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Application of Microstructure Based Brittle Fracture Model to Biaxial Strength of Graphite Materials

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ABSTRACT

From a viewpoint of advanced design method of graphite components, it is important to apply the realistic fracture model in the design method. The applicability of the microstructure based brittle fracture model under multiaxial stress condition was, therefore, investigated. The fracture model is possible to treat grain size as well as pore size with fracture mechanics approach taking account of the crystal structure of the graphite. The model was applied to the biaxial strength prediction of near isotropic nuclear graphite using grain/pore related microstructural parameters. Prediction results were compared with biaxial strength data obtained by simultaneous loadings of inner pressure and longitudinal load with thin-walled cylindrical specimen. From this study, it was found that the fracture model predicted fairly good not only mean strength but also strength distribution under biaxial stress condition, and it was concluded that the microstructure based brittle fracture model would be applicable as the advanced design method.

KEY WORDS: biaxial strength, brittle fracture, graphite, fracture modeling, microstructure, probability, structural design

INTRODUCTION

Graphite materials are used in the High Temperature Gas-cooled Reactor and so on for its excellent thermal resistivity. In the structural design, stress in the structure is limited with several categorized stresses such as membrane stress, bending stress and/or peak stress as well known. These stress limits are stipulated from an empirical viewpoint of the structural integrity so that the structure does not fracture under stress gradient condition. However, large safety margin is introduced in the design criteria, in which the maximum principal stress theory is adopted as multiaxial fracture theory [1]. Therefore, it is important to eliminate the needless safety margin in the advanced component design philosophy.

From a viewpoint of the component design method, there are several promising models to evaluate the strength of graphite with statistical approach; e.g. the Weibull theory[2,3], competing risk-based fracture model[4] etc. The Weibull theory treats only macroscopic strength with probabilistic procedure considering the stress profile. The competing risk-based fracture model is similar to the Weibull strength model. The advantage of this model is to treat different macroscopic fracture models; e.g. surface flaw dominated fracture mode as well as internal flaw dominate fracture mode is possible to treat by the model. Several researchers also proposed a microstructure based fracture theory of graphite applicable to tensile strength prediction [5-8] as well as to bending strength prediction [9]. The authors also have studied the applicability of the microstructure based brittle fracture model to predict the tensile and bending strengths [10,11]. Moreover, the authors have studied the grain size effect on strength, volume effect on strength and so on[12], and clarify the applicability to the structural design of graphite components under uniaxial condition[13].

From a standpoint of the advanced component design philosophy, it is expected that the more realistic fracture model will eliminate the needless safety margin. Therefore, the applicability of the multiaxial strength prediction by the microstructure based brittle fracture model is studied. In the paper, we apply the fracture model to the multiaxial condition, and model performance is examined from comparison with experimental data.

ANALYSIS

Microstructure Based Brittle Fracture Model for Uniaxial Strength [8]

The grain of graphite consists of a stack of parallel hexagonal net planes as schematically shown in Fig.1. Since weak van der Waals force operating in the c-axial direction and strong covalent bonds connection in the a-axial direction, cleavage within the grain occurs easily in the c-axial direction. If the inherent crack in the graphite body faces a grain having inclination angle θ as shown in

Fig.1, the crack would deviate from its extension direction. When the uniform stress σ acts inherent crack size c, probability of one grain will fracture, P_f, is:

$$P_f = \frac{4}{\pi}\cos^{-1}\left(\frac{K_{IC}}{\sigma\sqrt{\pi c}}\right)^{\frac{1}{3}} \tag{1}$$

where, K_{IC} is a fracture toughness of grains, so-called particle K_{IC}. If there are n grains in the entire low with width b, probability of all grains will fail, $P_n(\sigma,c)$, is:

$$P_n(\sigma,c) = \left[\frac{4}{\pi}\cos^{-1}\left(\frac{K_{IC}}{\sigma\sqrt{\pi c}}\right)^{\frac{1}{3}}\right]^n \tag{2}$$

Here, it is assumed that the crack will extend at a grain size, a, if the entire low with n grains fail. The probability that the crack will extend from c to c+ia, i.e. fracturing i rows of grains, is then expressed as:

$$\ln P_n(\sigma,c) = n\int_0^i \ln\left[\frac{4}{\pi}\cos^{-1}\left(\frac{K_{IC}}{\sigma\sqrt{\pi(c+ia)}}\right)^{\frac{1}{3}}\right]di \tag{3}$$

Then, the survival probability P_s is given as:

$$P_s = 1-\int_0^\infty f(c)\cdot p_n(\sigma,c)dc \tag{4}$$

where, f(c) means initial crack size distribution function.

