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Study on Fatigue Crack Closure in High Strain Region of Engineering Structure*

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ABSTRACT

Fracture and fatigue in the high strain region of engineering structures have called a great deal of attention in these years. Additional crack closure resulting from plastic zone in structures plays an important role in the estimation of the fatigue life of engineering structures with high strain region. The fatigue crack closure in high strain plastic zone of the structure is studied. The plastic deformation in the stress concentration region of the structures can result in the additional crack closure. The fatigue crack closure results from the plastic deformation of both the structure and the fatigue crack tip, when the fatigue crack propagates in the plastic zone of the structure. The additional crack closure factor U_p, resulting from high strain plastic zone in the structure, is defined. The fatigue crack closure factor U in the high strain plastic zone of the structures can be calculated by $U = U_e U_p$, where U_e is the crack closure factor because of the fatigue crack itself, that is the normal crack closure, and U_p is the additional crack closure factor resulted from high strain plastic zone in the structure.

A model analyzing the additional crack closure factor U_p is established, and a method estimating U_p is proposed. According to the analysis of the elastic-plastic response for the crack tip in the high strain region of the structures, the equivalent loading sequence for the elastic-plastic response for the crack tip in the high strain region is proposed, in which the additional crack closure factor U_p can be calculated.

The results show that the U_p of the crack in the plastic deformation zone of the structure is less than 1.0, and makes fatigue crack growth rate decreasing, and that out of the plastic zone is greater than 1.0, and makes the fatigue crack growth rate increasing. The results for fatigue test of the SCT specimens show that the U_p gotten from the analytical model proposed in this paper accords with the U_p from the tests.

KEY WORDS: high strain region, plastic zone, fatigue crack closure, additional crack closure factor, SCT specimen

INTRODUCTION

Fatigue crack closure is a very important factor to affect fatigue crack growth. It has called a great deal of attention in these years and much study for it has been done by the researchers all over the world[1-5], since it is proposed by Elber[1]. Most of researches are associated with the crack growth in elastic zone, so the crack closure results from only the crack itself. The plastic deformation in the stress concentration region of the structures can result in the additional crack closure, when the fatigue crack propagates in high strain plastic zone. Additional crack closure resulting from plastic zone in the structures plays an important role in the evaluation of the fatigue life of the engineering structures with high strain region. The plastic deformation in high strain region makes the fatigue crack postpone to open. The fatigue crack closure results from the residual plastic deformation of both the structure and the fatigue crack tip, when the fatigue crack propagates in the plastic zone of the structure. The fatigue crack in high strain plastic zone closes more evidently than that in elastic zone. An analysis model is established and the fatigue crack closure in the high strain plastic zone of the structure is studied for the reasonable estimation of the fatigue life of the structures with high strain region.

ELASTIC-PLASTIC ANALYSIS OF CRACK TIP IN HIGH STRAIN REGION

1) Residual stresses in high strain region of structures

When an engineering structure is acted on by load P_1, the elastic stress distribution in the high strain region of the structure can be expressed as:

$$\frac{\sigma_e(x)}{\sigma_{ys}} = c_0 + c_1(x/x_0) + c_2(x/x_0)^2 + c_3(x/x_0)^3 \tag{1}$$

where x_0 is an characteristic size of the structure. x_0 is the inner nozzle diameter for the nozzle of a pressure vessel. The elastic stress is $\sigma \geq \sigma_{ys}$ in the zone $x \leq x_{p1}$. Supposing the relationship between stress and strain of the material to be elastic-perfectly plastic relation, the size of plastic zone will extend to x_{p2} because of stress loose in the yielded zone

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of the structure. The elastic-plastic stress distribution is shown in the upperest curve in Fig.1 under plane stress condition.

The residual stresses are produced in the structure, when the structure is unloaded from P_1 to zero. The stresses in the high strain region descend rapidly during unloading. Stresses in local area are lower than minus yielding stress of the material and the structure in this area reaches reverse yielding. The stress increment resulting in reverse yielding is $2\sigma_{ys}$ according stress-strain response curve shown in Fig.1. The size of the reverse yielded zone is x_{p4}, when the load is researched zero during unloading. The stress increment distribution is shown in the lowest curve in Fig.1 under unloading $\Delta P = P_1$.

