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Multiaxial Low Cycle Fatigue Life Prediction Criteria: Comparisons and Results

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ABSTRACT

In this paper the suitability of several multiaxial low cycle fatigue (LCF) criteria is discussed by applying these criteria to the experimental data of 08X18H10T stainless steel, two titanium alloys (BT9 and BT1-0) and Inconel 718. The study has shown that "classical" multiaxial fatigue criteria (the maximum-normal-stress criterion, the maximum-shear-stress criterion and the von Mises' criterion) might be applied successfully for life prediction of the alloys in case of proportional loading. But the best correlation between the experimental and calculated data under proportional loading will be achieved when calculations are made according to the two-parameter criteria of Pisarenko-Lebedev and Coulomb-Mohr.

The influence of non-proportional loading on LCF life won't be considered if "classical" criteria are used. Therefore, two multiaxial LCF criteria for non-proportional loading are discussed too.

The investigation has shown that Garud's energy approach and non-proportional strain parameter by Itoh T, Sakane M, Ohnami M. and Socie D.F. are able to predict non-proportional lives with various strain histories. The data scatter appears to be somewhat smaller in the correlation with non-proportional strain parameter.

On the one hand, the calculating by Garud's approach is harder then calculating of non-proportional strain parameter. But on the other hand, the estimation of low-cycle fatigue life by Garud's approach doesn't need non-proportional testing. It should be noted that the non-proportional strain parameter has the material constant, which is determined by the stress range ratio under 90 degrees out-of-phase and proportional loadings.

KEY WORDS: multiaxial low cycle fatigue, fatigue criteria, tension-torsion loading, non-proportional loading, equivalent strain, fatigue life prediction, 08X18H10T stainless steel, BT9 titanium alloy, BT1-0 titanium alloy.

INTRODUCTION

The safety and durability of structures is an important issue because the sudden failure of complex system such as nuclear power plants, automobiles, aircraft and pressure vessels may cause many injuries, much financial loss and even environmental damage. Evaluation of low-cycle fatigue (LCF) becomes one of the major considerations in the design structures, because many of these systems are subjected to the repeated multiaxial loading,

Many multiaxial fatigue theories have been developed since 1900. At present, more than 60 criteria of fatigue strength for multiaxial loading are known [1]. In many cases, when calculating the limiting state of structure materials under multiaxial low cycle loading the same criteria are used as under static loading. For example, the Maximum Shear-Stress Criterion (Tresca's criterion) is used in a standard of strength calculation of equipment and pipelines for nuclear power installations [2] and the Maximum Distortion-Energy Criterion (von Mises' criterion) is recommended in ASME code [3].

Both under static loading and at high cycle fatigue, criteria of limiting state are generally formulated through equivalent stresses:

$$\varepsilon_I(t) = \begin{cases} |\varepsilon_1(t)| & \text{for} \quad |\varepsilon_1(t)| \geq |\varepsilon_3(t)| \\ |\varepsilon_3(t)| & \text{for} \quad |\varepsilon_1(t)| < |\varepsilon_3(t)| \end{cases}, \qquad (1)$$

where ε_{Imax} are principal stresses; $\varepsilon_I(t)$ are material constants.

According to the assignment of the criterion, the value of limited stress ε_{Imax} can be equal to ultimate strength σ_u, yield stress $\sigma_{0.2}$, endurance limit $\varepsilon_I(t)$. Material constants $m_1, m_2, ...$ are determined experimentally for different types of stress state.

If the determination of criterion's parameters needs carrying out one basic test (testing is being realized only under one type of stress state), this criterion will be one-parametric. The criterion needed two basic tests is two-parametric, etc. Increasing a number of basic tests improves an approximation property of criteria though, it greatly complicates their using. Therefore, the number of basic tests doesn't usually exceed three tests.

Amplitudes of plastic and total strain is used for describing the curves of low cycle fatigue. Therefore, the criteria of multiaxial low cycle fatigue should be formulated as equivalent strains.

In the present work, the multiaxial low cycle fatigue criteria are formulated as the strain analogies of known static strength criteria. The advantage of these criteria is its simplicity.

Also, two multiaxial low cycle fatigue criteria for nonproportional loading are discussed. Multiaxial low cycle fatigue criteria are applied to the experimental data and the suitability of the criteria is examined.

PROPORTIONAL LOADING

One-parameter criteria

The Maximum Shear-Stress Criterion of Fatigue Failure and the Maximum Distortion-Energy Criterion of Fatigue Failure are extensions of the Tresca and Von Mises static theories, respectively.