When N is the number of pores per unit volume and V is the specimen volume, the total survival probability of the volume V under stress σ having 2NV flaw tips is $(Ps)^{2NV}$. Therefore, the total fracture probability of the specimen is given as:

$$P_{ftot} = 1-(P_s)^{2NV} = 1-\left[1-\int_0^\infty f(c)\cdot p_n(\sigma,c)dc\right]^{2NV} \tag{5}$$

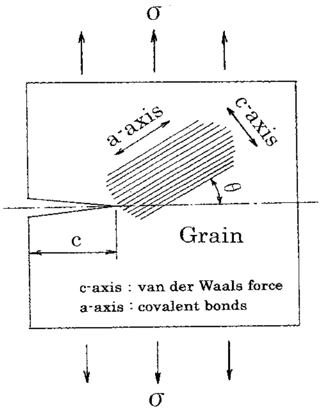


Fig. 1 Crack extension treatment in the microstructure based brittle fracture model.

Multiaxial prediction Modeling by Microstructure Based Brittle Fracture Model

If survival probabilities for principal stresses are $P_{stot1}(\sigma_1)$, $P_{stot2}(\sigma_2)$ and $P_{stot3}(\sigma_3)$, the total survival probability under multiaxial condition is represented by these products as[14, 15]:

$$P_{stot}(\sigma_1,\sigma_2,\sigma_3) = P_{stot1}(\sigma_1)\cdot P_{stot2}(\sigma_2)\cdot P_{stot3}(\sigma_3) \tag{6}$$

where σ_1, σ_2, σ_3 are the principal stresses. Here, we consider the initial crack size distribution function f(c), and assume that the fracture under σ_1, σ_2, σ_3 stresses are independent event each other. The total survival probability under the biaxial stress conditions is given by:

$$P_{stot} = \left[1-\int_0^{\infty} f(c)\cdot p_n(\sigma_1,c)dc\right]^{2NV}\cdot\left[1-\int_0^{\infty} f(c)\cdot p_n(\sigma_2,c)dc\right]^{2NV}\cdot\left[1-\int_0^{\infty} f(c)\cdot p_n(\sigma_3,c)dc\right]^{2NV} \tag{7}$$

Then, the total fracture probability P_{ftot} is written as:

$$P_{ftot} = 1 - P_{stot} \tag{8}$$

RESULT AND DISCUSSIONS

Uniaxial Strength Prediction

For the uniaxial stress condition, tensile strength of near isotropic nuclear graphite, PGX graphite, was predicted using Eq.(5). The input parameters used in this prediction are summarized in Table 1 [9]. Here, it is assumed that the pore acts as crack [8-13]. Namely, the initial crack size distribution function is defined by the pore size distribution. Moreover, it is assumed that the pore distribution is to be a log-normal statistical distribution [8-13]. The prediction result is compared with experimental data [9] in Fig. 2. We can see from this figure that the predicted result has a good agreement with experimental data.

Table 1 Input parameters of PGX graphite[9].

Parameter	value
Mean grain size (μm)	762
Bulk density (g/cm^3)	1.74
Mean pore size (μm)	238
Standard deviation parameter of pore size	1.73
Number of pores per volume (m^{-3})	0.187x10^8
Grain fracture toughness (MN/m$^{3/2}$)	0.225

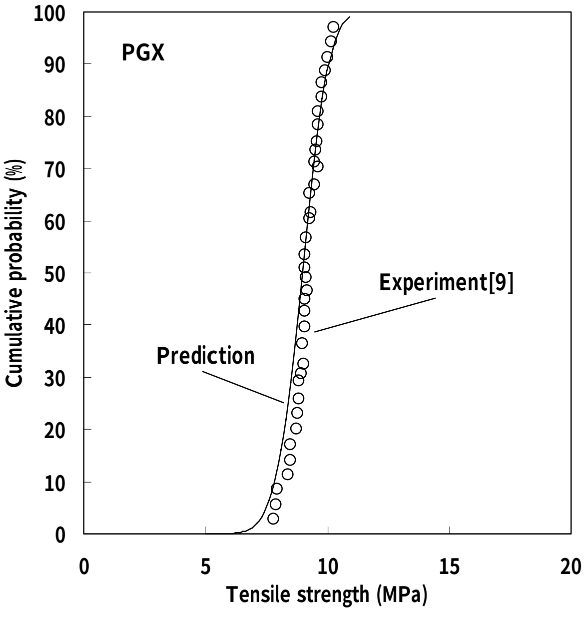


Fig.2 Comparison between experimental data and analytical results of tensile strength under uniaxial stress condition.

