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Limit Analysis of Pipe Tee Connection

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ABSTRACT

In the papers presented during SMIRT 14 and 15, we presented a few results obtained with a new numerical calculation method for the limit load. This method of limit analysis, known as the Elastic Compensation Method (ECM) was implemented in the CASTEM 2000 finite element code and it was generalised from solid elements to shell and beam elements.

The present paper is a follow-on from that work and includes a full parametric study of pipe tees and branch connections.

The geometric parameters for this study are the ratio of the radius to the thickness of the run, the ratio of the radii of the run relative to the branch and the ratio of the thickness of the run to that of the branch. The loadings calculated consist of seven unit loadings of pipe tees and branch connections : bending in the plane of the branch and of the run, bending out of the plane of the branch and run, torsion of the branch and run and internal pressure.

This study thus enables the calculation of 672 values of limit loading in a wide range of geometries.

The results of this study are of great significance in all domains where limit analysis is required : design of pipes and determination of reference stresses for fracture mechanics or for large scale plasticity criteria in pipes.

INTRODUCTION

Limit analysis has been used for a long time in the design codes to evaluate the margins due to plastification in elastic analyses. More recently, it has enabled definition of reference stresses for large scale plasticity models and for simplified methods in fracture mechanics.

Limit loading calculations were first of all performed analytically on simple structures, in order to obtain often very rough values. Advances in numerical calculation methods have enabled the development of methods which lead to improved evaluations of the limit loading on complex structures. The Elastic Compensation Method (ECM) used here is one such method. Its notable qualities observed with many applications are : simplicity, robustness and reduced calculation time.

Pipe tees and branch connections are important components in pipe lines. Due to their complex geometry, they often demand fine analysis, because the simplified methods are often too optimistic. Knowledge of the limit loadings for such structures, which are now easy to obtain using the new numerical methods, would enable a substantial improvement in the simplified analysis methods both in conventional design or in fracture analysis.

We therefore present below, the results of a parametric study of the limit loadings for pipe tees and branch connections. This study follows on from the considerable work on the theme of ECM in this field, presented in references [1] to [6] and performed on the basis of the ECM proposal in reference [7].

CALCULATION METHOD USED

The limit loading calculations presented here were therefore performed using the Elastic Compensation Method.

This Elastic Compensation Method for limit analysis makes it possible, from a series of incompressible elastic finite element variable Young's modulus calculations, to converge on a perfectly plastic solution to the problem (reference [1] and [2]).

This method was elaborated, validated and implemented in CASTEM 2000 (procedure @ANALIM) by D. Plancq as part of his thesis (references [1] to [5]). In this context, this method was extended to cover shells using an Ilyouchine type plasticity criterion, and it was shown (references [1], [4] and [5]) that on pipe branch connections, the results obtained were very similar to those obtained with solid elements calculation under the same method.

The calculations presented here were therefore performed using 3-node shells in large scale plasticity and the CASTEM 2000 shell plasticity criterion (Ilyouchine without crossed terms).

The mesh run and branch lengths as well as the mesh size, particularly in the vicinity of the joint, were optimised to obtain stable results according to the mesh size both with respect to stress and to displacements. The mesh refinement can be seen at the joint on the graph of deflection with bending in the plane of the branch, in figure 1 : to improve the legibility of the graph, the meshed run and branch lengths are much shorter than for the limit loading calculations for which these lengths have been optimised.