In terms of principal strains ε_1, ε_2 and ε_3, the Maximum Shear-Stress Criterion of Fatigue Failure may be written as

$$\varepsilon_{eqM} = \frac{1}{\sqrt{2}\cdot(1+\nu)} \left\{ (\varepsilon_{1a}-\varepsilon_{2a})^2 + (\varepsilon_{2a}-\varepsilon_{3a})^2 + (\varepsilon_{3a}-\varepsilon_{1a})^2 \right\}^{1/2} \le \varepsilon_{fs}, \quad (2)$$

where $\varepsilon_{1a}, \varepsilon_{2a}, \varepsilon_{3a}$ are principal strain amplitudes; ν - elasto-plastic value of Poisson's ratio.

Another equivalent strain formula is based on the Tresca's criterion, giving

$$\varepsilon_{eqT} = \frac{1}{(1+\nu)} (\varepsilon_{1a}-\varepsilon_{3a}) \le \varepsilon_{fs}. \quad (3)$$

For elastic conditions, the maximum principal stress is given by

$$\sigma_1 = E\left\{\varepsilon_1(1+\nu_e) + \nu_e(\varepsilon_1+\varepsilon_2+\varepsilon_3)\right\} / \left\{(1+\nu_e)(1-2\nu_e)\right\}, \quad (4)$$

where E and ν_e are Young's modulus and the elastic Poisson's ratio. For proportional loading conditions, Eq. (4) may be extended to elasto-plastic conditions by replacing the modulus E with the secant modulus E_s, and the elastic Poisson's ratio ν_e with the elasto-plastic Poisson's ratio ν. Thus, according to the Maximum-Normal-Stress Criterion, an equivalent strain may be defined as σ_1 / E_s. The strain analogy of the Maximum-Normal-Stress Criterion (Renkine's criterion) is this

$$\varepsilon_{eqR} = \frac{1}{(1+\nu)} \left\{ \varepsilon_{1a} + \frac{\nu}{1-2\nu}(\varepsilon_{1a}+\varepsilon_{2a}+\varepsilon_{3a}) \right. \le \varepsilon_{fs}. \quad (5)$$

The Poisson's ratio can be obtained in the elastic-plastic range from the following relation [1]:

$$\nu = 05 - 02 \cdot \frac{\sigma_{02}}{E\cdot\varepsilon_{eq}}, \quad (6)$$

where ε_{eq} is the amplitude of equivalent strain according to some criteria.

Note, that Eq. (2), (3) and (5) have been derived for the case of fully reversed strain controlled cycling.

In these criteria, there is only one material constant, i.e. uniaxial fatigue limit strain amplitude ε_{fs}. Therefore, they may be called one-parameter criteria. The value of ε_{fs} comes from the uniaxial strain-life curve.

Two-parameter criteria

Among the well-known two-parameter criteria of static strength, Coulomb-Mohr and Pisarenko-Lebedev's criteria were chosen for discussion. The strain analogy of Coulomb-Mohr's criterion can be expressed in the form:

$$0{,}5\cdot\gamma_{max} + S\cdot\varepsilon_n = C, \quad (7)$$

where S and C are material constants. This equation is equivalent to Brown-Miller criterion [4]. According to this, two parameters such as the maximum shear strain amplitude $\frac{\gamma_{\max}}{2}$ and the amplitude of the normal strain acting on the γ_{max} plane ε_n are important for the case of fatigue failure.

Using the equation (7) in uniaxial tension-compression and alternating torsion, expressions of S and C can be found in the following forms:

$$S = \frac{1}{(1-\nu)}\left\{\frac{\gamma_{fs}}{\varepsilon_{fs}} - (1+\nu)\right\}, \quad C = \frac{\gamma_{fs}}{2}, \tag{8}$$

where γ_{fs} is shear strain amplitude being equal to conventional endurance limit in the simple torsion for a given number of cycles.

The relation (7) can be rewritten as:

$$\varepsilon_{eqCM} = \frac{\varepsilon_{fs}}{\gamma_{fs}} \cdot \gamma_{\max} + \frac{2}{(1+\nu)} \cdot \left\{1 - (1+\nu) \cdot \frac{\varepsilon_{fs}}{\gamma_{fs}}\right\} \cdot \varepsilon_n \le \varepsilon_{fs}. \tag{9}$$

This equation is the strain analogy of Coulomb-Mohr's criterion.