Biaxial Strength Prediction

Ho et al. obtained the biaxial strength of PGX graphite with thin-walled cylindrical specimen as shown in Fig. 3 [16]. Simultaneous loading with internal pressure and longitudinal load produced the biaxial stress condition in this experiment. Figure 4 shows the prediction and experimental results under biaxial stress condition with stress ratio of $\sigma_1/\sigma_2=1.0$. The mean strengths of prediction and experiment are 8.5MPa and 8.7MPa, respectively. We can see from this figure that prediction has a good agreement with experimental data. Mean strength ratios of biaxial to uniaxial strength are summarized in Table 2. The strength ratios by prediction and experiment are the same value of 0.95. It is considered that a lot of pores are distributed randomly in the graphite body. If uniaxial stress σ_1 is applied, parallel cracks to σ_1 stress as shown in Fig. 5 do not cause the fracture. However, if biaxial stresses σ_1 and σ_2 are applied, parallel cracks to σ_1 stress act as perpendicular cracks to σ_2 stress. This is thought to be the reason why the biaxial strength is lower than the uniaxial strength.

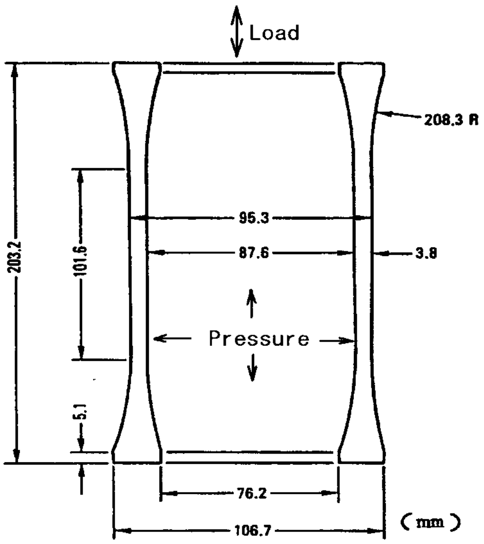


Fig. 3 Shape of the biaxial test specimen [16].

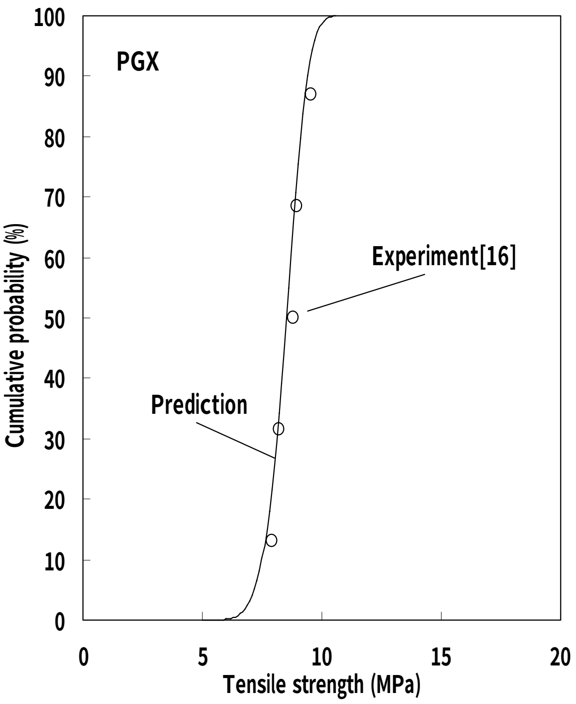


Fig. 4 Predicted biaxial strength of PGX graphite ($\sigma_1/\sigma_2=1.0$).

