The Energy Approach of Elastoplastic Fracture Mechanics Applied to the Problem of the Shallow Crack Effect

Wadier Yves 1) and Bonnamy Marc 2)

1) EDF – R&D, 1 avenue du Général de Gaulle, 92141 Clamart Cedex, France
2) AUSY France, 10 rue des acacias, 92134 Issy les Moulineaux, France

ABSTRACT

An experimental program was recently carried out at the CEA in order to study the shallow crack effect. Different tests have been performed on SENB specimens made of A58 forging steel and containing deep and shallow cracks, at different temperatures, in order to analyse the shallow crack effect with different approaches. At EDF, an energy approach has been developed and an energy release rate, called Gp, has been defined. Conversely to the J-approach, this energy approach is, in principle, valid in all situations, and can be used in particular to study the constraint effect.

The results obtained with this approach are compared to the experimental results. After having identified the critical value of the parameter Gp on the SENB specimens with large cracks, using finite elements calculations, and assuming that this value is a material characteristic, we have predicted the critical load of fracture of the specimens SENB containing short cracks. Then we have calculated the toughness corresponding to this load, and we have obtained a rather good agreement with the experimental results. The value of toughness of SENB specimens with small cracks is approximately 1.8 time the value of the toughness of SENB specimens with large cracks. A parametric study was performed to estimate the influence of the stress/strain curve representation, of small or large strains hypothesis and the influence of the choice of the value of “r”. The results obtained confirm the validity of the first prediction.

INTRODUCTION

The toughness “Kjc” of a material is determined by tests on CT specimens containing large cracks. When different specimens containing small cracks are considered, the toughness is found much higher. This effect has been observed experimentally by many laboratories, and is called “the shallow crack effect”. Two approaches of elastoplastic fracture mechanics are mainly used to analyse this effect : the two parameter approach and the local approach. The two parameter approach introduces a second parameter called “T” or “Q” in order to extend the classical J approach and is valid in the context of the deformation theory of plasticity. Then, the non proportional loading cannot be taken into account. The local approach mainly used for cleavage fracture is the BEREMIN model which considers the mechanism of fracture at the microstructure level. It is valid in the context of the incremental theory of plasticity but the parameters of the model must be very carefully identified and a mesh dependency can often be observed.

That is the reason why we have considered another approach called “the energy approach of elastoplastic fracture mechanics”. We start from Francfort and Marigo elastic fracture theory, based on a minimum energy principle, which generalises Griffith theory. A parameter “Gp” can then be defined as an energy release rate in the context of the incremental theory of plasticity, and is not mesh dependent. It can be used to analyse all the situations of cleavage fracture where the J-approach is not valid, such as problems with unloading, non proportional loading, residual stresses etc. In this paper, it is applied to the analysis of the shallow crack effect.

1. THE ENERGY APPROACH OF ELASTOPLASTIC FRACTURE MECHANICS APPLIED TO CLEAVAGE FRACTURE

We recall the basis of the energy approach, the definition and the calculation of the Gp parameter, and the main points and ideas of the approach.

1.1 Progressive or sudden propagation of a crack : the two paradoxes of the Griffith theory

An energy release rate called Gp(Δl) is defined for a crack represented by a notch, using Francfort and Marigo theory [1] and continuous damaged mechanics. The Francfort and Marigo theory generalises the Griffith theory and can predict initiation and sudden propagation of cracks, using a minimisation principle. This theory has been extended to plasticity [2] and has been applied to the problem of unloading [3] and to the analysis of the shallow crack and the warm pre-stress effects [4, 5]. The Δl propagation minimizes the total energy of the structure including the energy dissipated during the propagation. It is equivalent to maximize the function Gp(Δl). Then, two cases can be considered :

1. the maximum of G(Δl) is obtained for Δl = 0 : this case corresponds to progressive crack propagation,
2. the maximum of G(Δl) is not obtained for Δl = 0 : this case corresponds to sudden crack propagation,
In the first case, the usual energy release rate “G” or “Gp(0)” of Griffith theory, corresponding to progressive crack propagation, is retrieved. In the second case, the parameter Gp(Δl), corresponding to sudden crack propagation, is a mean energy release rate. Then, we have two paradoxes to consider:
1. the “Paradox of Rice”[6, 7] giving Gp(0)=0 for bounded flow materials but not in case of linear hardening [8, 9],
2. the “scale effects” of the Francfort and Marigo theory based on the Griffith hypothesis [10], induced precisely by this hypothesis, which cannot be acceptable.

