COUPLED DAMAGE MODES (CDM) PLASTICITY MODELS FOR THE SIMULATION OF COMPLEX MATERIALS USED IN REACTORS

J. DUBOIS, J. C. BIANCHINI, A. DE ROUVRAY
Engineering System International S.A.,
20 rue Saarinen, F-94150 Rungis Silic, France

Introduction.
Some materials which are used in nuclear power reactors or for their confinement, such as concrete, graphite, shock absorbing materials, rocks, etc..., often possess a complex constitutive behavior, which can however be characterized by a certain number of damage modes (i.e. cracking, shearing, compaction...), which can occur concurrently.

The CDM Family of constitutive models presented in this paper is a systematic and efficient procedure to assemble a plasticity model which can represent the main features of the material described by individual damage modes, damage parameters (representing the amount of damage in a particular mode) and by a consistent coupling between the damage modes.

Formulation.
The CDM models use the classical incremental theory of plasticity with the following modifications:

- Several yield functions can be used simultaneously.
- Koiter's generalization is used to activate several damage modes simultaneously.
- Damage differential equations are elaborated with damage functions in order to describe the evolution of each damage parameter.

Example of application.

Jointed rock. In order to evaluate the behavior of a large underground opening for a nuclear power plant, the specialized constitutive model UPLAS has been developed for the rock mass and is presented.

Concrete. The POSTRU (post rupture) model simulates the behavior of concrete. The inelastic modes used in the model correspond to tensile cracking, shear flow and hydrostatic compaction.

Conclusion.
The presented examples have shown that, for a wide range of materials, adapted CDM constitutive models can be generated by a systematic and efficient method which takes into account standard laboratory results characterizing individually the inelastic damage modes of the materials.
1. Introduction

Some materials, which are used in nuclear power reactors or for their containment, such as concrete, graphite, shock absorbing materials, rocks, etc..., often exhibit a complex constitutive behavior, which can however be characterized by a number of independent or simultaneous damage modes (i.e. cracking, shearing, compaction ...).

The CDM (Coupled Damage Modes) family of constitutive models presented in this paper is a systematic and efficient procedure to assemble a plasticity model which can represent the main features of a material described by:

- Individual damage modes
- Damage parameters representing the amount of damage in each mode
- A consistent coupling between the damage modes.

2. Formulation

2.1 Basic Framework. An extension of the theory of plasticity is used to simulate the inelastic behavior of each of the damage modes and the coupling of the modes. A yield function $\psi_n(c, h_n)$ is used to check the activation of the damage mode $n$:

$$\psi_n(c, h_n) < 0 \text{ the damage mode } n \text{ is not activated}$$

$$\psi_n(c, h_n) = 0 \text{ the damage mode } n \text{ is activated}$$

where $c$ are the stresses and $h_n$ is the damage parameter of mode $n$. The value of $h_n$ increases from a value of 0 (undamaged state), to a maximum value, or indefinitely, as the damage of the material increases. The total strain increment $\Delta e^t$ is classically divided into the sum of one elastic component $\Delta e^e$ and several inelastic components $\Delta e^m_n$ which are used when necessary. The increment of stress is due to the increment of elastic strain $\Delta e^e$ only: $\Delta c = D^e \Delta e^e$ where $D^e$ is the elastic matrix.

The satisfaction of equation (1) is obtained by the generation of a plastic strain $\Delta e^p = \lambda_n \Delta \delta_n / \delta c$ where $\psi_n(c, h_n)$ is the flow rule which gives the direction for the plastic strain and $\lambda_n$ is the amount of plastic flow of damage $n$. The parameter $h_n$ is defined, so that the increase of damage is proportional to the amount of plastic flow

$$\lambda_n \delta_n = \lambda_n L_n (h_n, c)$$

2.2 Coupling of the damage modes. When several damage modes are activated at the same time, the corresponding yield functions are simultaneously set to zero, by the generation of a set of additive plastic strains, corresponding to each of the activated damage modes:

$$\Delta e^t = \Delta e^e + \sum_n \lambda_n \Delta e^m_n$$

where $\Delta$ is the set of activated damage modes. This procedure known under the name of Koiter’s generalization, is shown on Fig. 5. The damage in one mode can be coupled with the damages of the other modes:

$$\Delta h_m = \frac{\Delta \psi_m}{\psi_m} / \nabla \lambda_m L_m (h_m, h_n, c)$$

