A NEW FINITE ELEMENT FOR STRUCTURAL ANALYSIS OF PIPING SYSTEMS

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SUMMARY

A few special finite elements have been proposed to analyse complex structural behavior of the piping systems. One of them has been proposed by Marcal et al. It is based on the modification of the strain displacement relations used for axisymmetric thin shell theory through the addition of longitudinal deformation mode of the beam. A ring element, which is based on the thin shell theory under the Kirchhoff-Love hypothesis, is proposed by Ohtsubo et al. It has two nodal points at center of both end sections. Displacement components in the element are approximated by the Fourier series arround the pipe section and by the Hermitian cubic function along the longitudinal direction. A special doubly curved quadrilateral element ELBOW6 under the Kirchhoff-Love hyposhesis, is proposed by Takeda et al. Piping system is divided into finite length straight and curved rings, and each ring is idealized with the elements around circumferential direction. The element has four nodal points at corners of quadrilateral to approximate circumferential and nomal displacements, and two nodes at center of both end sections of the ring to approximate longitudinal displacement. Circumferential and normal displacements are interpolated by the two dimensional Hermitian cubic function. Longitudinal displacement is expanded by the Fourier series around circumferential direction and each term is interpolated by the one dimensional cubic function along the longitudinal direction.

In this paper, we propose a new finite element ELBOW6R to analyse detailed structural behavior of piping systems in the three dimensional space with consideration of the interaction effects between straight and curved pipes. It is based on the thin shell theory including the effect of transverse shear deformations. A shell theory with transverse shear strain is applied to reduce the continuity conditions of the finite element displacement function. The geometrical configuration of the element is completely same as the ELBOW6 element. Circumferential and normal displacements and rotations are interpolated by the two dimensional Lagrangian bilinear functions. Longitudinal displacement is expanded by the Fourier series around circumferential direction and each term is interpolated by the one dimensional linear function along the longitudinal direction. Reduced integration technique is used to compute element stiffness matrices. It is relatively efficient and easy to implement this element into element libraries of finite element computer programs.

1. INTRODUCTION

In the design of the LMFBR piping systems, pipe bend or elbow has a meaningful role to adjust influence on another components, e.g. pressure vessel, by absorbing the thermal expansion of the system. Comparing with straight pipe, pipe bend is very flexible and its structural behavior is very complex. If we try to study detailed structural behavior of the pipe bend even if under simple mechanical loading, beam theory is not sufficient and it is necessary to consider the shell behavior of the pipe bend. In recent years, it is recognized that nonlinear behavior, such like elastic plastic, creep and large displacement effects, of pipe bend is very important to design piping system with large diameter pipe under high temperature condition.

Many finite element computer programs have been developed and almost of engineers are making used of such programs for the structural design of piping systems. A few special finite elements have been proposed to analyse complex structural behavior of the piping systems. One of them has been proposed by Marcal et al. [1,2]. It is based on the modification of the strain displacement relations used for axisymmetric thin shell theory through the addition of longitudinal deformation mode of the beam. A ring element, which is based on the thin shell theory under the Kirchhoff-Love hypothesis, is proposed by Ohtsubo et al. [3,4]. It has two nodal points at center of both end sections. Displacement components in the element are approximated by the Fourier series arround the pipe section and by the Hermitian cubic function along the longitudinal direction. A special doubly curved quadrilateral element ELBOW6 under the Kirchhoff-Love hyposhesis, proposed by Takeda et al. [5, 6]. Piping system is divided into finite length straight and curved rings, and each ring is idealized with the elements around circumferential direction. The element has four nodal points at corners of quadrilateral to approximate circumferential and nomal displacements, and two nodes at center of both end sections of the ring to approximate longitudinal displacement. Circumferential and normal displacements are interpolated by the two dimensional Hermitian cubic function. Longitudinal displacement is expanded by the Fourier series around circumferential direction and each term is interpolated by the one dimensional cubic function along the longitudinal direction.

