J-integral Solution for High Strain Region of SCT Specimen*

Gao Zengliang**, 1), Cai Guodong2), Jin Weiya1) and Fang Zhiming3)

1) Institute of Process Equipment and Control Eng., Zhejiang University of Technology, Hangzhou 310032, P. R. China
2) The department of Engineering Mechanics, The Pennsylvania State University, 16802, U.S.A
3) Zhijiang College, Zhejiang University of Technology, Hangzhou 310032, P. R. China

ABSTRACT

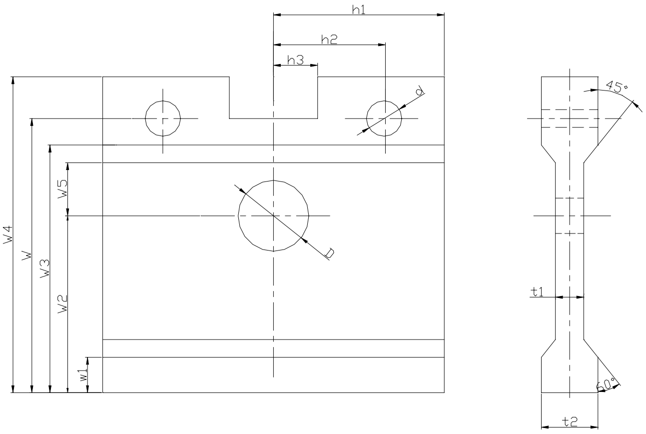
The special CT (SCT) specimen, as a test specimen for high strain region of engineering structures and equipments, has called a great deal of attention, since it has the character of stress fields of high strain region and is simple and very convenient for test of fracture and fatigue. J-integral is widely used in the world, which is the parameter to assess the intensity of elastic-plastic stress and strain field around crack tip. It is very important for the study of fracture and fatigue in high strain region that the J-integral of the SCT specimen can be calculated conveniently and exactly. There is no formula of J-integral calculation for the SCT specimens and the J-integral is gotten only by elastic-plastic finite element analysis, because the SCT specimens are not standard CT specimen. A great deal of finite element calculation of elastic and elastic-plastic J-integrals for the SCT specimens with different sizes, crack lengths and material properties are carried out. The elastic-plastic J-integral is divided into two parts, elastic part and plastic part, $J_{ep} = J_e + J_p$, where J_e is calculated based on the J-integral of CT specimen. $J_e = f_1 \cdot J_{CT}$, where f_1 is regressed with uniform design method created by a Chinese mathematician according to the finite element analysis results.

KEY WORDS: SCT specimens, J- integral, finite element analysis, regressive formula

INTRODUCTION

Li Zezeng[1] presented the special CT specimen, which is the improvement of CT specimen and has the characters of the high strain region stress field. Many researches done for the CT specimen may be used in the analyses of the SCT specimen. Furthermore, it is easy to prepare the SCT specimens and convenient to carry out the fracture and fatigue tests with it. Hence, the SCT specimen is an alternative way to replace the true vessel tests for the fracture and fatigue research at the pressure vessel nozzle region. The sketch of the SCT specimen is shown in the Fig.1.

J-integral is one of the most important parameters in elastic-plastic fracture mechanics to describe the crack tip stress and strain fields. Furthermore, J-integral theory is one of the fundamental principles of the elastic-plastic fracture mechanics. J-integral is widely used in various flaw assessment standards as the parameter of the elastic-plastic fracture. However, the exact J-integral solutions are only obtained by using the finite element methods, and there appears to be very little research reported in the literature on the engineering applications, except for EPRI, which obtains a series of similar engineering solutions through a lot of the FEM analyses. According to EPRI, the J-integral value can be determined as the sum of the elastic term J_e and the plastic term J_p. The plastic term J_p is given by the full plastic solution h_1, and various h_1 values for various kinds of two dimensional standard specimens are combined together by using FEM analysis, which is convenient to apply the J-integral in engineering structures. However, all of the specimens listed in EPRI are limited for simple structures. There are no theoretical equations to get the J value because of the complicated geometry of the SCT specimen. In the paper, authors try to get the J-integral elastic–plastic solution for the SCT specimen by splitting the elastic-plastic term into the elastic term and plastic term separately, then, the project of the paper is to obtain the J-integral empirical equation for the SCT specimen under the elastic-plastic conditions using the EPRI's meaning and the optimized mathematic method[2].