The linear variant of Pisarenko-Lebedev's criterion [5] has the following form:

$$\varepsilon_{eqM} = \lambda_1 \cdot \varepsilon_{eqR} + \lambda_0, \tag{10}$$

where λ_0, λ_1 are material constants obtained in the tension-compression and simple torsion tests:

$$\lambda_1 = \frac{2\varepsilon_{fs}(1+\nu) - \sqrt{3}\gamma_{fs}}{2\varepsilon_{fs}(1+\nu) - \gamma_{fs}}, \qquad \lambda_0 = \frac{(\sqrt{3}-1)\varepsilon_{fs}\gamma_{fs}}{2\varepsilon_{fs}(1+\nu) - \gamma_{fs}}. \tag{11}$$

After substitution the above-presented relation, we shall obtain Pisarenko-Lebedev's criterion in this form:

$$\varepsilon_{eqPL} = \chi_\varepsilon \cdot \varepsilon_{eqM} + (1-\chi_\varepsilon) \cdot \varepsilon_{eqR} \le \varepsilon_{fs} \tag{12}$$

where

$$\chi_\varepsilon = \frac{1}{\sqrt{3}-1} \cdot \left\{2 \cdot (1+\nu) \cdot \left(\frac{\varepsilon_{fs}}{\gamma_{fs}}\right) - 1\right\}. \tag{13}$$

Three-parameter criterion

Yu. I. Yagn [5] has proposed to find equations of the isotropic material limit surface in the form of a quadric polynomial, which is symmetric to three principal stresses:

$$\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right) + a \cdot (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) + b \cdot (\sigma_1 + \sigma_2 + \sigma_3) = c. \tag{14}$$

Buzhinsky [5] has obtained the same expression in another way. Thus, this relation is called Buzhinsky-Yagn's criterion. Material constants a, b, c are defined from three basic tests in the different stress state types. This criterion is three-parametrical.

The strain analogy of Buzhinsky-Yagn's criterion can be written in the form:

$$\frac{\mathrm{E}^2}{(1+\nu)^2}\left\{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right\} + \frac{\mathrm{A} \cdot \mathrm{E}^2}{(1-2\nu)^2} \cdot (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)^2 + \frac{\mathrm{B} \cdot \mathrm{E}}{(1-2\nu)} \cdot (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \mathrm{C}. \tag{15}$$

Material constants A, B, C can be determined like this:

$$A = \frac{3\cdot\left\{\left(1-\varepsilon_{pr}/\varepsilon_{fs}\right)\cdot\gamma_{fs}^2-\gamma_{pr}^2\right\}}{2\cdot(1+\nu)^2\cdot\varepsilon_{pr}\cdot\left(\varepsilon_{pr}-\varepsilon_{fs}\right)}-2,\ B = \frac{3\cdot E}{2\cdot(1+\nu)^2}\left\{\frac{\gamma_{fs}^2}{\varepsilon_{fs}}+\frac{\gamma_{fs}^2}{\varepsilon_{pr}}+\frac{\gamma_{pr}^2}{\varepsilon_{pr}\cdot\left(1-\varepsilon_{pr}/\varepsilon_{fs}\right)}\right\},\ C = \frac{3\cdot E^2\cdot\gamma_{fs}^2}{2\cdot(1+\nu)^2}. \quad (16)$$

After simple substitution Buzhinsky-Yagn's criterion may be expressed as

$$\varepsilon_{eqBY} = \frac{2\varepsilon_{fs}}{3\gamma_{fs}^2}\left\{(\varepsilon_{1a}-\varepsilon_{2a})^2+(\varepsilon_{2a}-\varepsilon_{3a})^2+(\varepsilon_{3a}-\varepsilon_{1a})^2\right\}+\frac{2\cdot A\cdot(1+\nu)^2\cdot\varepsilon_{fs}}{3\cdot(1-2\nu)^2\cdot\gamma_{fs}^2}(\varepsilon_{1a}+\varepsilon_{2a}+\varepsilon_{3a})^2+$$
$$+\frac{B\cdot(1+\nu)^2}{E\cdot(1-2\nu)}(\varepsilon_{1a}+\varepsilon_{2a}+\varepsilon_{3a})\le\varepsilon_{fs}. \quad (17)$$

This equation includes four material constants $\varepsilon_{fs}, \gamma_{fs}, \varepsilon_{pr}, \gamma_{pr}$. The last two constants are the limit amplitudes of an axial strain and shear strain for a certain type of proportional loading. These constants are interconnected.