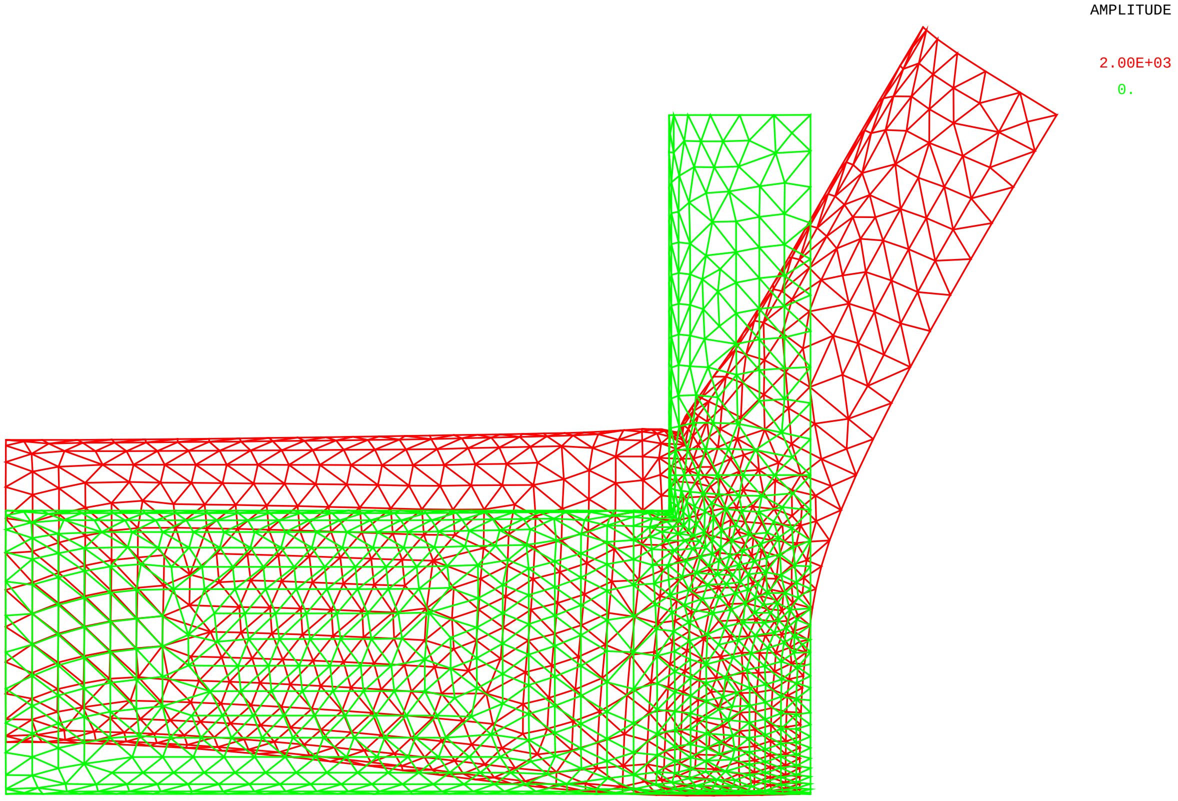


Figure 1 : Meshing and deflection with bending in the plane of the branch

For each type of geometry and loading, the calculation using CASTEM 2000 is performed as follows :

- A conventional elastic calculation is performed to evaluate for checking purposes :

- the elastic stress concentration factor C_2 (or C_1) relative to a straight pipe,
- the flexibilities k obtained by rotation of the end sections and by flexing of conventional "beams".

- A calculation at the limit load is then performed using @ANALIM starting with the loading corresponding to the limit loading for the straight portion. The calculation therefore directly provides the limit load for the pipe tee or branch connection in relation to the straight portion. The result in "conventional pipe" notation will therefore in fact be ($1/B_2$ or $1/B_1$) and must always be less than 1.

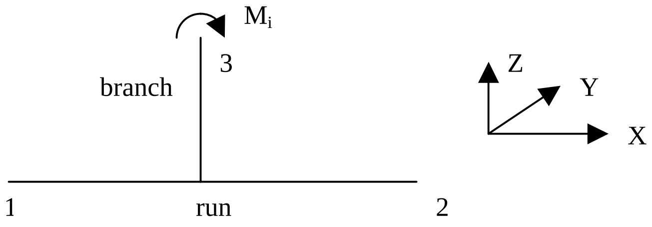
The C_2, C_1, B_2, B_1, and k notations used are the conventional notations for pipe design codes (ASME or RCC-M).

CASES COVERED

The 7 unit loadings

These cases were chosen to perform all the calculations on a quarter-tee with appropriate symmetries; anti-symmetries and limit conditions. These calculations are equivalent to the following « beam » loadings :

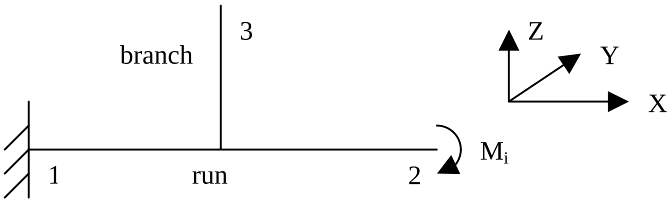
- 3 loadings applied to the branch



i = Y bending in the plane of the branch
i = X bending out of the plane of the branch
i = Z torsion of the branch

The limit conditions specified at 1 and 2 are such as to provide a constant beam moment M_i in the branch and constant moments $\pm M_i/2$ in both parts of the run.

