CONSTITUTIVE EQUATIONS OF VISCOPLASTICITY FOR NEUTRON IRRADIATED MILD STEEL

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Abstract

The strength analysis of reactor pressure vessel under prolonged irradiation is of principal concern of designers in the reactor technology. Therefore the suitable description of a behaviour of the neutron irradiated carbon steel is needed. The main effect of neutron bombardment damage in this material is an increase of yield strength and decrease of ductility. The sensitivity of radiation hardening to temperature and strain rate has been also detected experimentally. The aim of the paper therefore is to propose the constitutive equations of viscoplasticity for neutron irradiated mild steel in the wide range of the temperature /0 ÷ 700/°K and the strain rate $/5 \times 10^{-5} \div 5 \times 10^{5}\ /\ s^{-1}$.

On the basis of an analysis of the irradiation defects their interaction with dislocations and physical models of plastic deformation the mechanisms of radiation hardening are identified. The irradiation of mild steel causes the athermal radiation hardening in the ranges of the temperature and the strain rate corresponding to the mechanism of thermal activations for the virgin material. On the other hand, the thermally activated radiation hardening occurs in the ranges of the temperature and the strain rate pertaining to the athermal mechanisms before irradiation takes place.

The suitable equations of the one-dimensional macroscopic model concerning the polycrystalline material are derived on the basis of the physical theory of thermally activated dislocation motion in single slip system proposed by Seeger and Fleischer as well as the Taylor's averaging method of shear stresses in monocrystal combined with the Hall-Petch relation and generalized Orowan's relation. Then, the constitutive equations for complex state of stress are obtained. The effects of irradiation are described directly with use of the dose of irradiation.

Possible simplifications and approximations from the point of view of applications and specification of material functions are taken into consideration. The proposed equations are capable to predict the radiation hardening and saturation effect,as well.

1. Introduction

In the theory of viscoplasticity of irradiated materials the question arises what is the influence of neutron irradiation on the mechanisms of plastic deformation in an entire spectrum of strain rate and temperature, $\dot{\varepsilon}^p \in (10^{-5} \div 10^5) s^{-1}$, $\vartheta \in (0 \div 700)\,^\circ K$ (cf. PERZYNA [12]). Some results concerning this problem were obtained by PĘCHERSKI [13], [14] on the basis of available experimental data and the analysis of radiation hardening mechanisms.

The main effect of fast neutron irradiation is the production of vacancies and interstitials. Depending on the temperature they may freely migrate to grain boundaries and dislocations, anihitate, join in clusters and form complexes with impurities. The mechanism of radiation hardening in b.c.c. metals is very complicated and depends on many factors. The radiation defects may interact with dislocations as long range barriers or short range barriers. It depends on the test temperature, the strain rate, the content of impurities as well as the dose and the temperature of irradiation.

The important case, from practical point of view is when the material is irradiated in the temperatures $\vartheta_{irr} \in (313 - 573)\,^\circ K$ to the dose $\varphi \in (10^{17} \div 10) \frac{n}{cm^2}$. In such a case irradiation produces the complexes as the dislocation obstacles.

2. Mechanisms of radiation hardening in mild steel

The main experimental results are combined in Fig.1 *. The dashed line delimits four regions which reflect different mechanisms of plastic deformation before irradiation (cf. PĘCHERSKI [13], [14]).

The Region I is characteristic for weak dependence of the lower yield strength on the temperature and the strain rate. It is due to the athermal mechanism. The Regions II and IV correspond to the mechanisms of thermal activations and viscous drag respectively.

On the other hand, in the Region III plastic glide and twinning occur but the mechanisms of cleavage play prevalent role.

The mechanisms of radiation hardening are also marked in Fig.1. The uninterrupted and dashed line correspond to the athermal and the thermally activated radiation hardening respectively. Up to now, a suitable experimental results concerning the Region IV are not available. However, we may suppose that the athermal radiation hardening will be predominant here.

