A MODEL OF QUASI-STATIC CRACK GROWTH IN A DOUBLE CANTILEVER BEAM AT ELEVATED TEMPERATURE

L.S. FU

Ohio State University, Department of Engineering Mechanics, Columbus, Ohio 43210, U.S.A.

In this paper, the phenomenological theory of brittle creep rupture given by Kachanov and later generalized by Rabotnov is used to describe crack growth rates in a cracked body under steady state conditions. The quasi-static time-dependent crack growth is studied by using a double cantilever beam of arm length a, ligament length c, thickness b and height 2h. The material is assumed to follow Norton's creep law and basic material information from tension, creep, and creep rupture tests are required.

The beam on elastic foundation solution is used and the damage rate, $\dot{\psi}$, is found to be governed by a differential integral equation. Knowing that $\psi(x,0) = 1$, a numerical scheme is used for integration and for determining the crack growth rates. Predictions by this model at 1200°F (649°C) are presented. Results indicate that the stress intensity factor K is not a constant at initiation of crack growth and is not necessarily a good parameter for correlating quasi-static time-dependent crack growth under sustained load at elevated temperature in nickle-base superalloys.

1. Introduction

Requirements of high performance and long service life have led to increased attention in the design and life prediction of structures for energy systems and modern jet engines that enter military and commercial services. Prediction of life from extrapolating low cycle fatigue (LCF) data generated in laboratories has thus become of great concern as the traditional life prediction techniques have produced conservative underestimates of total useful life leading to early retirement of components such as heat exchangers, engine disks, etc.

At elevated temperature, fatigue crack growth (FCG) rates in technical alloys, e.g. Inconel 718, SS 316, Ni-alloy, 1 CR - Mo - V steel, are found to be frequency dependent, a phenomenon that is normally absent at ambient temperature. As frequency decreases, there is a time-dependent effect on FCG rates. Depending upon the specific alloy system under consideration the time-dependence of FCG rates may come from oxidation damage, creep, or a combination of both. Recently, the fracture mechanics approach has been investigated for predicting accurate total LCF life at elevated temperature. More detailed information can be found in a recent review. [1].

The quasi-static crack growth under sustained load at elevated temperature, also termed creep crack growth (CCG), is therefore an integral part of the determination of FCG rates. This paper takes this view and presents a model for the characterization of the CCG rates and times to fail under sustained load at elevated temperature.

It is assumed that creep takes place everywhere and crack slowly advances under sustained load when local rupture occurs. The local rupture is defined in terms of the damage parameter of Kachanov [2].

Crack growth rates and times to fail in a double cantilever beam (DCB) is studied in detail. A numerical procedure is used in determining the damage parameter as a point function of time. Basic material information from tension, creep, and creep rupture tests are required. Computations for a nickle-base superalloy at 1200°F are given.

2. Formulation

The phenomenological theory of brittle creep rupture given by Kachanov [2] and later generalized by Rabotnov [3] is used to describe crack growth rates and times to fail in a cracked body under steady state creep conditions. It is assumed that creep takes place everywhere in the solid and higher creep rates at the crack tip lead to local concentration of the creep strain. Crack slowly advances when local rupture occurs.

It is well-known that metals deteriorate at elevated temperature under creep conditions. Usually, voids develop in the metals at points of maximum tension thereby reducing the effective area across which tensile force is transmitted. Such a phenomenon is photographically displayed by Soderquist [4]. It is noted that the formation of cracks is always in the direction perpendicular to maximum tension.

Kachanov proposed a description of the brittle creep rupture by introducing the concept of continuity ψ, $(0 \leq \psi \leq 1)$, also called damage function. The deterioration is defined by $(1 - \psi)$. For the virgin state of the material the function is equal to one, $\psi = 1$, and it decreases to zero, $\psi = 0$, at total separation of material, or rupture. This concept of continuity was later generalized by Rabotnov to take into account of the coupling effect of deterioration and creep.

Based upon Kachanov and Rabotnov's theories of creep and creep rupture, the damage rate $\dot{\psi}$ can be written as

$$\dot{\psi} = -Ag(\sigma_1,\psi) \tag{1}$$

where

$$g = [\sigma_1/\psi\sigma_o]^\nu \tag{2}$$

in which σ_1 is maximum principal stress, A and ν are material constants obtained from creep rupture tests, and σ_0 is a reference stress, say yield stress. The constant ν is in general less than the creep exponent and is equal to the inverse slope of a conventional log-log plot of stress against creep rupture life.

Consider now the quasi-static but frequency or time-dependent crack growth in a double cantilever beam of arm length a and ligament length c, thickness b and height 2h, Fig. 1. Under steady state creep conditions, the nickel-base superalloys are found to follow Norton's creep law [5]

$$\dot{\varepsilon} = \alpha\sigma^n \tag{3}$$

where α and n are creep constants.