- 3 loadings applied to the run



i = Y bending in the plane of the run
i = Z bending out of the plane of the run
i = X torsion of the run

The built-in limit conditions at 1 provide zero moment in the branch and a constant moment M_i in the run.

- 1 loading by internal pressure (with end effects).

Any elastic loading can be obtained on the basis of these 7 unit loadings.

Range of geometries explored

A pipe tee or branch connection can be characterised by three geometric parameters.
In this case, the following three parameters were used :
r_r/e_r, r_b/r_r and e_b/e_r
where r radius
e thickness
and subscripts b and r indicate branch and run.
The calculations performed concerned the following geometries :

	Min. value	**Max. value**	**Number of points calculated within this range of values**
r_r/e_r	3.33	25	8
r_b/r_r	0.25	1	4
e_b/e_r	0.5	1	3

96 geometries were therefore calculated.
In each case, the limit loadings under 7 unit loadings were evaluated : 672 limit load calculations were therefore performed.

RESULTS AND COMMENTS

Figures 2 to 8 show the results of the calculations. Each figure provides the limit loadings relative to the straight portion, i.e. $1/B_2$ for moment loading and $1/B_1$ for pressure loading.
The figures are sorted as follows :

Bending in the plane of the branch	figure 2
Bending out of the plane of the branch	figure 3
Torsion of the branch	figure 4
Bending in the plane of the run	figure 5
Bending out of the plane of the run	figure 6
Torsion of the run	figure 7
Pressure	figure 8

In each loading case, the corresponding figure shows, $1/B_i$ in relation to r_r/e_r for 4 values of the ratio of radii r_b/r_r and for 3 values of the thickness ratio e_b/e_r. There are therefore 12 curves on each figure. The notations 1 to 12 are defined in the table below :

	1	2	3	4	5	6	7	8	9	10	11	12
e_b/e_r	1	0.75	0.5	1	0.75	0.5	1	0.75	0.5	1	0.75	0.5
r_b/r_r	1	1	1	0.75	0.75	0.75	0.5	0.5	0.5	0.25	0.25	0.25