Correlation of calculations with experimental data

Four kinds of materials were used to verify the selected criteria in this paper. They are Inconel 718, BT1-0 titanium alloy, BT9 titanium alloy and 08X18H10T steel. The results of Inconel 718 were derived from the Reference [6]. All the other results were taken from the Reference [7]. Mechanical properties of the materials are given in Table 1.

Table 1. Mechanical properties of the investigated materials

Nomenclature	Material			
	Inconel 718	BT1-0	BT9	08X18H10T
Young's module E, [GPa]	209	114	118	203
0.2% Yield Stress $\sigma_{0.2}$, [MPa]	1160	490	865	320
Ultimate Strength σ_u, [MPa]	1420	565	970	690
Elongation δ_5, [%]	-	26	17	40
Reduction of Area ψ, [%]	28	57	45	55
Poisson ratio ν, [-]	0,34	0,33	0,35	0,29

Biaxial in-phase fatigue test results and predicted lives using one-, two- and three-parameter criteria are compared in Table 2.

The mean relative error in fatigue life prediction was defined as

$$\bar{S} = \frac{1}{n}\left[\sum_{i=1}^{n}\left(\frac{N_{pi}-N_{fi}}{N_{fi}}\right)^2\right]^{0.5} \quad (18)$$

where n is number of observations, N_f and N_p are fatigue and predicted life, respectively.

According to Table 2, calculations by two-parameter criteria are much closer to the presented experimental results than calculations by one-parameter criteria in all cases. Buzhinsky-Yagn's criterion insignificantly improves the fatigue life prediction in comparison with the two-parameter criteria. Note, that the computation of ε_{eqBY} needs a definition of one more constant and additional tests for that. So the use of the two-parameter criteria under proportional low cycle loading is more preferable.

NON-PROPORTIONAL LOADING

Garud's energy approach

Garud [8] has suggested to correlate fatigue life to crack initiation N_f and the plastic work per cycle W_c, defined as:

$$W_c = \int_{cycle} \sigma_{ij} \cdot de_{ij}^p \text{ , } [\, MJ/m^3 \,] \tag{19}$$

where σ_{ij} is the stress tensor and de_{ij}^p is the plastic strain increments tensor.

Table 2. Mean relative errors in fatigue life prediction by one-, two and three-parameter criteria

Materials	Criterion	Mean relative errors in fatigue life prediction, $\overline{S}$		
		$10^2 < N_f < 10^3$	$10^3 < N_f < 10^4$	$10^4 < N_f < 10^5$
IN-718, T=20^{o}C, [5]	Renkine's	--	0.98	2.69
	Tresca's	--	0.36	0.30
	von Mises'	--	0.22	0.10
	Coulomb-Mohr's	--	0.12	0.12
	Pisarenko-Lebedev's	--	0.09	0.10
BT1-0, T=20^{o}C, [6]	Renkine's	0.84	1.26	--
	Tresca's	0.51	0.50	--
	von Mises'	0.40	0.39	--
	Coulomb-Mohr's	0.25	0.26	--
	Pisarenko-Lebedev's	0.24	0.25	--
BT9, T=20^{o}C, [6]	Renkine's	0.16	0.53	--
	Tresca's	0.38	0.67	--
	von Mises'	0.33	0.65	--
	Coulomb-Mohr's	0.25	0.28	--
	Pisarenko-Lebedev's	0.24	0.25	--
08X18H10T, T=20^{o}C, [6]	Renkine's	0.13	0.22	--
	Tresca's	0.26	0.56	--
	von Mises'	0.22	0.47	--
	Coulomb-Mohr's	0.12	0.19	--
	Pisarenko-Lebedev's	0.12	0.19	--
	Buzhinsky-Yagn's	0.09	0.14	--

In combined tension and torsion the general expression of the plastic work per cycle (19) is reduced to

$$W_c = \int_{cycle} \sigma_x \cdot d\varepsilon_x^p + r \cdot \int_{cycle} \tau_{xy} \cdot d\gamma_{xy}^p \text{ .} \tag{20}$$

In the equation r is a weighting factor at plastic shear strain energy determined in a test (for 08X18H10T steel r=0.5). The incremental theory of plasticity was used to describe relations between cyclic stresses and strains under multiaxial non-proportional loading. From the known load cycle and through a step-by-step procedure the parameter W_c is calculated. The following relation is postulated between the plastic work per cycle and fatigue life N_f:

$$N_f = F(W_c) = K \cdot (W_c)^{\eta} \text{ .} \tag{21}$$

The cyclic constants K, η were carried out from full reversed uniaxial fatigue tests. These values for 08X18H10T steel are: K = 5365 and η = -1.913. The predictions were made using the von Mises type of yield function.