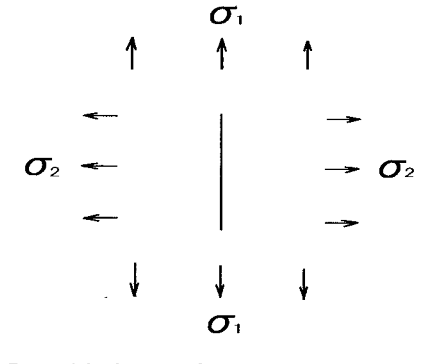


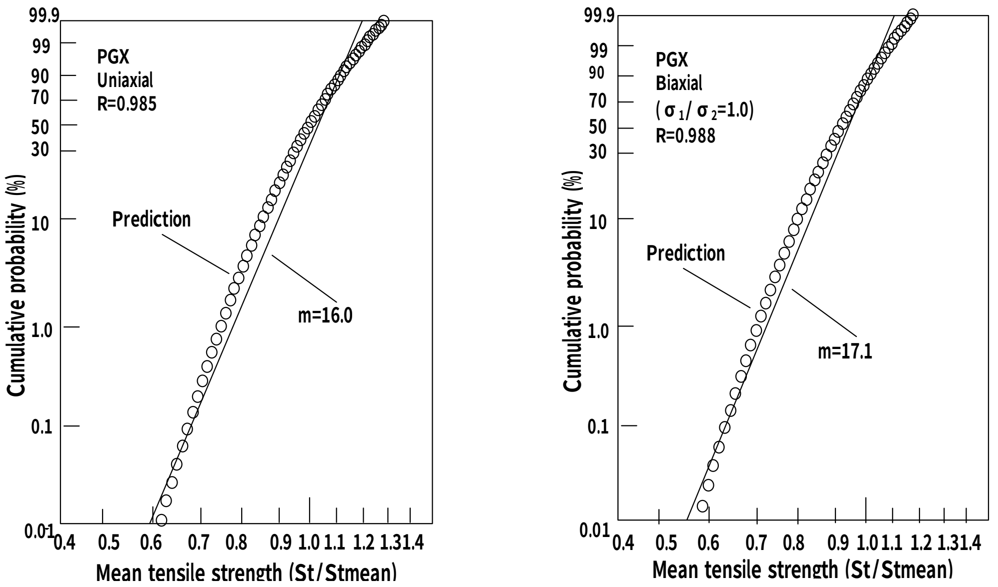
Fig. 5 Crack under biaxial stress condition.

Table 2 Strength ratio of biaxial to uniaxial strength ($\sigma_1/\sigma_2=1.0$).

	Mean strength under uniaxial conditions	Mean strength under biaxial conditions	Strength ratio (Biaxial/Uniaxial)
Prediction	9.0 MPa	8.5 MPa	0.95
Experiment	9.2 MPa	8.7 MPa	0.95

Figure 6 shows the Weibull probability plot of predicted results for uniaxial strength and biaxial strength of strength ration $\sigma_1/\sigma_2=1.0$. We can see from this figure that the Weibull modulus of the biaxial strength is somewhat larger than that of the uniaxial strength. Namely, scatter of the biaxial strength is smaller than that of the uniaxial strength. The reason of this is thought to be the same reason as the biaxial strength reduction; i.e. pores act more potentially to fracture under biaxial stress condition rather than uniaxial stress condition. Therefore, the scatter would be smaller under biaxial stress condition.

Figure 7 shows the predicted and experimental results of biaxial strength. The blacken circles mean predicted mean strengths and bars mean standard deviations. The minimum biaxial strength occurs at the stress ration of $\sigma_1/\sigma_2=1.0$. We can see from this figure that predicted results are agree well with the experimental data. It is, therefore, concluded that the microstructure based brittle fracture model would be applicable to the biaxial strength prediction.



(a) Uniaxial strength

(b) Biaxial strength ($\sigma_1/\sigma_2=1.0$)

Fig. 6 Weibull probability plot of predicted results.

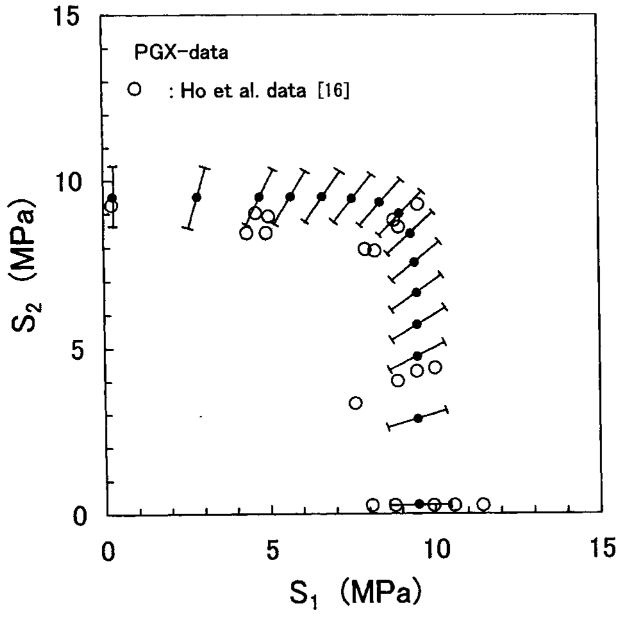


Fig. 7 Predicted and experimental results of biaxial strength.