$W=150mm \bullet W_2=100mm \bullet W_3=128mm \bullet W_4=188mm$
$h_1=90mm \bullet h_2=70mm \bullet h_3=31mm \bullet t_1=10mm \bullet d=12.5$

Fig.1 Sketch of SCT specimen

THE ELASTIC-PLASTIC SOLUTION OF J-INTEGRAL

1) EPRI Engineering Method

The method is adopted for the material obeyed the Ramberg-Osgood relationship, the relationship between stress and total strain is listed below:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n \qquad \bullet 1 \bullet$$

where $\varepsilon_0 \bullet \sigma_0 \bullet \alpha$ and n are the material constants.

Here, J-integral is postulated to be the sum of the elastic term and plastic term, and the two terms are considered separately•

$$J_{ep} = J_e(a_e) + J_p \qquad \bullet 2 \bullet$$

$$J_e(a_e) = \frac{[K(a_e)]^2}{E^{/}} \qquad \bullet 3 \bullet$$

$$J_p = \alpha\varepsilon_0\sigma_0 c h_1 (\frac{\sigma}{\sigma_0})^{n+1} \qquad \bullet 4 \bullet$$

Where a_e is the crack length, J_p is the plastic term of J value. h_1 is related to the strain hardening exponent n. Hence, according to the h_1, the specific *J* value solution of the specific structure containing the crack is determined.

2) The Method Used in the Paper

In the paper, the J-integral is composed of the elastic term and plastic term too. First of all, the SCT specimens with different dimensions and different crack length are analyzed. The elastic term J_e is obtained from the FEM and the elastic constants of the material is indicated. By combining the empirical equation of standard CT specimens, the linear elastic *J* value equation for the SCT specimen is induced:

$$J_e = f_1 \cdot J_{CT} \qquad \bullet 5 \bullet$$

Where, f_1 is the geometric factor of the SCT specimen, and J_{CT} is the standard J value equation of the standard CT specimen.

Since there are various possible forms of the dimensions of the specimen and the crack length; a lot of numerical analyses need to be made. Hence, a new optimized mathematic method is adopted to arrange the number of numerical analysis. Combining with the material properties and the load, a lot of models are analyzed by the FEM. The number of models is determined by the mathematic method. The elastic-plastic *J* value (J_{ep}) for every load sub step is expressed by the EPRI means, according to Equation (2), the full plastic *J* value (J_p) is obtained:

$$J_p = J_{ep} - J_e(a_e) \qquad \bullet 6 \bullet$$

Then, using mathematic method to deal with the data, the empirical equation is obtained.

NUMERICAL ANALYSES

The numerical analyses are finished by the ANSYS software.

The finite element model is shown in Fig.2. The crack length is determined by defining a singular point away from the axis. For every model, six independent paths are chosen to calculate the *J* value, showed as the Fig.3. The average *J* value from the six independent paths is taken as the *J* value.

RESULTS AND DISCUSSIONS

1) Under the linear elastic condition, the geometric factor is the main effect factor to evaluate the J value of the SCT specimen. After optimizing the number of tests by uniform design method, 28 models with different dimensions are analyzed by FEM. Dealing with the data and combing with the empirical *J* equation of the CT specimen, the geometric factor is obtained. The empirical equation is listed as follows:

$$J = (\frac{P}{t_1})^2 \frac{W}{E_1} f(a_1/W)/(B*b) \tag{7}$$

Where, $f(a_1/W) = 2(1+\alpha)/(1+\alpha^2)$, $\alpha = \sqrt{X^2+2X+2}-(X+1)$, $X = 2a_1/(W-a_1)$, B the ligament width of SCT specimen, b the transient ligament width, a_1 the length of crack.