The damage parameters $h_n$ can be used for the classical isotropic and/or kinematic hardenings. The complete set of equations for a coupled damage mode model is shown on table 1.

3. UPLAS: a coupled damage mode model for stratified rocks.

The model UPLAS was developed for the analysis of underground nuclear power plants in stratified rock masses (See reference (1)). The rock mass of a stratified rock is composed of a relatively homogeneous rock matrix, and of one or several sets of quasi ubiquitous planes of weakness (joints). The damage modes of the rock mass can be: crack and/or shear damage in the matrix and crack and/or shear damage along the joint (see table2).
Fig 6 shows a satisfying comparison between the experimental and the model peak strength, which depends on the inclination of the joint plane. For additional examples, see Ref.1.

4. **REINCON**: a model for reinforced concrete.

REINCON is a constitutive model for reinforced concrete available in the PAM-system. The model computes the stresses $\sigma_c$ in the concrete and $\sigma_s$ in the steel and for the steel concrete interaction. The stresses in the composite reinforced concrete are $\sigma = \sigma_c + \sigma_s$. Large displacements are taken into account by a first order Jaumann rate. Several models are available at E.S.I., to compute the concrete stresses $\sigma_c$. Among them, the simple CD model PLAST is presented for demonstration purposes. Then, extensions are discussed concerning the phenomena of concrete behavior and steel concrete interaction.

4.1. **PLAST**: a CD model for plain concrete. In PLAST, the following damage modes and parameters have been used:

4.1.1. Tensile cracking. A maximum stress criterion is used to check crack formation: $\tau = \sigma_i - \sigma_c$, where $\sigma_i$ is a principal stress and $\sigma_c$ is the tensile strength. The plastic strain in direction $i$ is equal to: $\varepsilon_i = \lambda_c \sigma_i / \sigma_c$ where $\lambda_c$ represents the increase of crack width. When a crack forms, the direction of the crack is computed and stored. A second crack can form, perpendicular to the first one. When the crack is formed, the stresses on the crack face are set to zero, and the crack width is computed. When the crack closes, the stresses are restored.

4.1.2. Shear damage. For the shear damage, a Drucker-Prager or Mohr-Coulomb criterion is used:

$$3\sigma_m + \sigma_m - k = 0 \text{ or } \sigma_m \sin \phi + 3 \cos \phi = \frac{\sigma_m}{\sqrt{3}} \sin \phi \sin \phi - c \cos \phi = 0 \quad (4)$$

where $\beta(a)$, $\phi(a)$, $k(a)$, $c(a)$ are respectively the friction, friction angle and the two cohesion coefficients which depend on the shear damage parameter $a$, $\sigma_m$ is the mean stress, $\sqrt{3}$ is the quadratic mean of the deviatoric stresses and $\phi$ is Lode's angle. Idealized stress strain curves for concrete are shown on Fig. 1. In the strain softening part, straight lines issued from point 0 generate points like C and D having the same damage parameter $x = \varepsilon_1 / \varepsilon_{1,m}$ which varies from 0 at points A and B (no damage) to 1 at points E and F (full damage), where the material reaches its residual strength. The damage parameter is then generalized to three dimensional plastic strain states:

$$\bar{\varepsilon} = \frac{\int_{0}^{1} \left( \frac{1}{2} \varepsilon_1 \sigma_1 \right) \varepsilon_1^{1/2} dt}{\varepsilon_{1,m}}$$

It can be shown that, for Drucker-Prager

$$d a = \lambda c L_a (a) \quad \text{with} \quad L_a (a) = \frac{1}{2} \sqrt{1 + 2k^2 (a)}$$

From the input curves OAB and OBF, a preprocessor program computes $\beta(a)$ and $k(a)$, then $\beta(a)$ and $k(a)$ (or $\phi(a)$ and $c(a)$) for $0 < a < 1$. When the value of the damage parameter $a$ reaches 1, the program uses the residual strength $\beta(1)$ and $k(1)$. A detailed presentation of the preprocessor and a generalisation for stress-strain curves of arbitrary shape are discussed in Reference (3).