Then we have to consider the Francfort and Marigo theory using an hypothesis different from the Griffith one’s - as the Barenblatt hypothesis - or we can consider a modelisation of the crack by a notch.

1.2 Progressive or sudden propagation of a notch : definition of an energy release rate

We assume that the shape of the notch is a « cigar » shape, the notch tip, called « Γ » being represented by half a circle, « r » being the radius. The notch propagation area is noted « Ze » (Damaged Zone) and is Δl dependent, see the figure below.

We assume the minimisation principle of Francfort and Marigo with the following hypothesis:
1. the elastic energy is cancelled during the notch propagation at each point of the damaged area Ze,
2. there is no changes in plasticity in the damaged area Ze during the notch propagation,
3. the minimization is made with a displacement field prescribed on all the structure, which means that : on one hand, the solution of the problem on the new equilibrium configuration after propagation cannot be found, but on the other hand a « sine qua non » condition of propagation (including eventually dynamic effects) can be obtained.

The minimization with respect to Δl (crack propagation) is equivalent to the maximization of the energy release rate Gp(Δl). The progressive propagation case corresponds to the maximum obtained for Δl equal to 0, and Gp(0) is not zero, even in the situation of a bounded flow material, because it is equal to the flow of elastic energy through the notch tip. The sudden propagation case corresponds to the maximum obtained for Δl not equal to 0, and because we calculate the Gp parameter using only the Ze area, we can eliminate the parasitical scale effect. Then, the two paradoxes have been eliminated thanks to the notch model for the crack [11].

1.3 Calculation of the parameter :

The Gp parameter is calculated as the maximum (with respect to Δl, length of the notch propagation) of the integral over the area Ze (corresponding to the notch propagation) of the elastic energy, divided by Δl, according to the formula:

$$Gp(Δl) = \max_{Δl} \left[ \left( \int_{Ze} (we.dS) / Δl \right) \right]$$

1.4 Main points of the approach :

**General validity** : The physical interpretation of the parameter Gp is very clear. Gp is an energy parameter for all situations and in particular for all loading situations : proportional or not proportional (loading plus unloading, residual stress, thermo-mechanical loading etc.), which is not the case of J. So, we can consider that this parameter has a general validity in elastoplastic fracture mechanics.

**Transferability** : As a consequence of this first point, it seems that, in principle, the problems of transferability of the J-approach should not arise if the energy approach is used. Of course this point must be clearly verified by a lot of comparisons between the results of numerical simulations and those obtained by experimental works.

**Specific tests are not necessary** : The characterisation of the critical value Gpc of the parameter Gp is obtained using the standard tests on CT specimens, and finite element calculations of these tests giving the critical time of fracture (corresponding in principle to the critical load). Then we are able to calculate the critical value of the parameter Gp.

**The crack is modeled by a notch** : The choice of the ideal model of a sharp tipped crack involves a very large error for the numerical solution in the vicinity of the crack tip. Conversely, using the notch model, the numerical solution can be calculated accurately in this area. As a direct consequence of that, we will have no problems of mesh dependency, as soon as the mesh is sufficiently refined. But as the notch must be representative of a real crack, the radius “r” of the notch must be small enough, for example : 10 microns < r < 100 microns, and this choice implies very fine meshes.
The Griffith theory is extended: The Griffith theory is limited to progressive and continuous crack propagation. It is not possible to predict initiation or sudden crack propagation with this theory. In the case of cleavage fracture, the energy approach, based on a minimisation energy principle, can predict these different phenomena. This approach can then be applied to analyse some effects corresponding to progressive or sudden propagation as: “constraint effect”, “warm pre-stress effect”, or “pop in effect”, as well as situations corresponding to crack initiation as the Charpy tests.