Figure 2 : Branch in plane bending

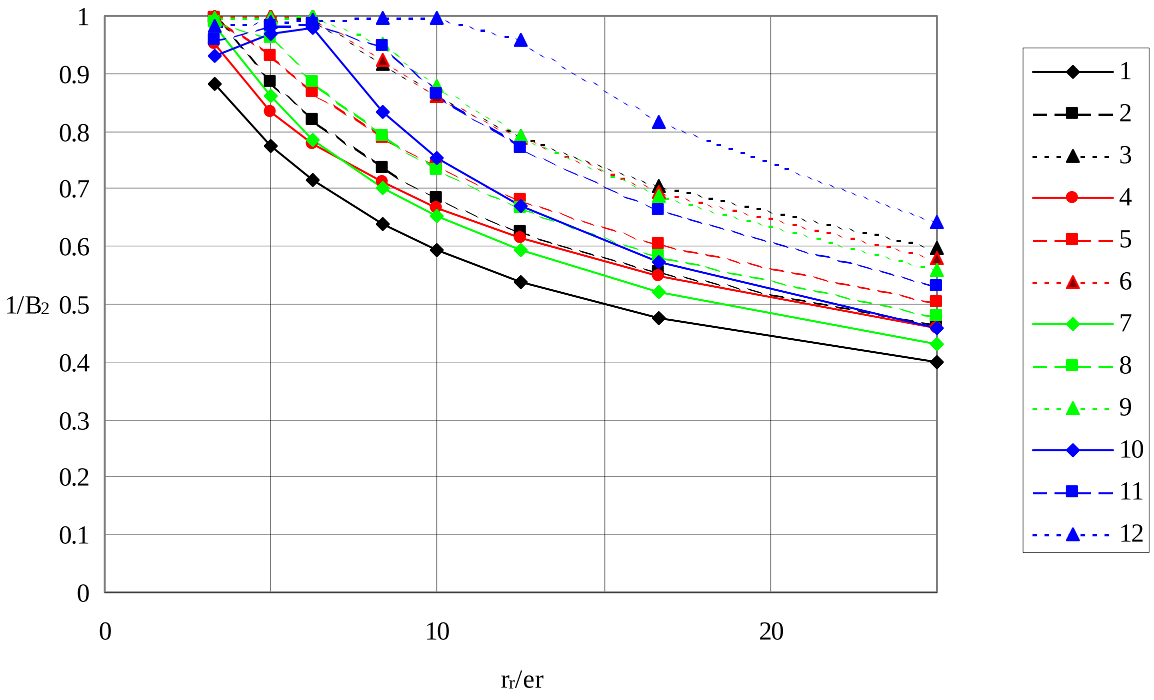


Figure 3 : Branch out of plane bending