From the analysis of the above mentioned experimental results following observation arises (cf. PĘCHERSKI [13], [14]):

The irradiation of mild steel causes the athermal radiation hardening in the ranges of the temperature and the strain rate corresponding to the mechanism of thermal activations for the virgin material. On the other hand,

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* In the paper of STEICHEN, WILLIAMS [16] the influence of irradiation on the upper yield strength was discussed only. However, it is possible using all their tabulated results to analyse the influence of irradiation basing on the lower yield strength, too.

the thermally activated radiation hardening occurs in the ranges of the temperature and the strain rate pertaining to the athermal mechanisms before irradiation takes place.

In higher temperatures the mechanisms of radiation hardening are limited by the process of annealing. In low temperatures the Region II is limited by the ductile-brittle transition temperature which inereases with irradiation.

In Fig.2 the regions of the plastic deformation mechanisms after irradiation are shown. We may observe that irradiation causes an extension of the Region III towards higher temperatures and generates the new thermally activated mechanism in the Region IIb. The Regions IIa and IV pertain to the athermal radiation hardening.

According to the PERZYNA's [12] concept the discussed deformation mechanisms spectrum makes a physical basis for the prediction of macroscopic behaviour of irradiated mild steel in an entire region of the strain rate and the temperature.

3. Physical model of radiation hardening

The identification of the mechanisms of radiation hardening as well as the physical theory of thermally activated dislocation motion and surmounting of obstacles in single slip system, proposed by SEEGER [15], FLEISCHER [5] and FRANK [6] (cf. also e.g. KUO, ARSENAULT [8] and THOMPSON 17) make a physical basis of constitutive equations of viscoplasticity for neutron irradiated mild steel.

The relation between the critical shear stress τ and the velocity of dislocation v in single slip system for the case of athermal radiation hardening corresponding to the Region IIb is given by the equation (cf. PĘCHERSKI [14]) :

$$v = A\vartheta_0 \exp\left\{-\frac{U\left[a\,\tau_A(\varphi)\left(\frac{\tau}{\tau_A(\varphi)}-1\right)\right]}{k\vartheta}\right\} \qquad (3.1)$$

where U is the activation energy, A is the activation area, ϑ_0 denotes the frequency coefficient, a is a structural constant, $\tau_A(\varphi)$ is the athermal stress, k is the Boltzman's constant and ϑ is the absolute temperature.

In the case of thermally activated radiation hardening corresponding to the Region IIa the relation $v-\tau$ is given by the following equation (cf. PĘCHERSKI [14]) :

$$v = v_0(\varphi)\exp\left\{\frac{-U\left[A^*(\varphi)\left(\frac{\tau}{\tau_A}-1\right)\right]}{k\vartheta}\right\} \qquad (3.2)$$

$$v_0(\varphi) = \begin{cases} \dfrac{\vartheta_0}{N_z(\varphi)} \\ \dfrac{\vartheta_0}{\sqrt{2x_0\,\varrho_b(\varphi)}} \end{cases}, \quad A^*(\varphi) = \begin{cases} \tau_A\left(\dfrac{4x_0 b}{U_0}\right)\left[\dfrac{Gb}{N_z(\varphi)}\right]^{1/2} & \text{Seeger's model} \\ \tau_A\big/\beta G b\sqrt{2x_0\,\varrho_b(\varphi)} & \text{Fleischer's model} \end{cases} \qquad (3.3)$$

The quantities $N_z(\varphi)$, $\rho_b(\varphi)$ denote the number of radiation obstacles per unit area and unit volume, respectively. The constants U_o, x_o characterize the radiation obstacle, G denotes the shear modulus and b is the length of the Burgers vector.