2. APPLICATION TO THE PROBLEM OF THE SHALLOW CRACK EFFECT

An experimental program was recently carried out at the CEA on SENB specimens made of A58 forging steel, in order to study the shallow crack effect. Ten tests were performed on specimens with large cracks and ten tests on specimens with short cracks, at the temperature of –90°C. The experimental results shows that the toughness of specimens with short cracks (181 MPa.m$^{1/2}$) is about 1.8 time the toughness of large cracks specimens (103 MPa.m$^{1/2}$).

2.1 Definition of the Problem

Material data:

In order to analyse the influence of the stress/strain curve representation - and precisely the influence of hardening - we have considered three cases, see Table 1:

1/ the case not bounded flow corresponding to the true stress / strain curve,

2/ the case bounded flow 20% corresponding to the true stress / strain curve with a limitation of the Von Mises stress for a strain higher than 20%.

3/ the case bounded flow 50% corresponding to the true stress / strain curve with a limitation of the Von Mises stress for a strain higher than 50%.

<table>
<thead>
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<th>Kc (MPa.m$^{1/2}$)</th>
<th>Strain</th>
<th>Stress</th>
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<tr>
<td>Not bounded flow</td>
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<td>571.32</td>
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<td>933.78</td>
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<td>0.50512</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>0.50512</td>
<td>1066.50</td>
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Table 1: different stress/strain curves considered

The mean toughness value (50% Master Curve value) is given in Table 2. The others material characteristics are:

- Young modulus: 208510 MPa
- Poisson’s ratio: 0.3
- Yield limit: 571 MPa

<table>
<thead>
<tr>
<th>Kc (MPa.m$^{1/2}$)</th>
<th>SENB1 large crack</th>
<th>SENB2 shallow crack</th>
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<tr>
<td></td>
<td>103.</td>
<td>181.</td>
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Table 2: material toughness

Geometry:

The geometrical dimensions of the SENB specimens (see Figure 1), are defined as follows:

- S = 370 mm, L = 420 mm, W = 505 mm, thickness = 50 mm
- for the large crack: a = 25 mm, in this case the specimen is called SENB1.
- for the shallow crack: a = 3.8 mm, in this case the specimen is called SENB2.

A parametric study will be performed on the value of the radius of the notch. Three values will be considered:

- r = 100 microns, r = 50 microns, r = 20 microns.

Loading and boundaries conditions:

A displacement is prescribed on the upper part of the specimen. For the boundaries conditions, see Figure 2:

Modelisation:

We consider a 2D problem assuming the plane strain hypothesis. The constitutive law is elastoplastic with an isotropic hardening. The crack is represented as a very thin notch, “r” being the radius of the circle at the notch tip.

We define the shallow crack effect “SCE” as Kjc (predicted on SENB2) divided by Kjc (identified on SENB1).

$$
SCE = \frac{K_{jc\ (SENB2)}}{K_{jc\ (SENB1)}}
$$
Figure 1 : SENB geometry

Figure 2 : boundaries conditions

Mesh:

In Figure 3 the mesh of half the structure is presented, taking advantage of the symmetry : it contains approximately 11000 nodes. Different zooms of the notch tip area are presented in Figures 4 and 5.

2.2 Analysis of the results

The numerical analysis was performed at EDF-R&D with the Code-Aster, the finite element code of EDF. The results predicted with the Gp criteria are analysed for the SENB2 specimen containing the shallow crack and compared to the experimental results. The influence of the stress strain curve representation as well as the influence of the notch radius on the shallow crack effect (parameter SCE) are investigated. But first of all, we have identified the critical value Gpc of the parameter Gp by using the SENB specimen with the large crack, the critical load (or time) corresponding to the critical value of Kj : $K_{jc} = 103 \text{ MPa.m}^{1/2}$.

2.2.1 Identification of Gpc

In principle we would have to consider a standard CT specimen in order to identify the critical value Gpc of Gp. But unfortunately the fracture tests on these specimens were not available in the context of the experimental program of the CEA. So we have performed a numerical comparison between a CT specimen and a SENB1 specimen with a large crack. We have noticed that the stress and strain fields are nearly identical for a given value of Kj. Therefore it is possible to identify Gpc directly with the SENB1 specimen instead of identify Gpc with the CT.

Therefore let's consider the SENB1 specimen. The J parameter is calculated using the theta method [12], and the critical time of fracture “tc” corresponds to $J = J_c$, $J_c$ being the toughness associated by the Irving formula to $K_{jc}$ given in Table 1.