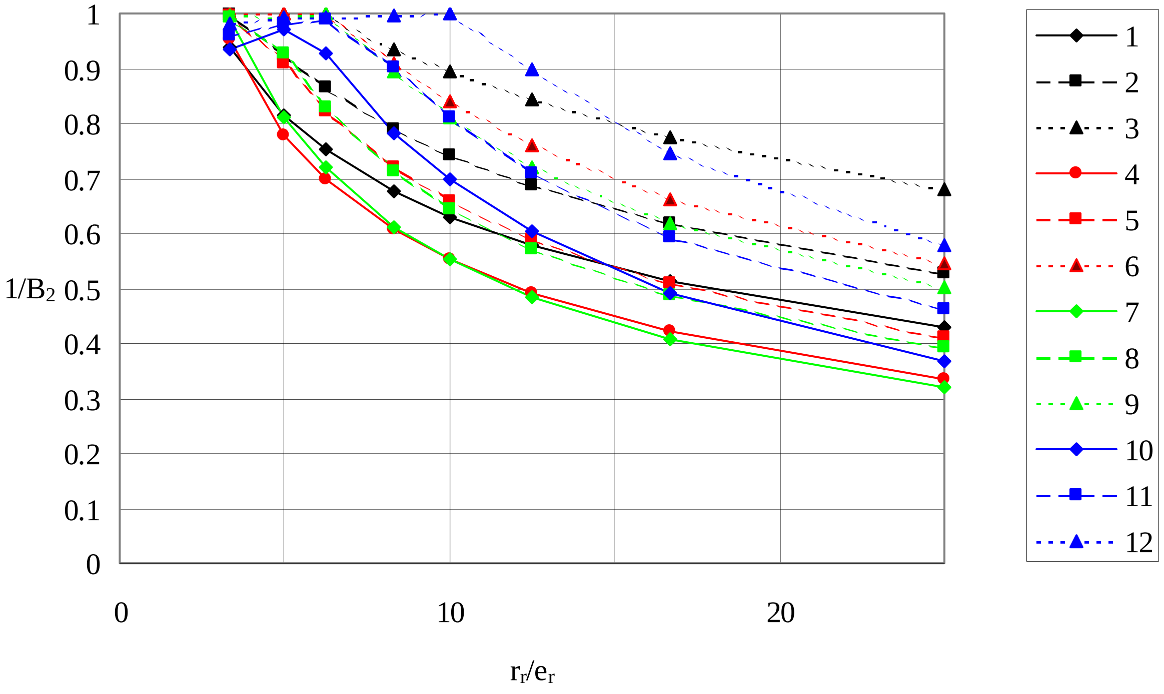


Figure 4 : Branch torsion

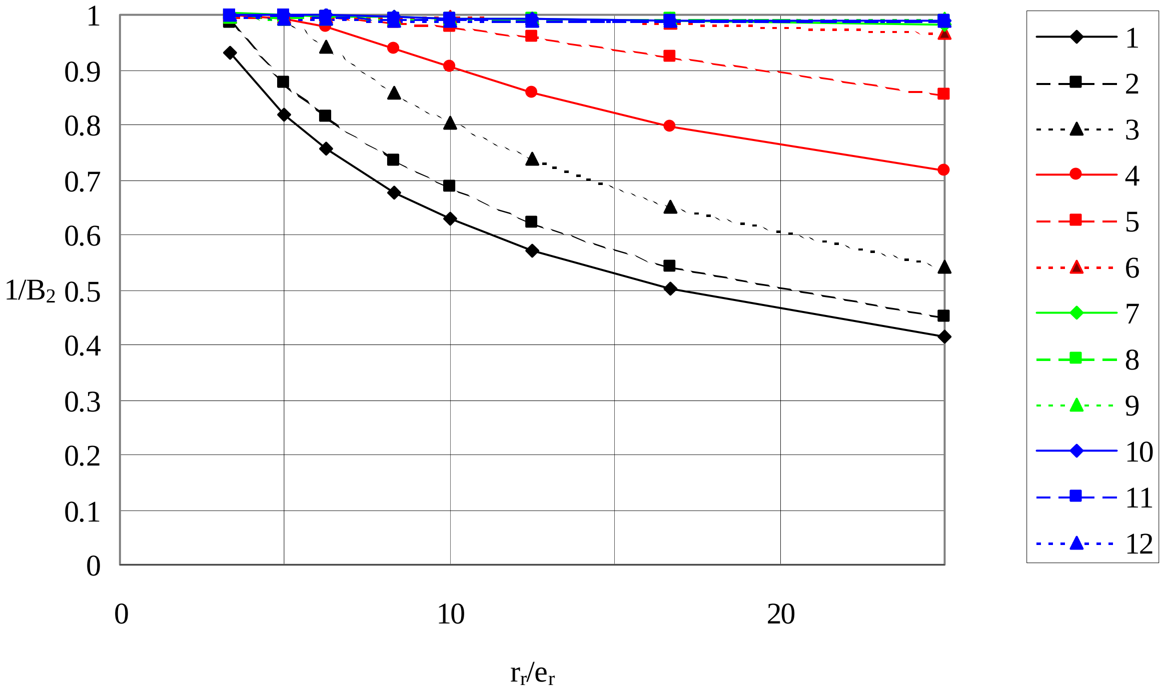


Figure 5 : Run in plane bending

