Stress-strain Relationship of Reinforced Concrete Subjected to Biaxial Tension

Jaeyeol Cho1), Namso Cho2), Namsik Kim2), Youngsun Choun3)
1) Seoul National University, Seoul, Korea
2) Hyundai Institute of Construction Technology, Yongin, Korea
3) Korean Atomic Energy Research Institute, Daejeon, Korea

ABSTRACT

One directional and biaxial tension tests of 13 reinforced concrete panels were conducted to derive a constitutive law of concrete. Based on the test results, a model equation is derived for the stress-strain relationship of concrete under tension. Main test variables are the reinforcement ratio and the applied load ratio in two directions. In addition, a failure envelope of concrete in the tension-tension region is suggested based on the initial crack occurrence. Test results show that the concrete carries substantial tensile stress even after cracking. However, the application of this proposed stress-strain relationship for concrete is limited to the case where the direction of reinforcement coincides with the direction of the applied principal stresses.

KEY WORDS: biaxial tension, reinforced concrete, constitutive law, stress-strain relationship, reinforcement ratio, applied load ratio, tensile stress.

INTRODUCTION

In general, high-rise buildings, box girder bridges, LNG containments, nuclear power plant containments, and offshore platforms are the shell type of reinforced concrete structures. These shell structures show two-dimensional behavior, that is, in-plane stress state.[1,2] Especially as a final goal of this study, an analytical model reflecting this stress state should be prepared in order to achieve a reasonable design and analysis method for containment building in nuclear power plants.

It is necessary to develop a stress-strain relationship for material satisfying the force equilibrium and strain compatibility so as to make the analysis close to reality. The commonly used stress-strain relationship obtained from each material test of concrete and reinforcement is quite different from that relationship. [3-5]

Introducing the concept of the average stress-strain relationship, therefore, this study dealt with the derivation of the constitutive law of material from biaxial tension tests for reinforced concrete panel specimens. This paper presents a stress-strain relationship of concrete and a failure envelope based on the cracking load.

Currently existing research related to this study mostly corresponds to the tension tests for plain concrete specimens [6,7] and uniaxial tests for reinforced concrete panels. [2,4,8] However, this output from biaxial tension tests is sure to reflect the characteristics of reinforced concrete structures. Through the extensive and intensive analysis of related references, the reinforcement ratio and load ratio in two orthogonal directions were selected for the main test variables and tests were conducted with 13 specimens.

BIAXIAL TENSION TEST OF REINFORCED CONCRETE PANEL

Specimen Design

The reinforcement ratio as a main test variable was selected in consideration of the relationship between this study and the present condition of containment in nuclear power plants. Fig. 1 shows the distribution of the reinforcement ratios of Korea’s existing nuclear power plant containment walls and those used in this study. Special attention was paid to the design of reinforced concrete panel specimens lest the cracking load of the concrete should exceed the yielding load of the reinforcement. Otherwise, the reinforcement’s yield would precede the crack occurrence, so that the tension stiffening effect could not be investigated.

Considering both the factors listed above and the test’s environmental conditions, the lower limit of the reinforcement ratio was about 0.008, that is, the specimen must be designed with a reinforcement ratio beyond 0.008. Accordingly, reinforcement ratio as a test variable was chosen as 0.009, 0.0135 and 0.0188. These values correspond to the higher region within the range of reinforcement ratios of existing containment walls of nuclear power plants. The above means that this study focused on the investigation into the tension stiffening effect of highly reinforced concrete members.

For the purposes of experimental convenience and the exclusion of other external effects, such as the reinforcement’s diameter and the concrete cover depth, reinforcement ratios were adjusted by changing the concrete’s cross-section area and the quantity of reinforcement, while the reinforcement’s diameter was fixed at 29 mm.
Table 1 shows the specimen’s design configuration in detail according to reinforcement ratio, where \( c \) and \( d_b \) are respectively the concrete’s cover depth and the reinforcement’s diameter. Fig. 2 illustrates the specimen name along with loading type, reinforcement ratio and applied load ratio in two orthogonal directions are included to promote cognition. As shown in Fig. 2, this study considered three types of biaxial tension loads, that is, applied load ratios \( P_1 : P_2 \) in two orthogonal directions are 1:1, 1:0.577 and 1:0.268. These ratios are expressed by an angle in the first quadrant of the tension-tension region, respectively 45°, 60° and 75°, and in Fig. 2 \( \alpha \) is \( P_1 / P_2 \).