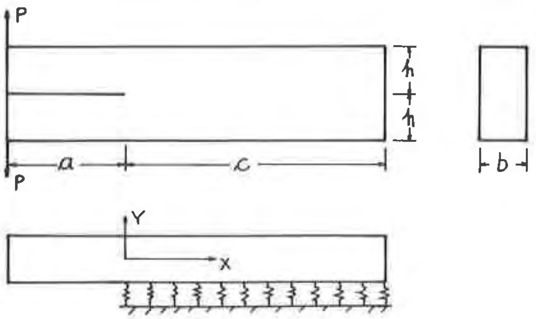

Fig. 1 DCB specimen

TABLE I

Data Used in Example

b	0.469 in.
h	1.2 in.
n	3.77
E	27×10^3 ksi
$\sigma_o = \sigma_{yp}$	160 ksi
ν	3.182
a_i	0.65 in.
c_i	(a) 1.35 in. (b) 3.35 in
P	(A) 4300 lb, (B) 2900 lb.

For the purpose of demonstrating the use of the model of material deterioration and creep in predicting crack growth, the DCB may be represented by a beam on elastic foundation [6]. Due to symmetry with respect to the mid-plane of the DCB, y = h/2, the principal stresses on that plane are σ_{yy}, 0, and σ_{xx}. The principal strain rates are given by

$$\dot{\varepsilon}_{yy} = -\dot{\varepsilon}_{xx} \tag{4}$$

where the dot denotes differentiation with time, t, and

$$\dot{\varepsilon}_{yy} = \dot{w}(x)/h$$

to simplify calculations, the relation defined as

$$\dot{\varepsilon}_{yy} = \dot{\varepsilon}_{yy}\,(0)\; w(x)/w(0)$$

is used.

Realizing that the y-direction is the first principal direction and employing equations (3), (4), and the equilibrium condition, after some algebraic manipulations, we obtain the governing equations

$$\partial\psi/\partial\tau = -[W(x)/W(0)]^{\nu/n} \left\{ \int_0^c \psi[W(x)/W(0)]^{1/n} dx/h \right\}^{-\nu} \tag{6}$$

for the damage rate and

$$\dot{\varepsilon}(0)/\varepsilon_0 = (P/bh\sigma_0) \left\{ \int_0^c \psi[W(x)/W(0)]^{1/n} dx/h \right\}^{-1} \tag{7}$$

for the strain rate at x = 0, and

$$\sigma_{yy}/\sigma_0 = \psi[W(x)/W(0)]^{1/n}\,[P/bh\sigma_0]\; x \left\{ \int_0^c [W(x)/W(0)]^{1/n} dx/h \right\} \tag{8}$$

for the normal stress σ_{yy} on the plane y = h/2, where

$$\tau = (P/bh\sigma_0)^{\nu}\,(\sigma_0/\sigma_k)\,(t/T_k) \tag{9}$$

Eq. (6) may simplified in the following manner. Let

$$\phi = \int_0^c \psi[W(x)/W(0)]^{1/n} dx/h \tag{10}$$

For fixed a and c, W(x) and W(0) do not change with respect to time and we have

$$d\phi/d\tau = \int_0^c \partial\psi/\partial\tau\,[W(x)/W(0)]^{1/n} dx/h \tag{11}$$

Substituting $\partial\psi/\partial\tau$ from equation (6), we obtain, after simple manipulation,

$$\phi^{\nu+1}\Big]_{\tau} = \phi^{\nu+1}\Big]_{\tau=0} - \tau \int_0^c [W(x)/W(0)]^{\nu/n} dx/h \tag{12}$$

Noting that at $\phi = 0$, $\psi = 1$, we have

$$\phi\Big]_{\tau=0} = \int_0^c [W(x)/W(0)]^{1/n} dx/h \tag{13}$$

$$\phi^{\nu+1} = \int_0^c [W(x)/W(0)]^{1/n} dx/h - \tau \int_0^c [W(x)/W(0)]^{\nu/n} dx/h \tag{14}$$

and, finally,

$$\partial\psi/\partial\tau = -[W(x)/W(0)]^{\nu/n}$$

$$\times \left\{ \int_0^c [W(x)/W(0)]dx/h - \tau \int_0^c [W(x)/W(0)]^{\nu/n} dx/h \right\}^{\nu/\nu+1} \qquad (15)$$

The above equation can be integrated numerically for any given W. A numerical scheme in integrating equation (6) is given next and used to determine $\partial\psi/\partial\tau$ and $\psi(x,\tau)$. A scheme for the prediction of crack growth rate based upon the concept of continuity is also obtained.