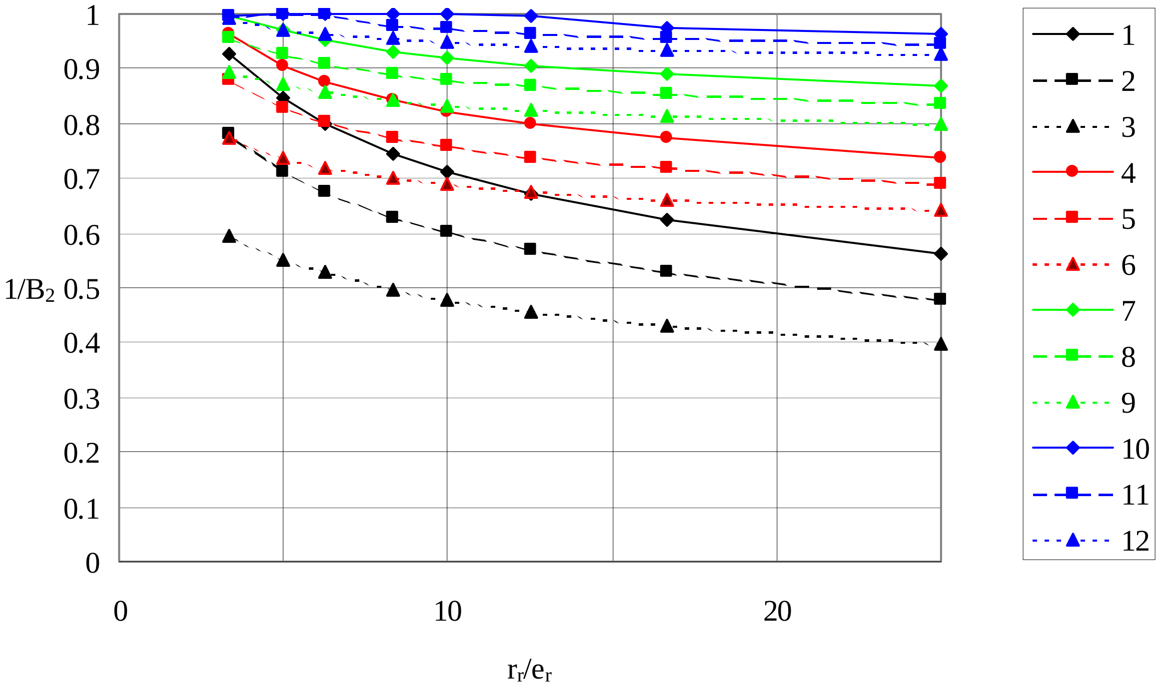


Figure 6 : Run out of plane plane bending

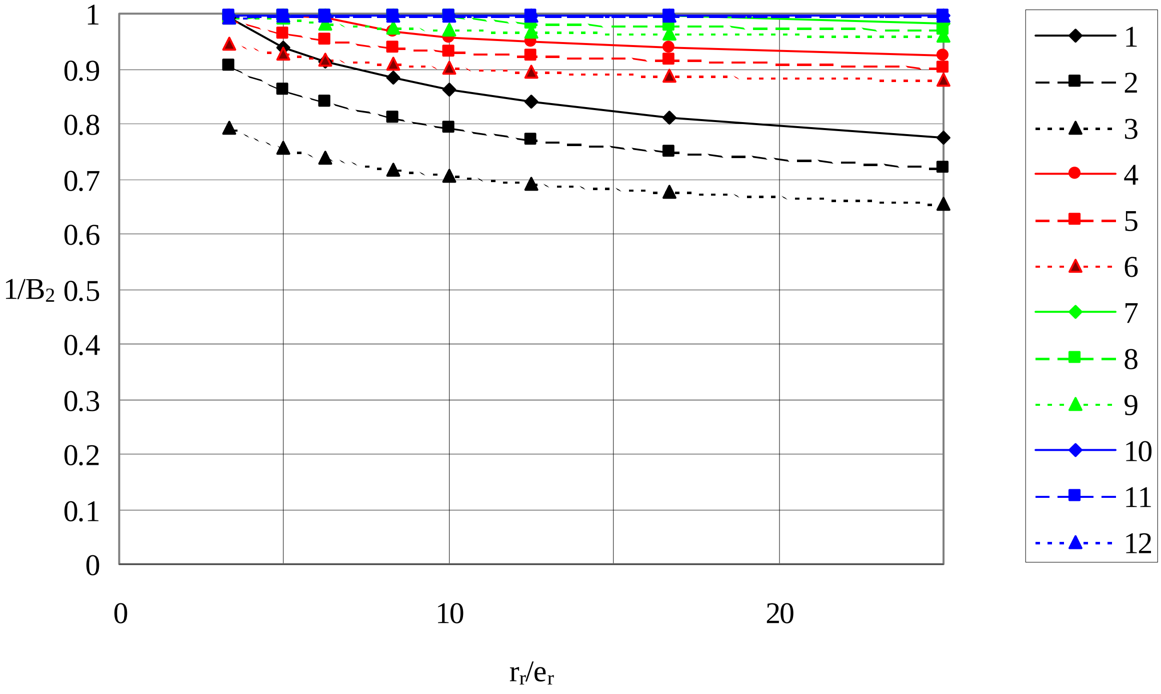