Let us remark that the only quantity dependent on the dose φ in the eq.(3.1) is the athermal stress τ_A, whereas in the eq. (3.2) irradiation affects the preexponential term ν_o and influences the activation energy U changing the excess stress multiplier A^*. The generalized Orowan's equation (cf. KRÖNER, TEODOSIU[7]:

$$\dot{\underset{\sim}{\varepsilon}}^P = \sum_{i=1}^{s} \rho_M^{(i)} b^{(i)} v^{(i)} \underline{g}^{(i)} \otimes \underline{n}^{(i)} \tag{3.4}$$

provides the averaging of microscopic quantities v and τ for all active glide systems in single crystal, where $\dot{\underset{\sim}{\varepsilon}}^P$ denotes the plastic strain rate tensor, $\rho_M^{(i)}$, $b^{(i)}$ are the density of mobile dislocations, the length of Burgers vector, respectively, and $v^{(i)}$ is the velocity of dislocation in the „i"th slip system defined by the normal $\underline{n}^{(i)}$ to the slip plane and the versor of slip direction $\underline{g}^{(i)}$.

From the eqs. (3.1), (3.4) the following relation corresponding to the Region IIb is obtained:

$$\dot{\underset{\sim}{\varepsilon}}^P = \sum_{i=1}^{s} \rho_M^{(i)} b^{(i)} A^{(i)} \nu_o^{(i)} \exp\left\{\frac{-U\left[\alpha \tau_A^{(i)}(\varphi)\left(\frac{\tau}{\tau_A^{(i)}(\varphi)} - 1\right)\right]}{k\vartheta}\right\} \underline{g}^{(i)} \otimes \underline{n}^{(i)} \tag{3.5}$$

If we assume that the critical shear stress is the same in each slip system the relation for the uniaxial tension in the direction $\underline{e}$ takes the form:

$$\dot{\varepsilon}^P = \gamma_1^* \exp\left\{\frac{-U\left[\alpha \tau_A(\varphi)\left(\frac{\tau}{\tau_A(\varphi)} - 1\right)\right]}{k\vartheta}\right\} \tag{3.6}$$

$$\dot{\varepsilon}^P = \underline{e}\,\dot{\underset{\sim}{\varepsilon}}^P \underline{e}, \quad M^{(i)} = \left(\underline{e}\cdot\underline{g}^{(i)}\right)\left(\underline{n}^{(i)}\cdot\underline{e}\right), \quad \gamma_1^* = \sum_{i=1}^{s} \rho_M^{(i)} M^{(i)} b^{(i)} \nu_o^{(i)} A^{(i)} \tag{3.7}$$

Similarly for the Region IIa the suitable equations are as follows:

$$\dot{\varepsilon}^P = \gamma_2^*(\varphi) \exp\left\{\frac{-U\left[A^*(\varphi)\left(\frac{\tau}{\tau_A} - 1\right)\right]}{k\vartheta}\right\}, \quad \gamma_2^*(\varphi) = \begin{cases} \dfrac{b}{N_z(\varphi)} \displaystyle\sum_{i=1}^{s} \rho_M^{(i)} M^{(i)} \nu_o^{(i)} \\ \dfrac{b}{\sqrt{2x_o \rho_b(\varphi)}} \displaystyle\sum_{i=1}^{s} \rho_M^{(i)} M^{(i)} \nu_o^{(i)} \end{cases} \tag{3.8}$$

The Hall-Petch relation:

$$\sigma_{LY} = \sigma_i + k_y d^{-1/2} \tag{3.9}$$

and Taylors averaging formula:

$$\sigma_i = m\left(\tau^* + \tau_A\right) \tag{3.10}$$

are used to obtain the transition from the single crystal description to the relations for polycrystalline material. Thus from the eqs. (3.4), (3.7), (3.8), (3.9), (3.10) the following relations were obtained:

$$\dot{\varepsilon}^P = \gamma_1^* \exp\left\{\frac{-U\left[\frac{\alpha}{m} Y(\varphi)\left(\frac{\sigma}{Y(\varphi)} - 1\right)\right]}{k\vartheta}\right\}, \quad \dot{\varepsilon}^P = \gamma_2^*(\varphi) \exp\left\{\frac{-U\left[\frac{Y}{m} A^*(\varphi)\left(\frac{\sigma}{Y} - 1\right)\right]}{k\vartheta}\right\} \tag{3.11}$$

for the Region IIa and the Region IIb, respectively, where Y denotes the quasi-static yield strength.