The finite element analysis of plates and shells have been studied by many researchers since the early days of the finite element method. An finite element analysis of thin shells including the effects of transverse shear deformation is developed by Key et al. [7]. A doubly-curved arbitrary quadrilateral element is developed with bilinear displacement assumptions for the inplane displacements and rotations and a bicubic variation in the out-of-plane displacement. For plate bending applications a simple and efficient finite element, based on a thick plate theory in which transverse shear strains are counted, is introduced by Hughes et al. [8]. The element is a four-node quadrilateral with bilenear displacement and rotations in conjunction with selective reduced integration technique. Recently studies of plate bending elements with reduced integration have been done to evaluate this technique [9, 10].

In this paper, we propose a new finite element ELBOW6R to analyse detailed structural behavior of piping systems in the three dimensional space with consideration of the interaction effects between straight and curved pipes. It is based on the thin shell theory including the effect of transverse shear deformations. A shell theory with transverse shear strain is applied to reduce the continuity conditions of the finite element displacement function.

The geometrical configuration of the element is completely same as the ELBOW6 element. Circumferential and normal displacements and rotations are interpolated by the two dimensional Lagrangian bilinear functions. Longitudinal displacement is expanded by the Fourier series around circumferential direction and each term is interpolated by the one dimensional linear function along the longitudinal direction. Reduced integration technique is used to compute element stiffness matrices. It is relatively efficient and easy to implement this element into element libraries of finite element computer porgrams.

2. KINEMATIC RELATIONS

The general treatment of a linearized thin shell theory including the effect of transverse shear deformations is presented by Washizu [11]. In this section we deal with basic linearized equations of curved and straight pipes with transverse shear strains.

Refering to Figure 1 which shows the geometry of a part of elbow, Cartesian components of the displacement vector are assumed by following equations:

$$\begin{aligned} u_\alpha(\alpha,\beta,\zeta) &= u(\alpha,\beta) + \zeta\,\theta_\beta(\alpha,\beta) \\ u_\beta(\alpha,\beta,\zeta) &= v(\alpha,\beta) - \zeta\,\theta_\alpha(\alpha,\beta) \\ u_\zeta(\alpha,\beta,\zeta) &= w(\alpha,\beta) \end{aligned} \tag{1}$$

The strain-displacement relations are given by

$$\begin{aligned} \varepsilon_\alpha &= \varepsilon_\alpha^o + \zeta\,\kappa_\alpha \\ \varepsilon_\beta &= \varepsilon_\beta^o + \zeta\,\kappa_\beta \\ \gamma_{\alpha\beta} &= \gamma_{\alpha\beta}^o + \zeta\,\kappa_{\alpha\beta} \\ \gamma_{\alpha\zeta} &= \gamma_{\alpha\zeta}^o \\ \gamma_{\beta\zeta} &= \gamma_{\beta\zeta}^o \end{aligned} \tag{2}$$

where

$$\begin{aligned} \varepsilon_\alpha^o &= \frac{1}{A}\frac{\partial u}{\partial \alpha} + \frac{v}{AB}\frac{\partial A}{\partial \beta} + \frac{w}{R_\alpha} \\ \varepsilon_\beta^o &= \frac{1}{B}\frac{\partial v}{\partial \beta} + \frac{w}{R_\beta} \\ \gamma_{\alpha\beta}^o &= \frac{1}{B}\frac{\partial u}{\partial \beta} + \frac{1}{A}\frac{\partial v}{\partial \alpha} - \frac{u}{AB}\frac{\partial A}{\partial \beta} \\ \gamma_{\alpha\zeta}^o &= \theta_\beta + \frac{1}{A}\frac{\partial w}{\partial \alpha} - \frac{u}{R_\alpha} \\ \gamma_{\beta\zeta}^o &= -\theta_\alpha + \frac{1}{B}\frac{\partial w}{\partial \beta} - \frac{v}{R_\beta} \end{aligned} \tag{3}$$

and

$$\begin{aligned} \kappa_\alpha &= \frac{1}{A}\frac{\partial \theta_\beta}{\partial \alpha} - \frac{\theta_\alpha}{AB}\frac{\partial A}{\partial \beta} \\ \kappa_\beta &= -\frac{1}{B}\frac{\partial \theta_\alpha}{\partial \beta} \\ \kappa_{\alpha\beta} &= \frac{1}{B}\frac{\partial \theta_\beta}{\partial \beta} - \frac{\theta_\beta}{AB}\frac{\partial A}{\partial \beta} - \frac{1}{A}\frac{\partial \theta_\alpha}{\partial \alpha} + \frac{1}{R_\alpha}\frac{1}{B}\frac{\partial u}{\partial \beta} + \frac{1}{R_\beta}\left(\frac{1}{A}\frac{\partial v}{\partial \alpha} - \frac{u}{AB}\frac{\partial A}{\partial \beta}\right) \end{aligned} \tag{4}$$