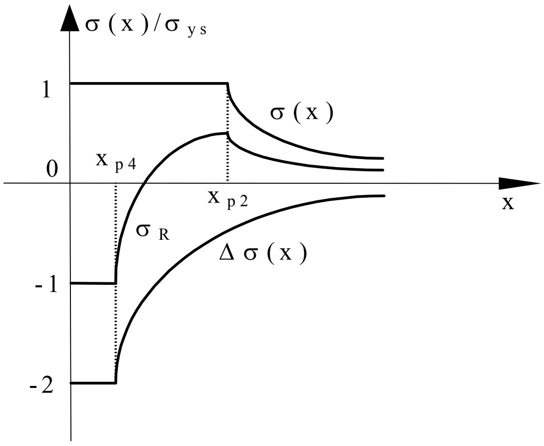

Fig.1 The residual stress distribution of structures with high strain region

The residual stress $\sigma_R(x)$ in the high strain region of the structure with no crack under full unloaded is constructed by superposition of the stress $\sigma(x)$ under full loaded and the stress increment $\Delta\sigma(x)$ under full unloaded according to plastic superposition method by Rice[6]. The $\sigma_R(x)$ is shown in the middle curve in Fig.1.

When $0 \le x \le x_{p4}$: $\sigma_R / \sigma_{ys} = -1$ (2)

When $x_{p4} \le x \le x_{p2}$: $\frac{\sigma_R}{\sigma_{ys}} = 1 - c_0 - c_1\left(\frac{x-\Delta x_{p2}}{x_0}\right) - c_2\left(\frac{x-\Delta x_{p2}}{x_0}\right)^2 - c_3\left(\frac{x-\Delta x_{p2}}{x_0}\right)^3$ (3)

where $\Delta x_{p1} = x_{p2} - x_{p1}$ • $\Delta x_{p2} = x_{p4} - x_{p3}$ • x_{p1} and x_{p3} are the monotony loading plastic zone and cyclic plastic zone without stress loose, respectively.

When $x \ge x_{p2}$:

$$\frac{\sigma_R}{\sigma_{ys}} = c_1\left(\frac{\Delta x_{p2} - \Delta x_{p1}}{x_0}\right) + c_2\left[\left(\frac{x-\Delta x_{p1}}{x_0}\right)^2 - \left(\frac{x-\Delta x_{p2}}{x_0}\right)^2\right] + c_3\left[\left(\frac{x-\Delta x_{p1}}{x_0}\right)^3 - \left(\frac{x-\Delta x_{p2}}{x_0}\right)^3\right] \quad (4)$$

2) Elastic-plastic response at crack tip in high strain region

If there is a fatigue crack whose length is *a* in a high strain region, the residual stress distribution is shown in eq.(2)-(4) before loaded. It is supposed that the residual stress distribution of the structure does not change, when the crack propagates in the high strain region. The structure with the crack is acted on by a cyclic load *0-P*. The stress intensity factor of the crack is *K*, when the load reaches *P*. The stress distribution around the crack tip at loaded *P* is gotten from the superposition of the residual stress distribution σ_R and the stress increment distribution around the crack tip resulting from the load *P*. The stress distribution of the structure comes back to the original residual stress distribution, when the structure is unloaded to zero.

The width of the cyclic yielding zone at the crack tip resulted from load increment acting on crack body is ω_0 for the plane stress condition.

$$\omega_0 = \frac{1}{\pi}\left(\frac{K}{2\sigma_{ys}}\right)^2 \quad (5)$$

The stress increment distribution $\Delta\sigma_p(x)$ along the crack tip resulting from the stress intensity factor increment is:

when $0 \le (x-a) \le \omega_0$: $\Delta\sigma_p(x) = 2\sigma_{ys}$ (6)

and when $(x-a)\geq\omega_0$: $\Delta\sigma_p(x)=\dfrac{K}{\sqrt{2\pi(x-a-\omega_0/2)}}$ (7)

When $0\leq a\leq(x_{p4}-\omega_0)$, the loading plastic zone is less than the unloading plastic zone. The unloading plastic zone decreases with the crack propagation, and the loading plastic zone equals the unloading plastic zone at $a=(x_{p4}-\omega_0)$. When $a\geq(x_{p4}-\omega_0)$, the loading plastic zone increases with the crack propagation, while the unloading plastic zone is always ω_0. Suppose the greater between the loading plastic zone and the unloading plastic zone as ω_m^e, and the smaller as ω_C, respectively.

