. B) the models describing the plastic properties in cyclic processes; then some information on the global shape of F enters the description. To illustrate the spectrum a few examples of the models are outlined here, their constitutive equations are stated, validity ranges qualitatively estimated and the needed identification tests and the corresponding inverse problems are very briefly conceptually analysed. A more detail description of the models of the branch A is in [3]. The model 2.1.4 and the branch B will be studied more fully in a forthcoming paper.

2.1 Monotoneous loading branch

2.1.1 The classical model of isotropic hardening

If in (3) only the lowest invariants are retained, i.e. $\varphi = \varphi(\alpha)$, $F = F(k, II_s)$ then from (1), (2) we get the classical model of isotropic hardening of von Mises type. ($f(\alpha, II_s) \equiv F(k(\alpha), II_s)$, $k(\alpha) = (\partial\varphi/\partial\alpha)/3$ is expressed as $f = II_s - h^2(\alpha)$)

$$\dot{\underset{\sim}{p}} = d(\bar{p})\, \dot{II}_s\, \underset{\sim}{s} \quad , \qquad \text{if} \qquad II_s = h^2(\bar{p}) \tag{4}$$

and $\dot{\underset{\sim}{p}} = 0$ if $II_s < h^2(\bar{p})$, where $d(\bar{p}) = [2\,h^2(\bar{p})(dh/d\bar{p})]^{-1}$. As it is proved for this model [3] that α can be identified with accumulated plastic strain $\bar{p} = \int \sqrt{tr\, \dot{\underset{\sim}{p}}^2}\, d\tau$ (or equivalently with plastic work), α in (4) is replaced by $\bar{p}$. To identify (4) it is sufficient to specify the material function $h(\bar{p})$. The identification test is thus reduced to a determination of the simple one-dimensional stress-strain diagram of the plastic material.

To estimate at least qualitatively the range of validity of the classical model two characteristic features of (4) are usually checked by experiments: the shape of the subsequent yield surface $II_s - h(\bar{p}) = 0$ and the coaxiality of $\dot{\underset{\sim}{p}}$ and $\underset{\sim}{s}$. Experimentally determined yield surfaces usually fit approximately the analytical form $II_s - h(\bar{p})$ only if very rough measure of the onset of yielding is used [4]. Non-coaxiality of $\dot{\underset{\sim}{p}}$ and $\underset{\sim}{s}$ appears after a sharp change of the direction of a loading path [5]. The direction of the plastic strain rate $\dot{\underset{\sim}{p}}$ delays from that of the deviatoric stress $\underset{\sim}{s}$ just after the change, but the delay disappears with continuing loading. It means that the non-coaxiality is sensitive to the local curvature of the loading path and does not occur if the curvature is small. Thus one can conclude that the model (4) is suitable for a rough estimate of a plastic response at a general type of loading or is suitable for a moderately accurate modelling of plastic behavior for a class of loading paths of a small curvature. For more adequate modelling some invariants of $\underset{\sim}{a}$, $\underset{\sim}{K}$ and higher invariants of $\underset{\sim}{s}$ have to be retained in (3).

2.1.2 Modelling of higher stress effects

If $\varphi = \varphi(\alpha)$ and $f(\alpha, II_s, III_s) \equiv F(k(\alpha), II_s, III_s)$ is of the form $f = g(II_s, III_s) - \alpha$, then (1), (2) yield

$$\dot{p} = c_1^2(\dot{II}_s/2)s + c_1c_2\,[(\dot{III}_s/3)\,s + (\dot{II}_s/2)\,r] + c_2^2(\dot{III}_s/3)\,r\,, \tag{5}$$

if $g = \alpha$, $\dot{p} = 0$ if $g < \alpha$, where $c_1 = 2(\partial g/\partial II_s)\sqrt{3\,d^2\varphi/d\alpha^2}$, $c_2 = 3(\partial g/\partial III_s)\sqrt{3d^2\varphi/d\alpha^2}$ and the tensor r denotes $r = s^2 - (II_s/2)\,1$. The evolution equation for α, which cannot be now replaced by $\bar{p}$, is gained by time differentiation of $\alpha = g(II_s, III_s)$.