4. Constitutive equations

Having practical aspects in mind as well as the possibility of experimental determination of the material functions we may replace the microscopic function $\exp\left[-\frac{U(\cdot)}{k\vartheta}\right]$ by the empirical excess stress function $\Phi(\cdot)$ (cf. PERZYNA [11]). We can also introduce macroscopic generalization of the microscopic quantities γ_1^*, $\gamma_2^*(\varphi)$

$\alpha, A^*(\varphi)$ and assume that the quasi-static yield strength Y depends on the plastic strain ε^p, the temperature ϑ and the dose φ :

$$Y = f_1(\varepsilon^p, \vartheta, \varphi) \quad , \quad Y = f_2(\varepsilon^p, \vartheta) \tag{4.1}$$

for the Region IIa and the Region IIb, respectively.
Finally, the one-dimensional macroscopic model of radiation hardening may be obtained (cf.PĘCHERSKI [14]).

The derived physical model and the above mentioned macroscopic one are capable to predict the radiation hardening and to determine the dependence of material functions on the irradiation. Other aspects of viscoplastic behaviour of irradiated material should be analyzed in the framework of the general thermodynamic theory of viscoplasticity with internal parameters (cf.PERZYNA [11], [12]). However, for the sake of simplicity, we shall concentrate on the special case pertaining to the mechanical theory of infinitesimal strains with temperature and radiation effects.

Let us assume the isotropic model of work-hardening and radiation hardening with quasi-static Huber-Mises yield condition:

$$\sqrt{J_2} = \varkappa(W^p, \vartheta, \varphi) \; ; \; W^p = \int_0^t \underset{\sim}{\sigma} \cdot \dot{\underset{\sim}{\varepsilon}}\, dt \; , \; J_2 = \tfrac{1}{2} \underset{\sim}{s}^2 \; , \; \underset{\sim}{s} = \underset{\sim}{\sigma} - \tfrac{1}{3} tr \underset{\sim}{\sigma}\, \underset{\sim}{1} \tag{4.3}$$

The material is considered as incompressible in the inelastic range. Assuming that:

$$\dot{\underset{\sim}{\varepsilon}} = \dot{\underset{\sim}{\varepsilon}}^e + \dot{\underset{\sim}{\varepsilon}}^p \tag{4.4}$$

the constitutive equations of elastoviscoplastic irradiated material pertaining the Region IIa take the form:

$$\dot{\underset{\sim}{e}} = \frac{1}{2\mu} \dot{\underset{\sim}{s}} + \gamma_1 \left\langle \Phi\left[\mathcal{A}_1'(W^p, \vartheta, \varphi) \left(\frac{\sqrt{J_2}}{\varkappa_1(W^p, \vartheta, \varphi)} - 1 \right) \right] \right\rangle \frac{\underset{\sim}{s}}{\sqrt{J_2}} \tag{4.5}$$

$$tr\, \dot{\underset{\sim}{\varepsilon}} = \frac{1}{3K} tr\, \dot{\underset{\sim}{\sigma}} + \alpha \dot{\vartheta} \tag{4.6}$$

$$\langle \Phi(\cdot) \rangle = \begin{cases} \Phi(\cdot) & \text{if } \sqrt{J_2} > \varkappa_1(\cdot) \\ 0 & \text{if } \sqrt{J_2} \leq \varkappa_1(\cdot) \end{cases} \; ; \; \dot{\underset{\sim}{e}} = \dot{\underset{\sim}{\varepsilon}} - \tfrac{1}{3} tr\, \dot{\underset{\sim}{\varepsilon}}\, \underset{\sim}{1} \tag{4.7}$$

where μ, K are elastic constants and α is the thermal expansion coefficient.