Non-Proportional Strain Parametr by Itoh T, Sakane M, Ohnami M. and Socie D.F.

The authors [9] have proposed the following strain parameter, which correlates with non-proportional low cycle fatigue lives:

$$\Delta\varepsilon_{NP} = (1 + \alpha \cdot f_{NP}) \cdot \Delta\varepsilon_I \quad (22)$$

where α is a material constant related to the additional hardening. f_{NP} is the non-proportional factor, which expresses the severity of non-proportional straining and is described by only the strain history; $\Delta\varepsilon_I$ is the maximum principal strain range. The value of α is defined as the ratio of stress amplitude under 90-degree out-of-phase loading (circular strain path in $\gamma/\sqrt{3} - \varepsilon$ plot, f_{NP} =1) to amplitude under proportional loading (f_{NP} =0). The 90-degree out-of-phase loading shows the maximum additional hardening among all the non-proportional histories. For 08X18H10T stainless steel, the stress amplitude under the 90-degree out-of-phase loading was increased up to 95% in comparison with the proportional loading, so the value of α is 0.95. For BT9 titanium alloy, the difference between the equivalent stress amplitudes of proportional and non-proportional loadings is equal to 5% that coincides with the inaccuracy of the experiment.

The maximum principal strain range is defined as

$$\Delta\varepsilon_I = Max[\varepsilon_{Imax} - \cos\xi(t) \cdot \varepsilon_I(t)] \quad (23)$$

where $\varepsilon_I(t)$ is the maximum absolute value of principal strain at time t and is given by Eq. 24.

$$\varepsilon_I(t) = \begin{cases} |\varepsilon_1(t)| & for \quad |\varepsilon_1(t)| \geq |\varepsilon_3(t)| \\ |\varepsilon_3(t)| & for \quad |\varepsilon_1(t)| < |\varepsilon_3(t)| \end{cases} \quad (24)$$

where $\varepsilon_1(t)$ and $\varepsilon_3(t)$ are the maximum and minimum principal strains at time t, respectively.

The maximum value ε_{Imax} of $\varepsilon_I(t)$ is expressed as

$$\varepsilon_{Imax} = max[\varepsilon_I(t)] \quad (25)$$

$\xi(t)$ is the angle between ε_{Imax} and $\varepsilon_I(t)$ directions and it expresses the variation angle of the principal strain direction.

The non-proportional factor is defined as

$$f_{NP} = \frac{k}{T \cdot \varepsilon_{Imax}} \cdot \int_0^T (|\sin\xi(t)| \cdot \varepsilon_I(t)) \cdot dt \quad (26)$$

T is the time for a cycle; k is a constant to make until under 90-degree out-of-phase loading and is $\pi/2$. The calculating of non-proportional factor f_{NP} are presented in detail in the Reference [9].

Comparison of the calculated and actual fatigue lives

The relationship between total strain amplitude and LCF life under full reversed tension-compression loading of 08X18H10T steel specimens was obtained by least squares method. The equation is expressed as

$$lg\, N = 2{,}838 - 3{,}461 \cdot (lg\, \varepsilon_a + 2{,}313) \quad (27)$$

For non-proportional loading ε_a is equal to ε_{NP}, and Eq. (27) becomes

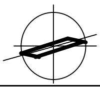
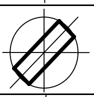
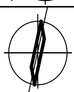
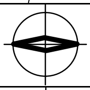
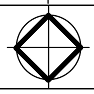
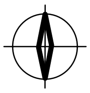
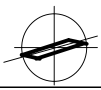
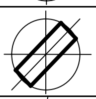
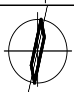
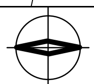
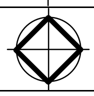
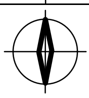
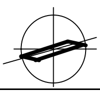
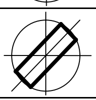
$$lg\, N = 2{,}838 - 3{,}461 \cdot (lg\, \varepsilon_{NP} + 2{,}313) \quad (28)$$

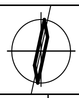
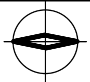
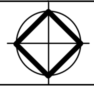
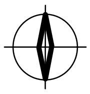
The comparison fatigue lives calculated by Eqs. 2, 21 and 28 with experimental data of Type 08X18H10T stainless steel are shown in Table 3.