4.1.3. Hydrostatic compaction. The idealized stress strain curve OABD of the compression damage mode is shown on Fig. 3. The corresponding yield surface and flow rule is:

$$\psi_b (\sigma^P, \sigma^F) = \sigma^P - p + \sigma^F = 0 \text{ where } \sigma^P = \int_{0}^{1} \varepsilon_1 \sigma_1 \varepsilon_1^{1/2} dt$$

--- 3 ---
is the compaction damage parameter. The damage function $L_b$ is equal to $1: d\theta_b = \lambda_b$
The corresponding hydrostatic test, pressure-volume change curve and yield surface are shown on Fig. 3.

4.1.4. Additional coupling for the damage modes. Some additional coupling between the damage modes can be added in the model. Among possible assumptions:
- if two cracks form, the shear strength parameters are set to their residual value
- if the normalized shear damage $a$ is greater than 0.5 the crack strength is set to zero.
The yield surface for the POSTM  model is shown on Fig. 4.

4.2. Addition of the steel generalization. For lack of space, the reader is sent to Ref. 4 for the description of the modelling of the steel bars. Paragraph 4.1. has shown the application of the CDM theory to a simple concrete model, for a demonstration purpose. For a more detailed simulation of concrete, the following features have been added in more advanced models of the PAM system:
- control of arbitrary shapes for the stress-strain curves of the shear damage (see ref. 3)
- control of dilatation by a coefficient $\chi(a,c)$ in the shear damage (see ref. 1)
- Addition of the aggregate interlock, the dowel action and the bond slip (ref. 5) across open cracks, by specialized yield functions and flow rules (instead of the usual crack criterion).
- Etc...

5. Example of application: Airplane crash on a nuclear power plant.

Fig. 7a shows an idealized airplane crash, which can be modelled by a rigid missile representing the engine and an equivalent pressure simulating the loads due to the highly deformable fuselage. The program HEMP/ESI with the concrete model REINCON computes the exit velocity of the rigid missile which perforates the external concrete containment, as well as the velocity and the volume of the concrete debris, in order to evaluate the damage on the internal concrete containment (Fig. 7b), (See ref. 2 for more details).

6. Conclusion.
The Coupled Damage Mode models possess a powerful framework which can be used to build up a constitutive model able to reproduce the various inelastic damage modes of a material described by representative laboratory or model experiments.

Acknowledgements.
Parts of this work have been financed by the DÉpartement des Etudes Mécaniques et Thermiques of the Commissariat à l’Energie Atomique (France), Electrobel (Belgium) and Ingenieur Buro Bung (Germany), which are gratefully acknowledged.

References
I. SYSTEM OF EQUATIONS

YIELD SURFACES:

\[ \psi_i \left( \sigma_0 \right) + \frac{\delta \psi_i}{\partial \sigma} \frac{\delta \sigma}{\sigma} + \frac{\delta \psi_i}{\partial h_i} \frac{\delta h_i}{h_i} = 0 \]

\[ \psi_j \left( \sigma_0 \right) + \frac{\delta \psi_j}{\partial \sigma} \frac{\delta \sigma}{\sigma} + \frac{\delta \psi_j}{\partial h_j} \frac{\delta h_j}{h_j} = 0 \]  \hspace{1cm} (1)

\[ \psi_N \left( \sigma_0 \right) + \frac{\delta \psi_N}{\partial \sigma} \frac{\delta \sigma}{\sigma} + \frac{\delta \psi_N}{\partial h_N} \frac{\delta h_N}{h_N} = 0 \]