In eqs. (3) and (4), A and B are the Lamé coefficients, and R_α and R_β are principal radii of curvature. The following equations to elbow are made

$$A = R + r \sin \beta$$
$$B = r$$
$$R_\alpha = \frac{R + r \sin \beta}{\sin \beta} \tag{5}$$
$$R_\beta = r$$

For straight pipe, prameter α is chosen according to the physical length along the longitudinal direction, and following equations are derived.

$$A = 1.0, \quad B = r$$
$$\frac{1}{R_\alpha} = 0, \quad R_\beta = r \tag{6}$$

3. FINITE ELEMENT APPROXIMATION

Piping system is divided hear into finite length straight and curved rings, and each ring is idealized with the elements around circumferential direction [see Fig. 1].

In a typical element, displacement variables v, w, θ_α and θ_β are approximated to vary bilinearly, defined by their values at the corner or surface nodes 1, 2, 4 and 5.

$$v = N_1 v_1 + N_2 v_2 + N_4 v_4 + N_5 v_5$$
$$w = N_1 w_1 + N_2 w_2 + N_4 w_4 + N_5 w_5$$
$$\theta\alpha = N_1 \theta\alpha_1 + N_2 \theta_{\alpha_2} + N_4 \theta\alpha_4 + N_5 \theta\alpha_5 \tag{7}$$
$$\theta\beta = N_1 \theta\beta_1 + N_2 \theta\beta_2 + N_4 \theta\beta_4 + N_5 \theta\beta_5$$

where

$$N_i = \frac{1}{4} (1+\xi\xi_i)(1+\eta\eta_i)$$

and u is expanded by the Fourier series around the circumferential direction β,

$$u = u^{(o)} + \sum_{k=1}^{n} [u^{(2k-1)} \sin k\beta + u^{(2k)} \cos k\beta] \tag{9}$$

and each term is approximated to vary linearly along the longitudinal direction ξ, defined by their values at the mid or elbow nodes 3 and 6,

$$u^{(i)} = \frac{1}{2} (1 - \xi) u_3^{(i)} + \frac{1}{2} (1 + \xi) u_6^{(i)} \qquad (i = 0 \sim 2n) \tag{10}$$

4. NUMERICAL EXAMPLES

The following numerical examples have been presented to verify the accuracy and adequacy of the proposed element. The FINAS program [12] was used in these analyses and all computations were performed on a CDC 6600 computer in single precision.

(1) Elbow with inplane bending by simplified elements

To study numerical integration rules around the circumferential direction, a simplified elbow with three nodes were specially developed under assumption of constant strain condition along the longitudinal direction. A Poisson's ratio of 0.304 and Young's modulus of 1.58×10^4 were used throughout.

A convergence study was carried out for an elbow which were subjected to inplane bending. The geometrical data employed were R = 300, r = 100, t = 5 and the pipe factor λ = 0.15. The results are depicted in Fig. 2. It may be remarked that uniform reduced integration element produces high convergent solutions. In Fig. 3 a convergence study for uniform reduced integration element is shown to different geometries.

(2) Elbow with straight pipes

The numerical studies and inelastic analysis for the IAEA International Piping Benchmark Problems [13] are presented in this section. The loading system and geometry are summarized in Fig. 4 and Table 1 respectively. A convergence study to elastic analysis is shown in Fig. 5. It is presented that the comparatively good behavior is presented by relatively coase mesh. Strain distributions by the elastic analysis is compared with experiment [13] and previous analysis [6] in Figs. 6 and 7. In Fig. 8 strain distribution of the elastic plastic analysis is compared with experimental results. The difference seen in Fig. 8 may be possibly due to the material properties employed in the analysis and the geometrical nonlinearity was neglected.