When $0\leq a\leq(x_{p4}-\omega_0)$: $\omega_c=\omega_0=\dfrac{1}{\pi}\left(\dfrac{K}{2\sigma_{ys}}\right)^2$ (8)

$$\omega_m^e=x_{p4}-a \tag{9}$$

When $(x_{p4}-\omega_0)\leq a\leq a_p$, ω_c is also calculated by Eq.(8). The a_p is the crack length when the front of the crack tip plastic zone is located in the point where the residual stress of the structure σ_R is zero.

$$x_p=a+\omega_m^e \tag{10}$$

When the structure is loaded from zero to *P*, the stress increment with reverse yielding at the front of the crack tip plastic zone is $(\sigma_{ys}-\sigma_R)$. The size of the plastic zone according to *Rice* analysis method is:

$$\omega_m^e=\frac{1}{\pi}\left(\frac{K}{\sigma_{ys}-\sigma_R}\right)^2=\frac{1}{\pi}\left(\frac{K}{2\sigma_{ys}}\right)^2\left(\frac{2\sigma_{ys}}{\sigma_{ys}-\sigma_R}\right)^2=4\omega_0\left(\frac{\sigma_{ys}}{\sigma_{ys}-\sigma_R}\right)^2 \tag{11}$$

When $(x_{p4}-\omega_0)\leq a\leq a_p$, x_p meets the condition $x_{p4}\leq x_p\leq x_{p2}$,

$$\omega_m^e\left[c_0+c_1\left(\frac{\omega_m^e+a'}{x_0}\right)+c_2\left(\frac{\omega_m^e+a'}{x_0}\right)^2+c_3\left(\frac{\omega_m^e+a'}{x_0}\right)^3\right]^2-4\omega_0=0 \tag{12}$$

where $a'=a-\Delta x_{p2}$. The plastic zone ω_m^e can be calculated with Eq.(12).

The ω_m^e increases with the increase of the crack length *a*. We know from Eq.(11) that $\omega_m^e=4\omega_0$, when the front of the crack tip plastic zone reaches the point $\sigma_R=0$. The monotony and the cyclic plastic zones of the crack in the high strain region of the structure are the same as those in the non-high strain region at this crack length, and the fatigue crack closure in the high strain region of the structure equals that in the non-high strain region. The characteristic crack size for the structure plastic zone affecting the fatigue crack closure is:

$$a_p=x_{p1}+\Delta x_{p2}-4\omega_0 \tag{13}$$

When the crack length $a\leq a_p$, the residual stress of the structure in the front of the crack tip plastic zone is compressive stress, that is $\sigma_R\leq 0$. The plastic zone of the structure results in the increase of the fatigue crack closure and the decrease of the effective stress intensity factor amplitude.

When $a_p<a\leq(x_{p2}-\omega_m^e)$, the front of the crack tip plastic zone reaches the positive residual stress region of the structure. That is, $\sigma_R>0\bullet\omega_m^e>4\omega_0$ at $x_p=a+\omega_m^e$, where σ_R and ω_m^e are calculated by Eqs.(9) and (12), respectively.

When $a>(x_{p2}-\omega_m^e)$, $x_p>x_{p2}$, $\omega_m^e\left(1-\dfrac{\sigma_R}{\sigma_{ys}}\right)^2-4\omega_0=0$ (14)

Where

$$\frac{\sigma_R}{\sigma_{ys}}=c_1\left(\frac{\Delta x_{p2}-\Delta x_{p1}}{x_0}\right)+c_2\left[\left(\frac{\omega_m^e+a-\Delta x_{p1}}{x_0}\right)^2-\left(\frac{\omega_m^e+a-\Delta x_{p2}}{x_0}\right)^2\right]+c_3\left[\left(\frac{\omega_m^e+a-\Delta x_{p1}}{x_0}\right)^3-\left(\frac{\omega_m^e+a-\Delta x_{p2}}{x_0}\right)^3\right] \tag{15}$$

ESTABLISHMENT OF THE EQUIVALENT STRESS SEQUENCE

Matsuoka supposed that cracks with the same cyclic plastic response propagate at the same rate [7-8]. The equivalent stress sequence of the fatigue crack propagation under a single overload in constant amplitude loading was established based on the supposition. The equivalent stress sequence effect on the plastic response at the crack tip is the same as the overload effect. The fatigue crack growth rate under the equivalent stress sequence equals that under a single overload in constant amplitude loading. They calculate the fatigue crack closure and the crack growth rate with the equivalent stress sequence method. The calculation results accord with test results of three steels and two aluminum alloys.