The above analysis is now used for crack growth characterization. It is observed that the damage function is a point function that depends on time, i.e. $\psi = \psi(x,\tau)$. Initially, at $\tau = 0$, $\psi = 1$ at all x. As time marches on, ψ decreases and the rate of decrease $\dot{\psi} = \dot{\psi}(x,\tau)$ is the largest at $x = 0$. The first point to reach $\psi = 0$, total damage or total separation of bonds, is the point at $x = 0$. As the bonds are broken, the crack length a increases while the ligament length c decreases and the crack velocity is $\dot{a} = da/dt$. We therefore observe that both a and c are functions of time.

Due to the difficulty in obtaining an analytic solution for $\psi(x,\tau)$ from the differential integral equation, equation (6), a scheme that determines the characterization of crack in a discrete manner is used. This scheme is described as follows:

Step 1. $\tau = 0$, $\psi = 1$ at all x

$$\dot{\psi}(x) = -[W(x)/W(0)]^{\nu/n} \Big/ \int_0^c \left\{ [W(x)/W(0)]^{1/n} dx/h \right\}^{\nu}$$

Step 2. determine $\psi(x,\tau+\Delta\tau)$ by

$$\psi(x,\tau+\Delta\tau) = \psi(x,\tau) + \dot{\psi}(x,\tau)\,\Delta\tau$$

Step 3. Determine $\dot{\psi}(x,\tau+\Delta\tau)$ from equation (6) by using $\psi(x,\tau+\Delta\tau)$. from step 2.

Step 4. When ψ at any step is found to be zero at certain nodes, a and c are changed to new values and coordinate origin is moved accordingly.

Step 5. τ_i = time needed for initial crack growth, or incubation time.

Step 6. Crack velocity is $\dot{a}(\tau) = \Delta a/\Delta\tau$

Step 7. τ_f = time interval when crack velocity approaches infinity, or time to fail.

Now that ψ is determined for all time, the strain rate at $x = 0$ and the stress σ_{yy}, the deflection and the stress intensity factor K can be obtained as functions of time. It is assumed that damage occurs only under tension situation.

3. Results

Crack growth rates as predicted by the above analysis is studied in detail by using the suggested numerical scheme. Material constants, specimen dimensions and loading conditions used are as those given in Table I. These are related to a nickle-base superalloy at 1200°F.

The solution for deflection of a beam on elastic foundation obtained by Kanninnen [6], is used in Eq. (6) to obtain $\psi(x,\tau)$. Results for two specific geometries are given in Tables II to V. Computer program is available upon request.

4. Discussion

The calculation results indicate that the crack growth under sustained load at elevated temperature is characterized by an incubation period, and a period of slow stable crack growth terminated by fast crack propagation. This is in general agreement with experimental observation of crack growth behavior in an advanced nickle-base superalloy. The incubation time and the crack growth rate can be determined.

From the predictions, it's observed that

(1) the incubation time increases with decreasing applied loading.

(2) creep crack growth (CCG) rates are first very high, they then decrease and finally increase very fast.

(3) the inverse of the slope of the creep rupture curve ν and the creep exponent n play important roles in the determination of incubation time and crack growth rate.

(4) CCG rates appear to correlate with K by a power law in the slow stable growth region, the coefficient and exponent, however, are stress dependent, suggesting that K is not a good parameter in correlating CCG rates.

(5) the model can be improved by seeking a solution where the creep singularity is determined.

(6) the DCB model gives a value of stress intensity factor K that is proportional to the totel strain at crack tip.

(7) further computations can be obtained with necessary material properties given.

(8) the value of ν is needed for accurate predictions.

(9) to compare with experimental results, nonlinear solutions for the pertinent specimen geometry must be obtained.

5. Conclusions and Suggested Future Research

(1) The stress intensity factor K is not a constant at initiation of crack growth and is not a good parameter to use in correlating CCG rate.

(2) The creep rupture theory can be used to predict CCG rates as well as determining which of fracture criterion should be used for predicting CCG.

(3) Material information from uniaxial tension test, creep test and creep rupture at temperature are needed.

(4) Crack blunting should probably be studied to model multiple crack initiation and coalescence.

References

1. L.S. Fu, Quasi-Static Crack Growth In Metals at Elevated Temperature - A Review, Tech. Report, AFML-TR-78-136, Air Force Materials Laboratory, (1978).

2. L.M. Kachanov, Izv. Akad. Nauk. USSR, Nr 8, 26 (1958) (in Russian); see also Problems of Continuum Mechanics, p. 202, SIAM, Philadelphia, (1961).

3. Y.N. Rabotnov, Joint International Conf. on Creep, paper 68, New York, Aut; London, Sept./ Oct. (1963).

4. B. Soderquist, Roy. Inst Tech. Stockholm, doctoral dissertation (1968); see also Acta Polytech. Scand. Phys. ser. Nr 53 (1968).