Figure 7 : Run torsion

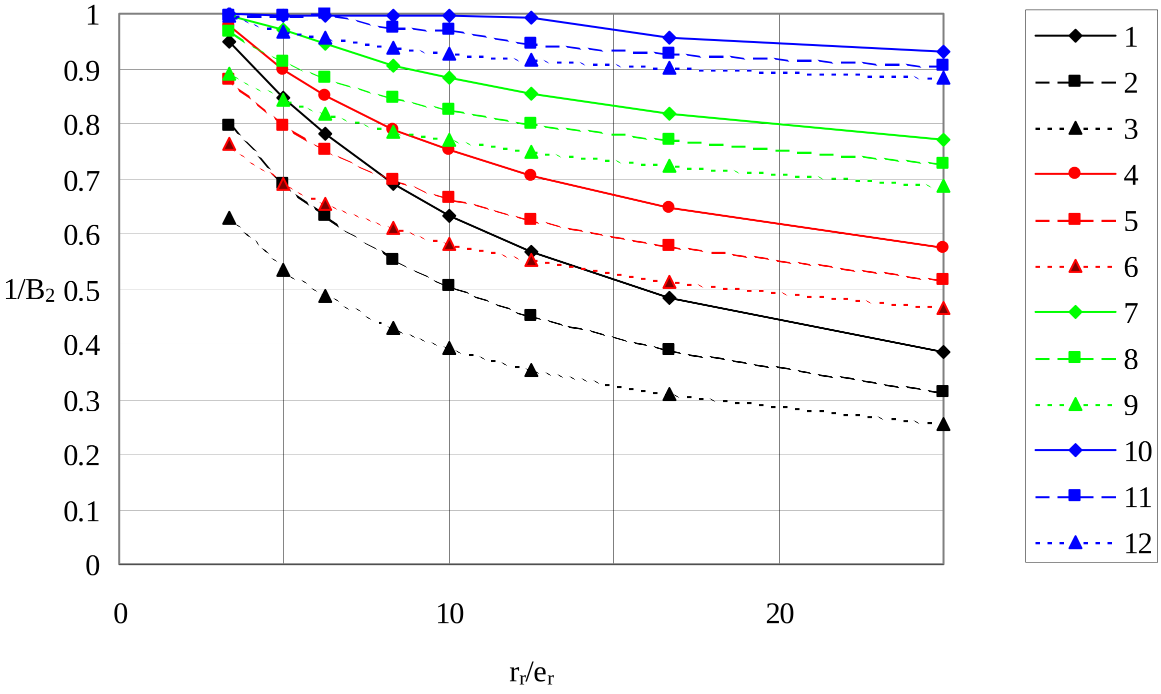
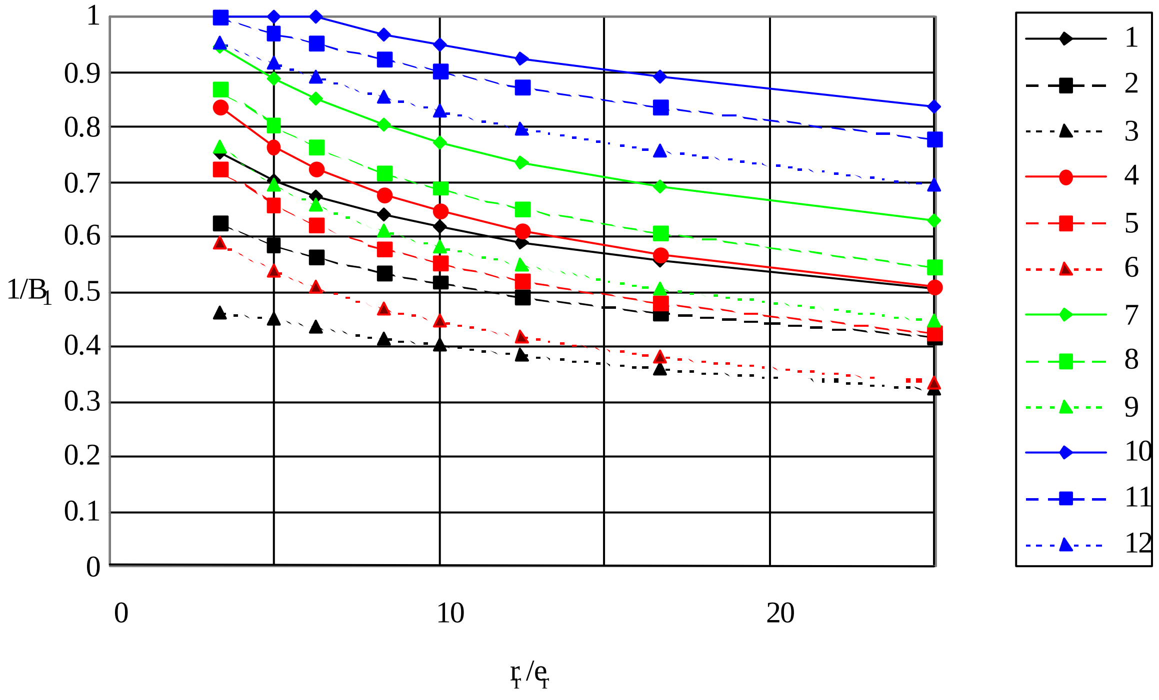