The supposition of Matsuoka is used to establish the equivalent stress sequence for the plastic response at the crack tip in the high strain region. Supposing an infinite plate with a *2a* long central through-thickness crack loaded by a constant amplitude stress, the cyclic stress range is $\Delta\sigma^e$ and the stress ratio is R_e. The monotony and the cyclic plastic zones at the crack tip in the infinite plate under $\Delta\sigma^e$ and R_e are equal to those in the high strain region of structures with the same crack length, therefore the $\Delta\sigma^e$ and R_e are determined by the plastic zones at the crack tip in the high strain region of the structures. The $\Delta\sigma^e$ and R_e are the equivalent stress range and the equivalent stress ratio of the crack in the high strain region of the structures. The $\Delta\sigma^e$ and R_e change generally with the crack length. The stress sequence with $\Delta\sigma^e$ and R_e is the equivalent stress sequence of the crack in the structure high strain region. The smaller ω_c and the greater ω_m^e of the plastic zone in loading and unloading are defined as the cyclic plastic zone ω_c^e and monotony plastic zone ω_m^e of the equivalent stress sequence for the fatigue crack in high strain region, respectively.

$$\Delta\sigma^e = 2\sigma_{ys}\sqrt{\omega_0 / a} \tag{16}$$

When $0 \le a \le \left(x_{p4} - \omega_0\right)$, the unloading plastic zone is larger than the loading plastic zone. ω_m^e is regarded as the monotony plastic zone resulted from compressive stress in the equivalent stress sequence. The absolute value of the compressive stress $\sigma_{\min}^e$ is larger than the tensile stress $\sigma_{\max}^e$, so the stress ratio of the equivalent stress cycle is $R_e \le -1$. $\omega_m^e = \omega_c^e = \omega_0$, $R_e = -1$ at the $a = \left(x_{p4} - \omega_0\right)$, the cyclic plastic zone at the crack tip equals the monotony plastic zone.

When $a \le \left(x_{p4} - \omega_0\right)$,

$$\omega_m^e = \frac{1}{\pi}\left(\frac{K_{\min}^e}{\sigma_{ys}}\right)^2 = \frac{1}{\pi}\left(\frac{\Delta K^e}{\sigma_{ys}}\right)^2\left(\frac{K_{\min}^e}{\Delta K^e}\right)^2 = 4\omega_0\left(\frac{\sigma_{\min}^e}{\sigma_{\max}^e - \sigma_{\min}^e}\right)^2 = 4\omega_0\left(\frac{R_e}{1-R_e}\right)^2 \tag{17}$$

$$\frac{1}{R_e} = 1 - \sqrt{\frac{4\omega_0}{x_{p4} - a}} \qquad \left(a \le x_{p4} - \omega_0\right) \tag{18}$$

If the residual plastic zone x_{p4} of the structure is large and the crack is short, the maximum stress in the equivalent stress sequence may be negative. This part of equivalent stress sequence is full compressive cycle, while R_e from Eq. (18) is positive. But it is different from the positive R_e with the full tensile cycle. If the fatigue crack closes totally under the compressive stress, the fatigue crack will not propagate in this part of the equivalent stress sequence.

When $a \ge \left(x_{p4} - \omega_0\right)$, the loading plastic zone is larger than the unloading plastic zone. ω_m^e is determined by $\sigma_{\max}^e$, and $R_e \ge -1$. If $a < a_p$, $R_e < 0$. If $a = a_p$, $R_e = 0$. If $a > a_p$, $R_e > 0$.

$$\omega_m^e = 4\omega_0\left(\frac{1}{1-R_e}\right)^2 \tag{19}$$

$$R_e = 1 - \sqrt{\frac{4\omega_0}{\omega_m^e}} \qquad \left(a \ge x_{p4} - \omega_0\right) \tag{20}$$

When $\sigma_R = 0$, $\omega_m^e = 4\omega_0$, We can get $R_e = 0$ from Eq.(20). Therefore, the equivalent stress sequence is tension ~ compression cycle or compression ~ compression cycle, if the front of the plastic zone at the crack tip in high strain region of the structure is located in the residual compressive stress region. The equivalent stress sequence is tension ~ tension cycle, if the front of the plastic zone at the crack tip is located outside the residual compressive stress region.

DETERMINING CRACK CLOSURE FACTOR *U* IN HIGH STRAIN REGION

Paris' law with the crack closure is:

$$\frac{da}{dN} = A\left(\Delta K_{eff}\right)^m = A\left(U\Delta K\right)^m \tag{21}$$

where U is fatigue crack closure factor, $U = \Delta K_{eff} / \Delta K$. For the fatigue crack growth in the area of structure without stress concentration, $U = U_e$, where U_e is the fatigue crack closure factor resulting only from the crack itself. The fatigue crack for $U_e = 1.0$ is called as ideal crack, and the fatigue crack does not close during the whole cycle. The opening stress and the closing stress is zero for the ideal crack with R=0.