CONCLUSIONS

The applicability of the microstructure based brittle fracture model under multiaxial stress condition was investigated. The fracture model was applied to the strength of graphite under biaxial stress condition. The obtained results in this study are summarized as follows:

- the fracture model predicted fairly good not only mean strength but also strength distribution under uniaxial and biaxial stress conditions,
- the biaxial strength was smaller than the uniaxial strength, and minimum biaxial strength occurred at stress ratio of $\sigma_1/\sigma_2=1.0$ in both prediction and experimental results,
- the scatter of the predicted biaxial strength was smaller than the uniaxial strength.

It is concluded from this study that the microstructure based brittle fracture model would be applicable to the prediction of biaxial strength of the graphite.

REFERENCES

1. Iyoku, T., Shiozawa, S., Ishihara, M., Arai, T. and Oku, T., “Graphite Core Structures and their Structural Design Criteria in the HTTR”, J. Nuclear Engineering and Design. 132 (1991) pp.23-30.
2. Weibull, W., “A statistical theory of the strength of materials”, Proceedings of Royal Swedish Institute for Engineering Research, No.151, 1953.
3. Price, R.J. and Cobb, H.R.W., “Application of Weibull statistical theory to the strength of reactor graphite”, CONF701105, 1970, pp.547-566.
4. Ishihara, M., Iyoku, T. and Shiozawa, S., ”New approach for fracture assessment of graphite”, Transactions of the 13th International Conference on Structural Mechanics in Reactor Technology, Vol.2. (1995) pp.491-496.
5. Buch, J.D., “The Mechanical Properties of Graphites and their Relation to a Physically Based Statistical Theory of Fracture”, CONF701105. (1970) pp.501-509.
6. McLachlan, N., Tucker, M.O., and Parry, M.J., ”Stress and the Microstructure of Graphite”, Proc. 19th Biennial Conf. On Carbon. (1989) pp.460-461.
7. Tucker, M.O., Rose, A.P.G. and Burchell, T.D., “The Fracture of Polygranular Graphites”, Carbon. Vol.24 (1986) pp.581-602.
8. Burvhell, T.D., ”A Microstructurally Based Fracture Model for Polygranular Graphites”, Carbon. Vol.34 (1996) pp.297-316.
9. Nakanishi, K., Arai, T., and Burchell, T.D., ”Assessment of Brittle Fracture Theory Based on Mesoscopic Fracture Mechanics of Multiple Grain/Pore System in Graphite and SiC”, Proc. Int.Sympo. on Carbon, Tokyo (1998) pp.332-333..
10. Takahashi, T., Ishihara, M., Baba, S., and Hayashi, K., “Effect of Grain and Pore Size Distribution on Strength of Graphite”, 1st World Conf. On Carbon, EUROCARBON2000, Berlin, Germany (2000) pp.397-398.
11. Ishihara, M. and Takahashi, T., “Proposal of a Brittle Fracture Model Based on Pore and Crack Interaction Effect”, Journal of the Society of Materials Science Japan. Vol.51 (2002) pp.425-430 (in Japanese).
12. Ishihara M., Takahashi, T. and Hanawa, Satoshi, “ Probabilistic Distribution of Tensile Strength of Nuclear Grade Graphite on the Basis of Mesoscopic Microstructure”, Proc. Asian Pasific Conf. On Fracture and Strength & Int. Conf. On Experimental Mechanics. Vol.1 (2001) pp.152-156.
13. Ishihara, M., Takahashi, T. and Hanawa, Satoshi., “ Applicability of Advanced Design Method of Graphite Components by Microstructure-based Brittle Fracture Model”, Trans. of 16th SMiRT conf. .Paper No.1114, Washington D.C. USA (2001).
14. Freudenthal, A.M. “ Statistical Approach to Brittle Fracture”, Chapter 6 of Vol.II, Fracture, edited by Liebowitz, H., Academic Press (1968).
15. Barnett, R.L. et. al, “Fracture of Brittle Materials Under Transient Mechanical and Thermal Loading”, Report AFFDL-TR, 66-220 (1966)
16. Ho., F.H., Vollman, R.E., Yu, H. and Adsit, N.R., “Biaxial Failure Surfaces of 2020 and PGX Graphites”, SMiRT L4/6, Chicago, (1983).