Under the assumption that:

$$\mathcal{A}_1'(W^p, \vartheta, \varphi) = \frac{\varkappa_1(W^p, \vartheta, \varphi)}{A_1(W^p, \vartheta)} \quad , \quad A_1(W^p, \vartheta) > 0 \tag{4.8}$$

$$\varkappa_1(W^p, \vartheta, \varphi) = k_0(W^p, \vartheta) + k_1(\varphi) \quad , \quad k_1(\varphi) = \frac{1}{\sqrt{3}} \Delta Y_1(\varphi) \tag{4.9}$$

the following relation for the dynamic yield condition takes the form:

$$\sqrt{J_2} = k_0(W^p, \vartheta) + \frac{1}{\sqrt{3}} \Delta Y_1(\varphi) + A_1(W^p, \vartheta)\, \Phi^{-1}\left(\frac{\sqrt{I_2^p}}{\gamma_1} \right) \tag{4.10}$$

where I_2^p is the second invariant of the strain rate tensor.

The quantity γ_1 and the material functions k_0, A_1, Φ may be determined from the known experimental data for unirradiated material. The radiation hardening $\Delta Y_1(\varphi)$ is obtainable from the known empirical relations (cf. e.g. BEMENT et al. [1] :

$$\Delta Y_1(\varphi) = C \varphi^{1/2} \quad , \quad \Delta Y_1(\varphi) = c_1 \left(1 - e^{-c_2 \varphi} \right)^{1/2} \tag{4.11}$$

where C , c_1 , c_2 are the material constants. The latter formula takes into account the saturation effect of radiation hardening.

Similarly for the Region IIb the constitutive equation for $\dot{\underset{\sim}{e}}$ takes the form:

$$\dot{\underset{\sim}{e}} = \frac{1}{2\mu}\dot{\underset{\sim}{s}} + \gamma_2(\varphi)\left\langle \Phi\left[\mathcal{A}_2'(W_i^p,\vartheta,\varphi)\left(\frac{\sqrt{J_2}}{k_o(W_i^p,\vartheta)} - 1\right)\right]\right\rangle \frac{\underset{\sim}{S}}{\sqrt{J_2}} \qquad (4.12)$$

Assuming that:

$$\mathcal{A}_2'(W_i^p,\vartheta,\varphi) = \frac{k_o(W_i^p,\vartheta)}{A_2(W_i^p,\vartheta,\varphi)} \quad , \quad A_2(W_i^p,\vartheta,\varphi) > 0\,, \quad \lim_{\varphi\to 0}\gamma_2(\varphi) = \infty \qquad (4.13)$$

we obtain the relation for the dynamic yield condition:

$$\sqrt{J_2} = k_o(W_i^p,\vartheta) + A_2(W_i^p,\vartheta,\varphi)\,\Phi\left[\frac{\sqrt{I_2^p}}{\gamma_2(\varphi)}\right] \qquad (4.14)$$

The first term denotes the quasi-static shear yield strength for unirradiated material, whereas the second one is responsible for the radiation hardening dependent on strain rate and temperature. The eqs. (4.12) , (4.16) lead in the limit $\varphi\to 0$ to the Prandtl-Reuss equations for virgin material (cf. OLSZAK, PERZYNA [10] . In this case the determination of material functions is more difficult because of the complexity of radiation hardening and the limited number of proper experimental data.

In many practical problems suitable approximations of the proposed constitutive equations are necessary. The example of the approximate description of irradiated materials in both regions IIa and IIb is given by the relations:

$$\dot{\underset{\sim}{e}}^p = \gamma\left\langle\left[\frac{k+\Delta k(\varphi)}{k}\left(\frac{\sqrt{J_2}}{k+\Delta k(\varphi)} - 1\right)\right]^{\delta}\right\rangle\frac{\underset{\sim}{S}}{\sqrt{J_2}} \; , \; \sqrt{J_2} = k\left[1 + \left(\frac{I_2^p}{\gamma}\right)^{1/\delta}\right] + \Delta k(\varphi) \qquad (4.15)$$

where the non-workhardening material is assumed. The approximation is proposed for the simplest case in which the radiation hardening $\Delta k(\varphi)$ is determined in quasi-static test only. The other material constants are independent on the dose φ and are the same like for the unirradiated material.