Due to the fact that the flexibility of the model (5) with respect to hereditary properties remains the same as for the classical model (4) (just one internal variable α is retained) we expect that the range of validity for both models are the same. Only difference is that for small curvature loading paths the model (5) enables more accurate description of stress dependence of plastic behavior. The reason is that effects connected with sudden changes of a loading direction, description of which requires a flexibility provided by a and K, are at small curvature paths supressed. Hence a and K are dummy in φ and F and (5) is then the most general model (within the considered framework) for such type of loading.

As at small curvature loading paths no unloading occurs, the yield condition is always satisfied and need not to be considered. Then to identify the model (5) a specification of the material functions c_1, c_2 of the variables II_s, III_s is fully sufficient. As an identification test consider a family of small curvature loading paths $s_1(s,l)$, $s_2(s,l)$ such as $II_s(s,l)$ and $III_s(s,l)$ are invertible, i.e. we can get $s(II_s, III_s)$, $l(II_s, III_s)$; s_1, s_2 are principal components of s, s means the arc length of a loading path and l distinguishes the loading paths within the family. Further supose that the principal components $p_1(s,l)$, $p_2(s,l)$ of p can be measured along these paths. Then (5) provides the system of two equations for c_1, c_2. If the system has for a range of s, l a solution $s_1(s,l)$, $s_2(s,l)$ the material functions are $c_i = c_i(s(II_s, III_s),\quad (II_s, III_s))$, $i = 1, 2$.

2.1.3 Modelling of a delay effect

Now a model which retains the lowest invariants of a and K is conceptually analyzed. If $\varphi = \varphi(II_a)$, $F = F(II_{sK}, II_K) \equiv II_{sK} - L^2(II_K)$ the relations (1), (2) yield

$$\dot{p} = \frac{H_2\,\mathrm{tr}\,(\dot{s}\,a)}{II_s + H_1}\,a\,, \qquad \dot{a} = \frac{\mathrm{tr}\,(\dot{s}\,a)}{II_s + H_1}\,(-s + H_3\,a) \tag{6}$$

if $II_{sK} = L^2$ and $\dot{p} = 0$, $\dot{a} = 0$ for $II_{sK} < L^2$; note that the yield condition $II_{sK} = L^2(II_K)$ is represented by a plane in the deviatoric stress space. The material functions $H_i = H_i(\bar{p})$, i = 1,2,3 are expressible through derivatives of φ and L, where $II_a = II_a(\bar{p})$ is utilized; $\bar{p}$ is accumulated plastic strain $\bar{p} = \int\sqrt{\mathrm{tr}\,\dot{p}^2}\,d\tau$. We get the relation $II_a = II_a(\bar{p})$ for fixed initial conditions as a solution of the differential equation of the type $dII_a/d\bar{p} = P(II_a)$ which follows from (1), (2) for the considered model.

From (6) we see that $\dot{p}$ and s need not to be coaxial. Moreover, within the power of the evolution equation for $\dot{a}$ in (6) is to simulate a „fading memory", i.e. the effect that a non-coaxiality of $\dot{p}$ and s caused by a sharp change of the direction of a loading path diminishes with continuation of small curvature loading. In this way some kinds of delay effects can be modelled.