5. CONCLUSIONS

In this paper we have proposed an finite element to analyse detailed structural behavior of piping systems. It is based on the thin shell theory including the effect of transverse shear deformations. The shell theory is applied to reduce the continuity conditions of the finite element displacement functions. A complex situation by relatively few elements with uniform reduced integration rule is adequately represented in numerical studies. It is relatively economical and easy to apply the proposed element by means of standard finite element computer programs.

Acknowledgements

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Table 1. Geometry of the elbow-pipe assembly

Outer diameter	D_0	4.5 in.
Elbow radius	R	6.0 in.
Wall thickness	t	0.121 in.
Pipe factor	λ	0.15012
Pipe leg length	L	12.75 in

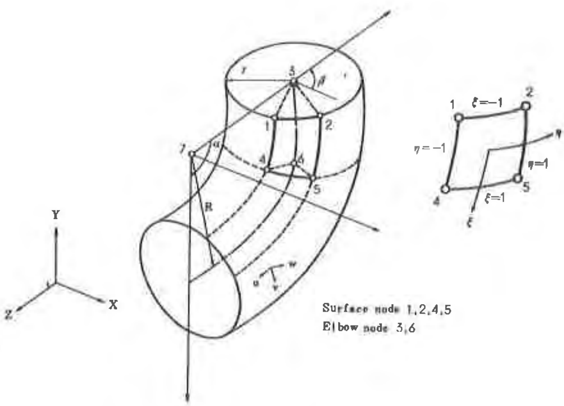


Figure 1.Geometry and coordinates of an elbow.

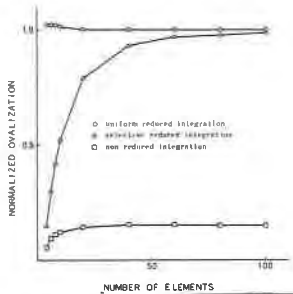


Figure ?(a). Convergence study for integration rules (displacement).

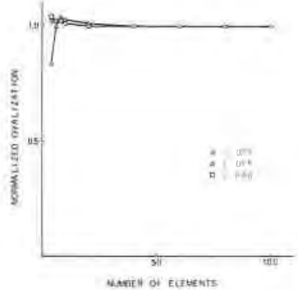


Figure 3(a). Convergence study for different geomet ies (displacement).

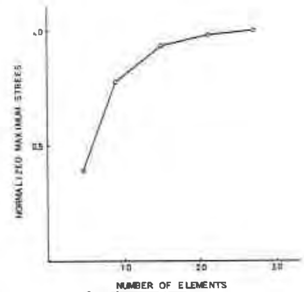


Figure 5(a). Convergence study for the circumferential idealization.

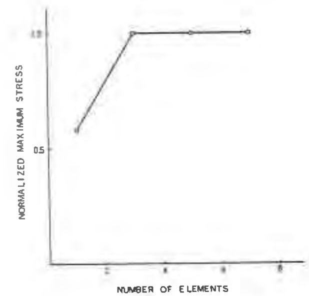


Figure 5(b). Convergence study for the longitudinal idealization.

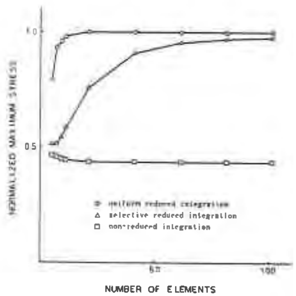


Figure 2(b). Convergence study for integration rules (stress).

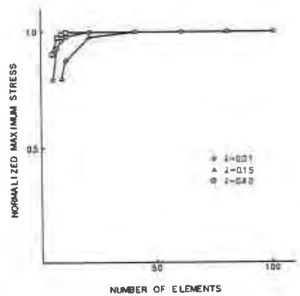


Figure 3(b). Convergence study for different geometries (stress).

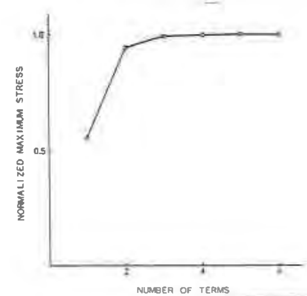
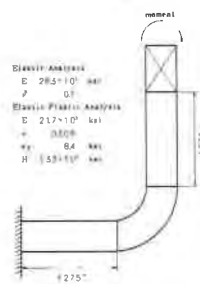


Figure 5(c). Convergence study for the Fourier series.