5. Concluding remarks

The other, more general approach to the description of the behaviour of irradiated materials was proposed earlier by PERZYNA [11], [12]. The main difference lies in the way of description of radiation effects. PERZYNA introduces an internal parameter ξ denoting the density of radiation defects. However, in the simplest case the evolution equation:

$$\dot{\xi}(t) = \Xi_o\,\phi(t) \quad , \quad \Xi_o = const \quad , \qquad (5.1)$$

where $\phi(t)$ denotes the radiation flux, leads to the equivalent description.

The approach with direct description by the dose φ enables to use the existing experimental data and is capable to describe radiation hardening and saturation effects, as well. However, it fails to consider annealing process. The proposed equations as well as their simplifications can be useful in the strength analysis of reactor structures working in moderate temperatures. The approximate eqs. (4.15) with $\delta = 1$ were applied to the problem of a spherical thick-walled container subjected to inner pressure, difference of temperature and irradiation. The proposed equations are also applicable in the analysis of the ductile-brittle transition in the structural elements of mild steel (cf. PĘCHERSKI [13], [14]).

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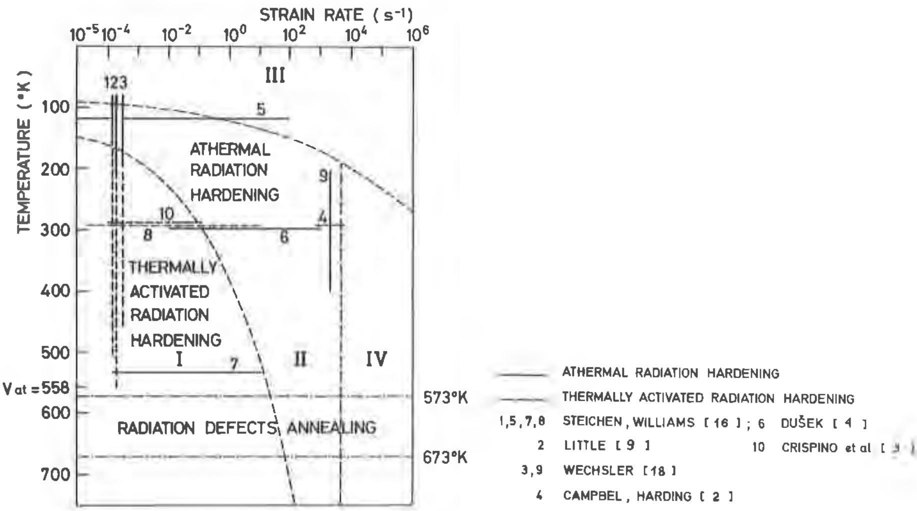
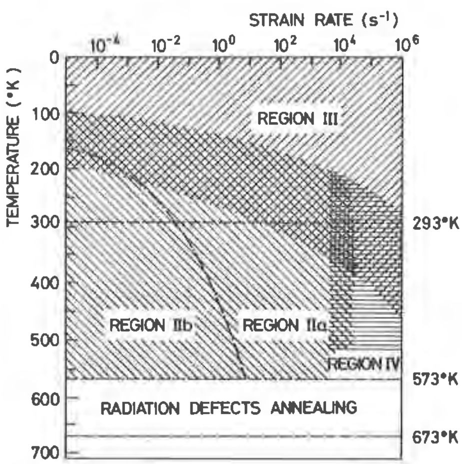


1. Regions of the temperature - strain rate spectrum of unirradiated mild steel that reflect different mechanisms of plastic deformation and main experimental results pertaining the radiation hardening mechanisms.



2. Regions of the temperature - strain rate spectrum of irradiated mild steel that reflect different mechanisms of plastic deformation.