FLOW RULES:

\[ d \sigma_i^P = \frac{\delta \phi_i}{\delta \sigma}, \ldots, d \sigma_n^P = \frac{\delta \phi_n}{\delta \sigma}, \ldots, d \sigma_N^P = \frac{\delta \phi_N}{\delta \sigma} \]  \hspace{1cm} (2)

ELASTIC STRESS-STRAIN LAW:

\[ d \sigma = D^e d e^e \]  \hspace{1cm} (3)

STRAIN EQUATION:

\[ d e = d e^t + \sum d e_m^P \]  \hspace{1cm} (4)

DAMAGE EQUATIONS:

\[ d h_m = \sum h_m \lambda_m L_{mn} (h_m, h_n, \sigma) \]  \hspace{1cm} (5)

UNKNOWNS:

\[ \begin{align*}
\sigma_0 & \quad \text{initial stress} \\
d \sigma & \quad \text{increment of stress} \\
D^e & \quad \text{Hooke matrix} \\
A & \quad \text{set of activated damage modes} \\
T & \quad \text{index for transverse} \\
N & \quad \text{number of activated damage modes} \\
P & \quad \text{number of possible damage modes}
\end{align*} \]

TABLE 1: THE COMPLETE SET OF EQUATIONS FOR THE COUPLED DAMAGE MODES (CDM) PLASTICITY MODEL

\[ \begin{align*}
\text{EQUATIONS:} & \quad (1) \quad (2) \quad (3) \quad (4) \quad (5) \quad \text{Total} \\
N & \quad \text{number of damage modes} \\
P & \quad \text{number of possible damage modes}
\end{align*} \]
<table>
<thead>
<tr>
<th>EXAMPLE OF LABORATORY TEST</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DAMAGE MODE</td>
<td>CRACK DAMAGE IN ROCK MATRIX</td>
<td>SHEAR DAMAGE IN ROCK MATRIX</td>
<td>SHEAR DAMAGE IN JOINT</td>
</tr>
<tr>
<td>STRESS-STRAIN PATH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STRESS PATH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PLASTIC PARAMETERS</td>
<td>Peak</td>
<td>residual</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PLASTICITY CRITERION</td>
<td>MOHR COULOMB</td>
<td>MOHR COULOMB</td>
<td>MOHR COULOMB</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADDITIONAL PARAMETERS OF MODEL</td>
<td>Orthotropic elastic moduli</td>
<td>Angles to define the orientation</td>
<td>Coefficient of control of dilatancy</td>
</tr>
</tbody>
</table>

**Table 2: Damage Modes, Plasticity Criteria of the UPNASA Model**

**Fig. 1: Shear Damage**
(Model POSTRU)

**Fig. 2: Cracking Damage**
(Model POSTRU)
Fig. 3: HYDROSTATIC COMPACTION DAMAGE IN MODEL POSTRU

INTERSECTION OF THE YIELD SURFACE WITH THE PLANE II

WESTERGAARD SPACE OF PRINCIPAL STRESSES

Fig. 4: DESCRIPTION OF THE YIELD SURFACE IN MODEL PROTRU (MOHR COULOMB)
Fig. 5: KONIT'S GENERALIZATION FOR SEVERAL COACTIVE YIELD SURFACES

Fig. 6: MODEL UNPLAS: Uniaxial yield stress plotted as a function of joint plane angle. Comparison with experimental results.

Fig. 7: CALCULATION OF AN AIRPLANE CRASH ON A NUCLEAR POWER PLANT CONCRETE CONTAINMENT PROGRAM HEMP/ESI

Phase 1: Crash of the front part of the fuselage.

Phase 2: Penetration of the rigid engine and crash of the center and the rear of the fuselage.

Fig. 7.a: Crash on the external containment

7.b: Crash on the internal containment