At monotoneous loading modes L need not to be known and to specify fully the model (6) one has to determine only three material functions $H_i(\bar{p})$. The equation for $\dot{\underset{\sim}{p}}$ in (6) requires $\underset{\sim}{a}$ to be a tensor coaxial with $\dot{\underset{\sim}{p}}$. To determine fully for known $\underset{\sim}{p}$ the tensor $\underset{\sim}{a}$ it suffices to fix the value of II_a. Hence, if we choose a particular function $II_a(\bar{p})$, $\underset{\sim}{a} = \underset{\sim}{a}(\bar{p})$ is determined by $\dot{\underset{\sim}{p}}$ and $\bar{p}$ and $\underset{\sim}{a}$ can be, in principal, measured. (In [3] the model of a delay effect similar to (6) is analysed, but the condition imposed on $\underset{\sim}{a}$ has not an invariant form, moreover the variables II_{sa}, II_a are not independent; it can be shown that $II_{sa} = II_{sa}(\bar{p})$, $II_a = II_a(\bar{p})$ for this model.) If the principal components of $\underset{\sim}{s}$, $\dot{\underset{\sim}{s}}$, $\dot{\underset{\sim}{p}}$, $\underset{\sim}{a}$, $\dot{\underset{\sim}{a}}$ as functions of $\bar{p}$ are experimentally determined along a loading path where sudden changes of the loading direction is now permitted, the equations (6) for the principal components $\dot{p}_1$ (the equation for $\dot{p}_2$ gives equivalent information) $\dot{a}_1$, $\dot{a}_2$ represent the system for three unknown material functions $H_i(\bar{p})$. This identification procedure, which rests on measurement along the single loading path (similarly as for the classical model), hints that the adjusted model (6) is suitable for loading paths of shapes similar to the identification test. Moreover, as $\underset{\sim}{s}$ enters the model only through II_{sa} no higher order stress effects are covered by the model. A description of delay effects for a whole family of loading paths of various shapes can be achieved by utilizing a special case of the more flexible model $\varphi = \varphi(\alpha, II_a)$, $F = F(k, II_s, II_{sK}, II_K)$ (see the note at the end of 2.2.1) or theoretically also by the model 2.1.4.

2.1.4 Modelling of complex delay effects

To model delay effects in more details a model of higher flexibility is needed. As an example we consider in (3) $\varphi = \varphi(\alpha, II_a, III_a)$ and $F(k, II_s, II_K, II_{sK}, III_s) \equiv$ $\equiv f(\alpha, II_s, II_a, II_{sa}, III_s) = g(II_s, II_a, II_{sa}, III_s) - \alpha$. In terms of the principal components of $\underset{\sim}{s}$ and $\underset{\sim}{a}$, which substantionally simplify the notation (however, the representation in the principal components is equivalent to the invariant representation only if $\underset{\sim}{a}$ and $\underset{\sim}{s}$ can be simultaneously diagonalized), we get from (1), (2)

$$\dot{p}_i = (c_j \dot{s}_j)\, c_i \quad , \quad \dot{a}_i = (c_j \dot{s}_j)\, d_i \quad , \quad i = 1, 2 \; , \tag{7}$$

if $g = \alpha$, and $\dot{p}_i = 0$, $\dot{a}_i = 0$ if $g < \alpha$; c_i, d_i, $i = 1, 2$, are functions of s_1, s_2, a_1, a_2 and can be related to φ and g through the derivatives of g and φ .

For monotoneous loading modes it is sufficient to specify in (7) four material functions c_i, d_i. Practically that is, however, very difficult problem complicated further by the fact that we do not known what $\underset{\sim}{a}$ means and how to measure it. Encouraged by the interpretation of $\underset{\sim}{a}$ in 2.1.3 we try to proceed as follows. From two equations for $\dot{p}_1$, $\dot{p}_2$ in (7) the values of the functions c_1, c_2 can be easily expressed in terms of values of $\dot{p}_i$ and $\dot{s}_i$. If we choose $c_i(s_1, s_2, a_1, a_2) = a_i$ we get $a_i = \dot{p}_i / \sqrt{\dot{p}_j \dot{s}_j}$. The special choice of the functions c_i may restrict the flexibility of the model. However, it provides a convenient interpretation of $\underset{\sim}{a}$ (invalid for non-monotoneous loading) such that $\underset{\sim}{a}$ can be in principal measured and the model (7) may be at least conceptually identified.