At the time step “tc” of the calculation, we are then able to calculate the critical value Gpc of the Gp parameter. The results obtained are presented in the Tables 3 and 4 corresponding to different values of the radius “r” and different representations of the stress/strain curves. The values of Gpc are small compared to $J_c$ because they are related only to the elastic energy.

The Gpc values depend of the radius “r”, because the surface of integration is proportional to “r” for a given value of $\Delta l$. We can see that the difference between the case “complete” and “bounded 50%” is very small. The values of Gpc are given at the experimental critical time (or load). The difference between SENB1 and SENB2 is 20% approximately.
which is reasonable if Gpc pretend to be a material characteristic, and if we remember that the difference on toughness is about 80%. The Gpc values for the SENB1 specimen given in Table 3 are the reference values which will be used to predict the critical load of the SENB2 specimen.

The variation of Gp as function of the distance Δl is presented in Figure 6, for the two specimens SENB1 and SENB2, at the experimental critical time tc (or load), and for the different values of “r” considered. We find : tc = 30 for SENB1 and tc = 95 for SENB2. The maximum of the curve correspond to the values of Tables 3 and 4. We can notice that this maximum is obtained for different values of the distance Δl from the notch tip : between 0.1mm and 0.5mm for SENB1, and between 0.05mm and 0.2mm for SENB2. It seems that this distance is nearly proportional to the radius “r”.

<table>
<thead>
<tr>
<th>«r» (mm)</th>
<th>Stress strain curve</th>
<th>Gp max (kJ/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>complete</td>
<td>1.0318</td>
</tr>
<tr>
<td>0.05</td>
<td>complete</td>
<td>0.6117</td>
</tr>
<tr>
<td>0.025</td>
<td>complete</td>
<td>0.3674</td>
</tr>
<tr>
<td>0.025</td>
<td>bounded (50%)</td>
<td>0.3663</td>
</tr>
</tbody>
</table>

Table 3 : Gp for SENB1 at the experimental critical load ( = Gpc)

The same curves are presented in Figure 7 but at the experimental critical time (or load) only for SENB1 : tc = 30. The time (or load) corresponding to SENB2 is a “predicted” time calculated assuming that the Gpc values are identified on SENB1 and consequently are the same for SENB2. Then, we have the same maximum values for a given value of “r”. Instead of tc = 95, we find : tc = 72 for r = 100 microns, tc = 67 for r = 50 microns, tc = 62 for r = 25 microns.

The procedure used to identify the critical time (or load) is presented in Figure 8. The variation of Gpc as function of time is given for SENB1 and SENB2 and for the different values of “r”. If we take for example the value r = 50 microns associated to the curve with the sign “+”, we have first to consider the curve with the dashed lines corresponding to SENB1. The critical time (t = 30) gives : Gpc = 0.61. Then we have to consider the curve with the continuous line corresponding to SENB2. As the value of Gpc is the same for SENB2, we are able to identify tc (tc = 67) for SENB2.

### 2.2.2 Estimation of the shallow crack effect “SCE”

The estimation of the shallow crack effect, given by the parameter SCE, must be compared to a reference value of SCE. The first idea might be to consider the experimental results of Table 2, giving : SCE(exp) = 1.76

Nevertheless, we have to consider that the finite elements modelisation implies different hypothesis and errors, and that these errors can counter-balance. As we want to validate the energy approach, and not the different hypothesis assumed, it is preferable to consider that the reference value of SCE is given by the 2D finite elements calculation (used in the analysis) at the critical experimental time, without any fracture criteria. We find : SCE(fem-2D) = 2.12

The 3D FE calculation made by the CEA give a result for SCE very close to the experimental one, see Table 5. The 2D calculation gives a 20% higher result, because a much stiffer structure is obtained using the plane strain hypothesis. It is preferable to compare results obtained with the same hypothesis if we want to point out the effect of the criteria.