5. A.E. Gamma, Hold-time Effect of a Single Overload on Crack Retardation at Elevated Temperature, Engineering Fracture Mechanics, in print.
6. M.F. Kanninen, An Augmented Double Cantilever Beam Model for Studying Crack Propagation and Arrest, Int. J. Fracture, Vol. 9, No. 1, (1973), 83-92.

TABLE II

τ	c	a	da/dτ	K x10^5	$\dot{\varepsilon}(0)$ x10^-1
0.	1.35	0.65	0	.3391	.5028
0.4		.65	0	.3391	.5028
0.405		.655	0.9983	.3414	.7634
0.41		.6838	5.7592	.3554	.7730
0.42		.7395	5.5727	.3852	.7953
0.44		.8445	5.2517	.4540	.8386
.46		.9533	5.4384	.5496	.8998
.48		1.0683	5.7494	.6918	.9865
.50		1.1917	6.1723	.9221	1.120
.52		1.3252	6.6704	1.336	1.358
.54		1.4939	8.4352	2.4223	1.900
.56		1.6963	10.1217	6.9414	3.390
.58		1.8482	7.5927	28.4851	8.147
.60	.1440	1.8560	0.3922	31.7153	8.456
.62		1.8560	0.303×10^{-3}	31.7180	8.456
.64		1.8560	$.76 \times 10^{-6}$	31.7180	8.456
.74		1.8560	$.3863 \times 10^{-9}$	31.7180	8.456

a+c = 2.0 in., p = 3400#

TABLE III

τ	c	a	da/dτ	K x10^5	$\dot{\varepsilon}(0)$ x10^-1
0	3.35	0.6500	0	0.3426	.3328
2.2		.6500	0	.3426	.3328
3.2		.6587	$.869 \times 10^{-2}$	.3447	.4086
3.22		.6061	.3719	.3465	.40864
3.24		.6735	.3690	.3483	.4087
3.26		.6808	.3664	.3501	.4087
3.30		.6952	.3600	.3536	.40877
3.36		.7163	.3517	.3587	.4089
3.42		.7370	.3447	.3637	.4091
3.46		.7507	.3413	.3670	.4093
3.50		.7642	.3373	.3703	.4095
3.55		.7808	.3319	.3743	.4098
3.60		.7971	.3274	.3783	.4100
3.70		.8298	.3165	.3859	.4107
3.80		.8597	.3091	.3934	.4115
3.90		.8899	.3025	.4007	.4124
4.10		.9471	.2858	.4146	.4144
4.30	2.9976	1.0024	.2764	.4280	.4168

a+c = 4.0 in., p = 4300#

TABLE IV

τ	c	a	da/dτ	K x10^5	$\dot{\varepsilon}(0)$ x10^{-1}
0.	3.35	0.65	0	.2709	.2631
2.2		0.65	0	.2709	.2631
3.2		0.6587	.0087	.2726	.3231
3.22		0.6661	.3719	.2740	.32114
3.24		0.6735	.369	.2754	.323164
3.26		0.6808	.3664	.2768	.3231516
3.30		0.6952	.3600	.2796	.3232
3.36		0.7163	.3517	.2836	.3233
3.42		.7370	.3447	.2876	.3235
3.46		.7507	.3413	.3413	.3236
3.5		.7642	.3373	.3373	.3238
3.55		.7808	.3319	.3319	.3240
3.60		.7971	.3274	.3274	.3242
3.7		.8288	.3165	.3165	.3247
3.8		.8597	.3091	.3091	.3254
3.9		.8899	.3025	.3025	.3261
4.1		.9471	.2858	.2858	.3277
4.3	2.9976	1.0024	.2764	.2764	.3296

a+c = 4.0 in., p = 3400 lb.

TABLE V

τ	c	a	da/dτ	K x10^5	$\dot{\varepsilon}(0)$ x10^{-1}
0.00	1.35	0.65	0	.4288	.6340
0.4		0.65	0	.4288	.6340
0.405		0.655	0.9983	.4317	0.966
0.41		0.6838	5.7592	.4495	.9776
0.42		0.7395	5.5727	.4872	1.003
0.44		0.8445	5.2517	.5740	1.06063
0.46		0.9533	5.4384	.6950	1.138
0.48		1.0683	5.7494	.87497	1.2477
0.50		1.1917	6.1723	1.166	1.416
0.52		1.3252	6.6704	1.689	1.718
0.54		1.4939	8.4352	3.06	2.4023
0.56		1.6963	10.1217	8.779	4.287
0.58		1.8482	7.5927	36.03	10.303
0.60	0.1440	1.8560	0.3922	40.11	10.7
0.62	0.14499	1.8560	$.3030\times10^{-3}$	40.114	10.7
0.64		1.85601	$.76\times10^{-6}$	40.114	10.7
0.74		1.85601	$.3863\times10^{-9}$	40.114	10.7

a+c = 2.0 in., p = 4300#