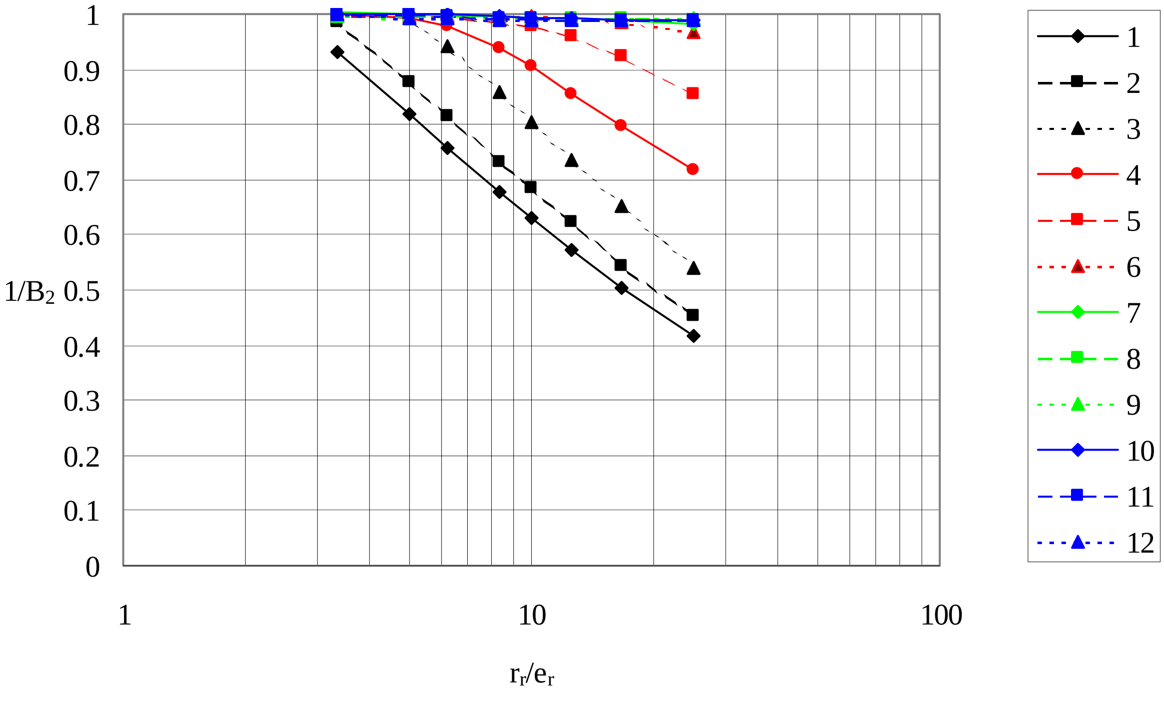
Figure 8 : Pressure



A comparative study of these figures leads to the following comments :

- Always check that $1/B_2$ is ≤ 1
- $1/B_2$ decreases as r_r /e_r increases and saturation at 1 occurs for low values of r_r/e_r.
- The curves obtained for r_r/e_r on a logarithmic scale, are similar to straight line segments, as shown in figure 9 below :

Figure 9 : Branch torsion



- For the loadings in the run, the thickest branch is the most favourable (largest value of $1/B_2$)
- For the loadings in the branch and under pressure, the thin branch is the most favourable, which is what one would expect, since in this case, the failure is related to that of the branch.
- The values of $1/B_2$ are between 0.3 and 1, except for the pressure loading case.
- When comparing the unit loadings in particular for an equal tee ($r_r = r_b$, $e_r = e_b$), these moment loadings are in the following order :

run torsion <	branch in plane bending	< run out of plane bending
	≈ branch out of plane bending	
	≈ branch torsion	
	≈ run in plane bending	
Min. value	Min. value	Min. value
≈ . 3	≈ . 4	≈ . 6

- This order is generally applicable to pipe branch connections.
- Generally, except for the pressure case, $1/B_2$ increases as the ratio of radii decreases, which is what one would expect.

CONCLUSION

Very thorough calculations of the limit loads in pipe tees and branch connections were performed using an elastic compensation method.
The geometric parameters for this study were the ratio of the radius over the thickness of the run, the ratio of radii for the run and the branch and the ratio of the thicknesses of the run and the branch. The loadings calculated were unit loadings for pipe tees and branch connections : bending in the plane of the branch and the run, bending out of the plane of the branch and the run, torsion of the branch and the run, and internal pressure.
This study therefore covered the calculation of 672 limit loadings in a wide range of geometries.
The results of this study are of great benefit in all fields where limit analysis is required : design of pipes and determination of reference stresses for failure mechanics or large scale plasticity criteria for pipes.

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