Figu e 4. IAEA benchmark problem.

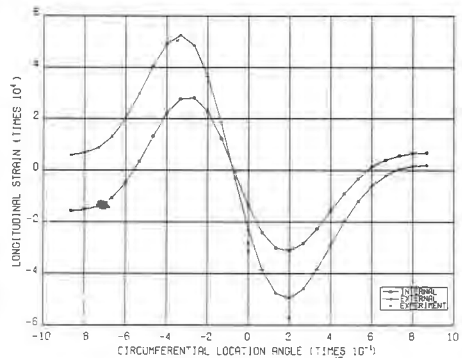


Figure 6(a). Longitudinal strain at the center of the elbow.

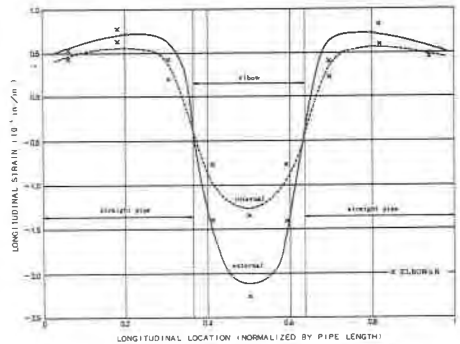


Figure 7(a). Longitudinal strain along the longitudinal direction.

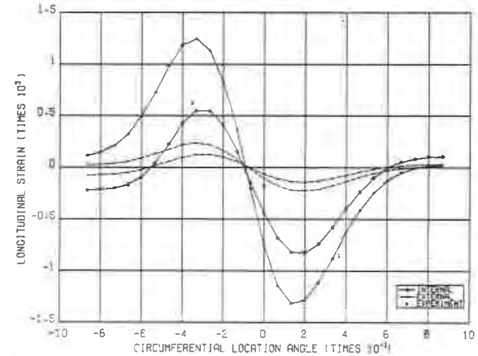


Figure 8(a). Longitudinal strain by the elastic plastic analysis.

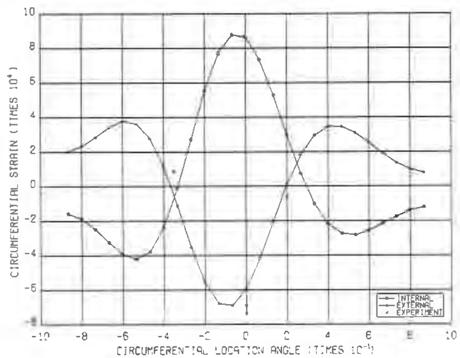


Figure 6(b). Circumferential strain at the center of the elbow.

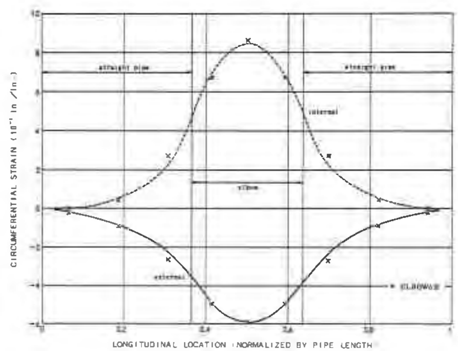


Figure 7(b). Circumferential strain along the longitudinal direction.

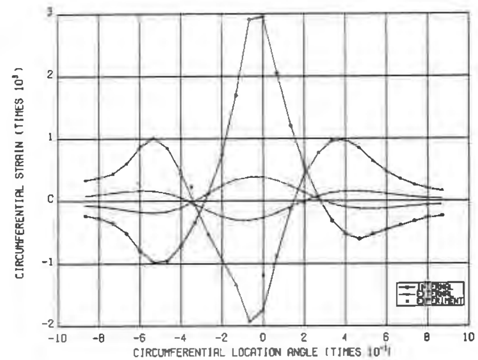


Figure 8(b). Circumferential strain by the elastic plastic analysis.