The estimation of the shallow crack effect SCE is given for different values of “r” and different stress / strain curve representation in Table 6. These values varies between 1.42 and 1.88, the mean value being 1.65. This value must be compared to the SCE(fem-2D) = 2.12, and not to SCE(exp) = 1.76, which would be more favourable but not convenient! We can conclude that the result of the energy approach is about 20% lower than the reference one.
The influence of “r” the notch radius as well as the influence of hardening on Gpc, critical value of Gp is presented in Table 7. We can remark that the influence of hardening increases when the value of the radius “r” decreases, because of course, smaller is “r”, higher are the strains at the notch tip. The Gpc value seems to be proportional to the radius.

![Figure 6: Gp at the experimental critical load](image1)

![Figure 7: Identification of Gp](image2)

![Figure 8: Identification of the critical time](image3)

The influence of the large strain hypothesis has been investigated in the case $r = 25$ microns. In Tables 6 and 7 we can notice that this influence is negligible.

<table>
<thead>
<tr>
<th></th>
<th>Kjc – SENB1 (Mpa.m$^{1/2}$)</th>
<th>Kjc – SENB2 (Mpa.m$^{1/2}$)</th>
<th>SCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental values</td>
<td>103.</td>
<td>181.</td>
<td>1.76</td>
</tr>
<tr>
<td>Numerical values at the experimental critical time (2D)</td>
<td>103.</td>
<td>219.</td>
<td>2.12</td>
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<tr>
<td>Numerical values at the experimental critical time (3D)</td>
<td>103.</td>
<td>185.</td>
<td>1.80</td>
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</table>

Table 5: References values for SCE
Radius « r »  

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<thead>
<tr>
<th></th>
<th>SCE at the “estimated” critical time</th>
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<tbody>
<tr>
<td></td>
<td>not bounded flow</td>
</tr>
<tr>
<td>100. µ :</td>
<td>1.64</td>
</tr>
<tr>
<td>50. µ :</td>
<td>1.51</td>
</tr>
<tr>
<td>25. µ :</td>
<td>1.42</td>
</tr>
<tr>
<td>25. µ (large strains) :</td>
<td>1.87</td>
</tr>
</tbody>
</table>

Table 6 : Estimated values of SCE

2.2.3 Parametric study

The influence of “r” the notch radius as well as the influence of hardening on the shallow crack effect are analysed in more details in this paragraph, considering the variation of the SCE parameter as function of time (or loading).

The influence of the notch radius is presented in the Figure 9, for the case “not bounded” hardening. It can be observed that the differences between the three curves corresponding to the three values of “r” are not very important. Then, the value of the notch radius has only a small effect on the shallow crack effect estimation.

Table 7 : Influence of “r” and hardening on Gpc

The influence of the hardening is presented in the Figure 10, for the case “not bounded” hardening. It can be observed, once again, that the differences between the three curves corresponding to the three values of hardening are not very important, (but not negligible in the case “bounded flow” at 20% of deformation, because the strains are much more important). Then, we can conclude that the hardening has not a significant influence on the shallow crack effect estimation.

3. CONCLUSION

An energy approach has been developed at EDF-R&D and an energy release rate called Gp has been defined. Conversely to the J-approach, this energy approach is, in principle, valid in all situations, and can be used in particular to study the constraint effect. This approach was recently applied to interpret tests on SENB specimens made of A58 forging steel and containing deep and shallow cracks, in order to analyse the shallow crack effect in the context of an experimental program realized at the CEA.

The numerical analysis was performed at EDF-R&D in order to analyse these tests with the Code-Aster, the finite elements code of EDF. First of all, we have identified the critical value Gpc of the parameter Gp, and then we have applied the energy approach to the interpretation of the tests.

The results obtained by this approach are compared to the experimental results. After having identified the critical value of the parameter Gp on the SENB1 specimens with large cracks, using FE calculations, and assuming that this value is a material characteristic, we have predicted the critical load of fracture of the specimens SENB2 containing short cracks. Then, we have calculated the toughness associated with this load, and we have obtained a rather good agreement with the experimental results. The estimated toughness of SENB2 specimens with small cracks are approximately 1.65 time the value of the toughness of SENB1 specimens with large cracks, to be compared to 2.12 (obtained at the experimental critical time with a 2D model). A parametric study was performed to estimate the influence of the stress/strain curve representation, the influence of small or large strains hypothesis and the influence of the choice of the value of “r”. All the results obtained confirm the validity of the first prediction.
References