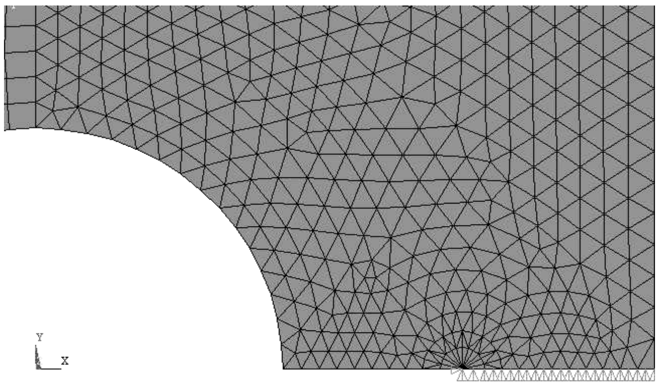


Fig.2 mesh pattern of the SCT specimen

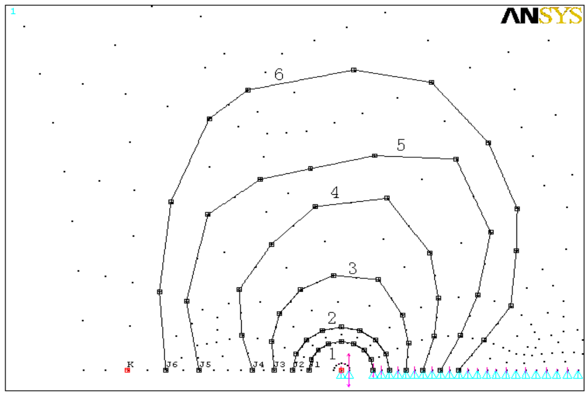


Fig.3 J-integral paths

For a complicated SCT specimen, the geometrical parameters are taken as the hole diameter (*D*), the length of crack (a_1), the thickness of reinforced region (t_2)•the width of reinforced region (w_1). Therefore, the geometric factor induced (f_1) may be related to the various parameters as follows:

$$f_1 = f(D, a_1, t_2, w_1) \tag{8}$$

Since f_1 is dimensionless, dimensionless analysis is performed only for the right-hand side of the equation. There are four parameters involved. Here the following are chosen:

$$f_1 = f(D/W, a_1/W, t_1/t_2, w_1/W) \tag{9}$$

Where, *W* is the width of the specimen, t_2 is the thickness of the middle reduced region of the specimen. The physical definition of every group is not illustrated well. The four groups may be called, respectively, the dimensionless hole diameter, dimensionless crack length, dimensionless thickness of the specimen, dimensionless width of the specimen.

Based on the above dimensionless argument, some simple analyses of the numerical results were obtained. Strictly speaking, in order to seek the dependence of one dimensionless group upon the others, one must keep the values of the remaining three groups constant and plot the curve of one group versus the others. In the paper, author took the whole groups as the comprehensive parameters by using the optimized mathematic method. After numerical analyses, the empirical equation is obtained:

$$f_1 = 0.416 + 5.39\left(\frac{a_1}{W}\right) + 12.49\left(\frac{D}{W}\right) \times \left(\frac{a_1}{W}\right) - 10.3\left(\frac{D}{W}\right) \times \left(\frac{W_1}{W}\right) - 18.6\left(\frac{a_1}{W}\right)^2 + 8.15\left(\frac{a_1}{W}\right) \times \left(\frac{t_1}{t_2}\right) \tag{•10•}$$

2) Comparing with the linear elastic condition, the difference is lied in the stress-strain curve under the elastic-plastic conditions. In the paper, it's supposed that the material obeys on yield criterion, so the material property is

defined as the bilinear kinematic hardening. That is, the relation between stress and strain is simplified as the two lines. The elastic strain is corresponding to the Young's modulus and the plastic strain corresponds to the motion of the slider up to the point of maximum strain. However, the relation between stress and strain is only corresponding to a straight line under the linear elastic condition. Hence, J_p value is not only related to the geometric factor, but also related to the material properties, load and so on. Big effort has been made to get the empirical equation under the elastic-plastic condition; however, there are so many factors that authors can't get a reasonable equation.

CONCLUSIONS

The J-integral of the SCT specimen is analyzed by using the FEM under the linear elastic and elastic-plastic conditions separately. Combining with the empirical equation of the standard CT specimen, a useful empirical equation is obtained in terms of the different dimension of the specimen and crack length, and a good try is made to get the empirical equation under the full plastic condition.

REFERENCE

1. Li Z. Z. and G. S., A new special compact tension specimen to simulate high strain gradient region at the nozzle of pressure vessel, *Pressure Vessel Technology*, Vol.6, No.3, pp.6-13 (1989) (in Chinese)
2. Fang K. T. • *Union design and the reference forms,* Science Press, Beijiang, China, 1994.6 (in Chinese)