The high strain plastic zone of the structure can change the fatigue crack closure. The crack closure is composed of two parts: one resulting from the fatigue crack itself and another from the strain plastic zone of the structure.

The $0 \bullet P$ fatigue cycle often used in engineering will be considered, so the common fatigue crack closure U_e without high strain plastic region of the structures is calculated with stress ratio R=0.

The U_p is the additional crack closure factor resulted from high strain plastic zone in the structure, which is calculated from the equivalent stress sequence of the high strain region of the structure. Supposing the crack to open totally at $\sigma^e = 0$ of the equivalent stress sequence for the calculation of U_P, the tensile stress of the equivalent stress sequence makes the fatigue crack propagation, so $\Delta\sigma^e_{peff} = \sigma^e_{\max}$.

For the compression ~ compression cycle in the equivalent stress sequence, U_p is $U_p = 0$.

For the tension ~ compression cycle in the equivalent stress sequence, U_p is

$$U_p = \frac{1}{1 - R_e} \tag{22}$$

If $R_e \le 0$, $U_p \le 1.0$, it means that the residual compressive stress in high strain region results in the fatigue crack propagation rate decreasing. If $R_e > 0$, $U_p > 1.0$, the residual tensile stress in high strain region results in the fatigue crack propagation rate increasing.

$$U_e = \frac{\Delta\sigma^e_{eff}}{\Delta\sigma^e_p} \tag{23}$$

$$U = \frac{\Delta\sigma^e_{eff}}{\Delta\sigma^e} = \frac{\Delta\sigma^e_{eff}}{\Delta\sigma^e_p} \cdot \frac{\Delta\sigma^e_p}{\Delta\sigma^e} = U_e U_p \tag{24}$$

Eq.(24) is the fatigue crack closure factor after the high strain plastic region of the structure is considered.

COMPARISON BETWEEN TEST RESULTS AND CALCULATION RESULTS

The fatigue crack closure and crack growth rate in high strain region of SCT specimens of 16MnR steel are measured[9]. The calculated and measured additional fatigue crack closure factors U_p are compared in Fig.2. It is shown that the calculated U_p is in accordance with the measured U_p in the high strain region. When the crack is short, the fatigue crack closure is not evident because the initial crack is a saw cut.

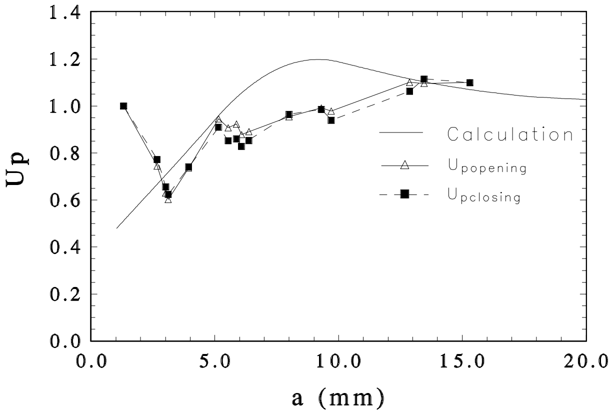

Fig.2 Comprison of the calculated and measured U_p of SCT specimens

CONCLUSION

1) The effect of the structure high strain plastic zone on the fatigue crack closure is studied. It is proposed that the high strain plastic zone in the structures can result in the additional fatigue crack closure. The crack closure is composed of two parts: one resulting from the plastic zone of fatigue crack tip itself and another from the strain plastic zone of the structures.

2) The additional crack closure factor U_p, resulting from high strain plastic zone in the structure, is defined. The fatigue crack closure factor U in the high strain plastic zone of the structures can be calculated by $U = U_e U_p$. An analysis model for calculating the additional crack closure factor U_p is established.

3) The calculation method of crack tip plastic zone of elastic-perfectly plastic materials is proposed based on elastic-plastic response at the crack tip in high strain region. The equivalent stress sequence for the plastic response at the crack tip in the high strain region is established, according to the supposition "cracks with the same cyclic plastic response propagate at the same rate".

4) The results show that the U_p of the crack in of the structure is less than 1.0, and the plastic deformation zone makes fatigue crack growth rate decreasing, and $U_p = 0$ and the crack will not propagate if the plastic region is large and the crack is small. The fatigue crack growth rate increases out of the plastic zone. The results for fatigue test of the SCT specimens show that the U_p gotten from the analytical model proposed in this paper accords with the U_p from the tests.

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