As an identification test a family of loading paths with four variable characteristics has to be chosen (e.g. for a path consisting of two joined small curvature segments the characteristics are: the arc lenght s, the angle γ between the tangents of the segments at the joining point, the arc length of the initial segment s_o and the orientation Θ of the path in the stress space). Two equations for $\dot{a}_i$ in (7) with experimentaly determined $a_i = c_i$, $\dot{a}_i$ and s_i as functions of the characteristics s, γ, s_o, Θ, represent the system for two remaining unknown functions $d_i(s, \gamma, s_o, \Theta)$. An invertibility of s_1, s_2, a_1, a_2 in s, γ, s_o, Θ is now required.

2.2 Notes on the cyclic loading branch

Models convenient for description of plastic behavior in cyclic processes may be generated (at least for models of a lower flexibility) by replacing in spectrum A the stress invariant II_s (resp. III_s) by $II_{s-K} = II_s + II_K - 2\,II_{sK}$ (resp. III_{s-K}). It is very easy to show that for $\varphi = \varphi(II_a)$, $F = F(II_{s-K})$ the relations (1), (2) yield the classical model of kinematic hardening; we get $\dot{\underset{\sim}{a}} = \dot{\underset{\sim}{p}}$, hence, we may take $\underset{\sim}{a} = \underset{\sim}{p}$ and the tensor $\underset{\sim}{K}$, which determines the position of the yield surface $II_{s-K} = \text{const}$ in the stress space, is $\underset{\sim}{K} = \underset{\sim}{K}(II_p)$. The model which admits a more general but still rigid shape of the yield surface $g(II_{s-K}, III_{s-K}) = \text{const}$ is gained, if $\varphi = \varphi(II_a, III_a)$, $F = F(II_{s-K}, III_{s-K})$; from (1), (2) we get $\dot{\underset{\sim}{a}} = \dot{\underset{\sim}{p}}$, $\underset{\sim}{K}$ is then $\underset{\sim}{K} = \underset{\sim}{K}(II_p, III_p)$.

2.2.1 Discrete memory parameters

A more interesting and useful model arises, if $\varphi = \varphi(\alpha, II_a)$, $F = F(k, II_{s-K}) = II_{s-K} - L^2(k)$, from (1), (2) we have

$$\dot{\underset{\sim}{p}} = \dot{\underset{\sim}{a}} = 2\lambda(\underset{\sim}{s} - \underset{\sim}{k}) \quad , \qquad \dot{\alpha} = (2/3)\,\lambda\, L(dL/dk), \tag{8}$$

if $II_{s-K} = L^2$, and $\dot{\underset{\sim}{p}} = \dot{\underset{\sim}{a}} = 0$, $\dot{\alpha} = 0$ for $II_{s-K} < L^2$; λ is determined by $\dot{g} = 2L\dot{L}$. From (8) and $II_{s-K} = L^2$ we get $\dot{\alpha} = b(\alpha, IIp)\dot{\bar{p}}$, where $b = (2/3)\, dL/dk$, $k(\alpha, II_p) = \partial\varphi/\partial\alpha$ and $\bar{p}$ is accumulated plastic strain. As $\bar{p}$ represents the arc length of a plastic strain path, we have for a particular loading process $II_p(\bar{p})$, hence, $\dot{\alpha} = b(\alpha, II_p(\bar{p}))\,\dot{\bar{p}}$ yields a functional dependence of α on $\bar{p}$ (this is reduced to a function $\alpha(\bar{p})$, if k is independent of II_p, e.g. if $\varphi(\alpha, II_p) = \varphi_1(\alpha) + \varphi_2(II_p)$). An interesting situation arises, if the plastic strain paths can be parametrized, i.e. II_p is determined by $\bar{p}$ and parameters n_i. Then $\dot{\alpha} = \bar{b}(\alpha, \bar{p}, n_i)\dot{\bar{p}}$ may have solution $\alpha = \alpha(\bar{p}, n_i)$, where n_i are the discrete memory parameters intuitively introduced e.g. by [6, 7]. For a simple symmetric plastic strain controlled cycling the plastic strain amplitude suffices to parametrize $II_p(\bar{p})$. The model (8) then falls within Eisenberg s theory [6] and the idenfication technique from [6] may be utilized. Note also that in the case of monotoneous loading the characteristics of segmented paths may serve as the discrete memory parameters and then delay effects can be modelled by (8).