The Table 3 shows that the Mises' equivalent strain doesn’t correlate with the non-proportional LCF lives. Garud's energy approach and non-proportional strain ε_{NP} proposed by Eqs. 21 and 28 are able to predict non-proportional lives with various strain histories. The data scatter appears to be somewhat smaller in the correlation with Non-Proportional Strain Parameter by Itoh T, Sakane M, Ohnami M. and Socie D.F.

On the one hand, the calculating by Garud's approach is harder then calculating of non-proportional strain ε_{NP}. But on the other hand, the estimation of LCF life by Garud's approach doesn't need non-proportional testing. It should be noted that the Eq. 22 has the material constant α which is determined by the stress range ratio under 90-degree out-of-phase and proportional loadings.

Table 3. Comparison of calculated fatigue lives with experimental data for 08X18H10T steel specimens [7]

Loading conditions					Test results	Predicted fatigue lives N_p [cycles] by equations:		
Paths	ε_{eqM}, [-]	ε_a, [-]	γ_a, [-]	f_{NP}, [-]	N_f,[cycles]	(21)	(28)	(2)
	0,006	0,00595	0,00258	0,151	143	259	209	333
	0,00445	0,00445	0,00775	0,435	221	161	193	937
	0,004	0,00099	0,00688	0,091	2094	2686	1493	1355
	0,005	0,00500	0,00217	0,210	276	415	333	626
	0,004	0,00400	0,00693	0,772	259	225	202	1355
	0,006	0,00150	0,01039	0,201	437	764	309	333
	0,004	0,00397	0,00172	0,151	655	796	852	1355
	0,00537	0,00537	0,00929	0,435	176	85	101	492
	0,005	0,00124	0,00855	0,091	1079	1390	698	626
	0,006	0,00600	0,00260	0,210	248	250	177	333
	0,005	0,00500	0,00866	0,772	191	104	93	626
	0,004	0,00100	0,00693	0,201	1102	2484	1260	1355
	0,005	0,00496	0,00215	0,151	403	429	394	626
	0,00358	0,00358	0,00620	0,435	462	348	409	1989

	0,006	0,00149	0,01032	0,091	801	810	365	333
	0,004	0,00400	0,00173	0,210	627	772	722	1355
	0,006	0,00600	0,01039	0,772	116	56	50	333
	0,005	0,00125	0,00866	0,201	855	1279	581	626
$\overline{S}$						0,117	0,079	1,800

CONCLUSIONS

The following conclusions may be drawn.

1. An analysis of the strain analogies of known static strength criteria to predict the multiaxial LCF life indicated that the case of proportional tension/torsion loading could be satisfactorily analyzed with the two-parameter criteria of Pisarenko-Lebedev and Coulomb-Mohr. The investigated one-parameter criteria of Renkine, Tresca and von Mises are very conservative.

2. In the case of non-proportional loading the best results are obtained by Garud's energy approach and non-proportional strain parameter by Itoh T, Sakane M, Ohnami M. and Socie D.F. These approaches have got both advantages and disadvantages. On the one hand, the calculating by Garud's approach is more difficult then calculating of non-proportional strain parameter. But on the other hand, the estimation of LCF life by approach of Itoh T, Sakane M, Ohnami M. and Socie D.F. needs non-proportional testing.

NOMENCLATURE

σ	stress, MPa;
E	Young's modulus, MPa;
ε_n	amplitude of the normal strain acting on the γ_{max} plane, - ;
$\overline{S}$	mean relative errors in fatigue life prediction, - ;
N	number of cycles;
W	plastic work, MJ/m^3
γ	shear strain, - ;
ε	axial strain, - ;
ν	Poisson's ratio, - ;

Subscripts

a	amplitude;
BY	Buzhinsky-Yagn;
eq	equivalent;
f	failure;
fs	fatigue strength at N cycles;
CM	Coulomb-Mohr;
M	von Mises;
NP	non-proportional;
p	predicted;
pr	proportional;
PL	Pisarenko-Lebedev;
R	Rankine;
T	Tresca;
t, e, p	total, elastic, plastic;
1,2,3	principal values

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