3. Outlook

The constitutive modelling outlined in the previous sections should be understood as a program rather than a tool ready for practical applications. Still at least three difficult problems have to be overcome.

3.1 Iteration procedure

The identification of a plastic model and the solution of an elastic-plastic stress analysis problem cannot be decouplet in general [8] . For adequate modelling the range of loadings paths has to be known. However, the loading paths are usually unknown unless the problem is solved, but to solve it an adequate model is needed.

To overcome this difficulty a following iteration procedure can be suggested.

Using first the classical model of isotropic (or kinematic) hardening adjusted from stress-strain diagram the stress analysis problem has to be solved and the loading paths family determined. If only loading paths of a small curvature occur a difference in the material function of the classical model for two loading paths which are most appart has to be determined. When the inaccuracy in the solution caused by this difference cannot be tolerated the model 2.1.2 has to be adjusted and used. If sharp changes of loading directions occur, the intensity of delay or Baushinger effects has to be estimated and a model which is able to incorporate this effect has to be adjusted and employed. The convergence of the iteration procedure means that the employed model covers the family of loading paths which occur in the solution of the stress analysis problem. An inaccuracy of the solution caused by the scatter in the material functions of the used model has to be within required limits. The scatter arises due to errors of the identification tests and the remaining inadequacy of the model to describe precisely the plastic response for the whole family of the loading paths of the problem.

3.2 Inhomogeneous identification tests

The identification tests are basically of two types. The classical approach is based on homogeneous tests. In section 2 we have seen that a large number of such tests would be needed even for identification of the models of a modest flexibility. Another, perhaps more economical, possibility is to utilize inhomogeneous identification tests each of which carries far more information. For a wide class of materials one can determine the stress field in an inhomogeneously deformed body from given boundary conditions and experimentally determined strain field without knowing the constitutive equations. Each point of the inhomogeneously deformed body then gives information as an individual homogeneous identification test.

Consider a class of materials for which always the principal directions of the stress $\underset{\sim}{\sigma}$ coincide with the principal directions of the strain $\underset{\sim}{\varepsilon}$ (e.g. the models described here fall within this class). Then at each point of such body stress $\underset{\sim}{\sigma}$ may be expressed as $\underset{\sim}{\sigma} = \underset{\sim}{Q}\, \underset{\sim}{\sigma}_D\, \underset{\sim}{Q}^T$, where $\underset{\sim}{\sigma}_D$ is the diagonal tensor of the principal values of stress and $\underset{\sim}{Q}$ is the orthogonal transformation which diagonalizes strain $\underset{\sim}{\varepsilon}$. The stress $\underset{\sim}{\sigma}$ has to satisfy the equilibrium conditions. In the practically important two-dimensional case the equilibrium equations reduce to a system of two hyperbolic equations for two unknown principal stress components which can be solved by integration along characteristics [9] .

3.3 Regularization

The described identification procedure has to be understood as a brief conceptual outline of types of needed identification tests and inverse problems. In an actual identification we face a crucial question of the stability of the solution of the inverse problems with respect to perturbations in experimental input data. There are indications that the instability we encounter is of a general nature and some regularization technique has to be employed. For internal variables models of plasticity a convenient way of regularization has been described by Pister [8] .

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