![Fig. 1 Reinforcement ratio](image)

![Fig. 2 Specimen identification](image)

### Table 1. Specimen details

<table>
<thead>
<tr>
<th>Reinforcement Ratio</th>
<th>Reinforcement Detail</th>
<th>Specimen Size [mm]</th>
<th>Cover Depth [mm]</th>
<th>( c / d_b )</th>
<th>Spacing [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 = 0.0090</td>
<td>8-D29</td>
<td>1,500×1,500×380</td>
<td>80</td>
<td>2.8</td>
<td>300</td>
</tr>
<tr>
<td>R2 = 0.0135</td>
<td>8-D29</td>
<td>1,000×1,000×380</td>
<td>80</td>
<td>2.8</td>
<td>200</td>
</tr>
<tr>
<td>R3 = 0.0188</td>
<td>10-D29</td>
<td>900×900×380</td>
<td>80</td>
<td>2.8</td>
<td>150</td>
</tr>
</tbody>
</table>

### Material Properties

In this test program, high strength concrete of 40 MPa was used. That is the most widely applied in prestressed concrete structures as a typical structure type for the containment wall of a nuclear power plant. The water-cement ratio, air-content and slump were respectively 32%, 2% and 120 mm, and unit quantities of water, cement, sand and coarse aggregate were respectively 172 kg/m³, 531 kg/m³, 681 kg/m³ and 1016 kg/m³. The compressive strength of the concrete was obtained by testing cylinders having a diameter of 100 mm and a height of 200 mm. At the curing age of 28 days and on the test date, the average compressive strength of the cylinders was respectively 40.1 MPa and 41.9 MPa. These results meet quite well the design strength of 40 MPa.

All of the reinforcements have a diameter of 29 mm with a specified yield strength of 400 MPa, a measured yield stress of 404 MPa, and a measured modulus of elasticity of 194,000 MPa.

With regard to the measurement scheme and loading system, refer to the reference in which miscellaneous test methods are illustrated in detail.[9]
Cracking Stress

There are several test methods in the determination of the tensile strength of plain concrete, such as the third point loading method (ASTM C78-02), the center point loading method (ASTM C293-02), the splitting method of cylinder (ASTM C496-96), etc. These test methods yield various tensile strengths for the same concrete; for instance, the tensile strength given from the flexural tensile test is 40%-80% higher than that from the splitting tensile strength test.[10] However, these indirect test methods have been frequently used for the sake of convenience.

A reliable tensile strength of concrete can be obtained from direct tension tests, but it is not easy to apply tension force to plain concrete specimens. In particular, it is nearly impossible to exclude eccentricity in applying tension force, so its standard test specification currently has not been prepared. Therefore, the most effective and reasonable test method for acquisition of the cracking stress and stress-strain curve of concrete is the direct test using reinforced concrete panels. In this case, the load applied to reinforcements is transferred into the concrete by a bond along the development length.

In this experimental program, 13 reinforced concrete panel specimens were tested with uniaxial and biaxial tensile load. The tensile strength of concrete can be given from the cracking load of each specimen. Cracking load \( P_{cr} \) and cracking strain \( \varepsilon_{cr} \) were determined at the point when the first through-crack occurred.

The following is the calculating procedure for the cracking stress from applied load. Assuming that the tension force is distributed in the reinforced concrete panel in proportion to the stiffness of the concrete and reinforcement, when the tensile strain of concrete is \( \varepsilon_c \), a total applied load is as follows:

\[
P = P_c + P_s = (E_c A_c + E_s A_s)\varepsilon_c
\]

where \( P_c \) and \( P_s \) are the load sustained by the concrete and reinforcement, respectively, and \( A_c \) and \( A_s \) are the cross-section areas of the concrete and reinforcement, respectively. At the time when the first crack occurs, \( P = P_{cr} \), \( \varepsilon_c = \varepsilon_{cr} \) and \( E_c \varepsilon_{cr} = f'_{cr} \). This can be written as

\[
P_{cr} = f'_{cr} A_c + E_s A_s \varepsilon_{cr}
\]

Eq. (2) can be retransformed as

\[
f'_{cr} = \frac{P_{cr}}{A_s} - \rho E_c \varepsilon_{cr}
\]

where \( \rho = A_s / A_c \), the reinforcement ratio for the net cross-section area of the concrete, that is, \( A_c = A_g - A_s \) and \( A_g \) is the gross cross-section area of the concrete. Table 2 shows the compressive strength of the concrete, the cracking load and cracking strain given from the test results, and the cracking stress and modulus of elasticity calculated by the above equations.
Fig. 4 is a failure envelope for the concrete in the tension-tension region plotted with the calculated cracking stresses. As shown in Fig. 4, with the increase of the reinforcement ratio, the cracking stress of the concrete increases, but the global shapes of the envelopes show a similar pattern. On the axis labeled \( f_0 \) is the uniaxial compressive strength of the concrete. In the finite element methods, the failure criterion of concrete in the tension-tension region is modeled with a straight line at \( f_0 \) like the dotted line in Fig. 4. In Fig. 4, a dotted line was given from the following equations with the average splitting tensile strength and the compressive strength of concrete.

\[
f_c = 0.9 f_{\text{sp}}
\]

(4)

\[
\frac{f_c}{f_0} = 0.79
\]

(5)

where the splitting tensile strength of concrete \( f_{\text{sp}} \) is the average of 3.67 MPa from the material test and the average compressive strength of concrete \( f_0 \) is 41.9 MPa. Fig. 4 apparently indicates that the commonly used failure envelope is quite different from that given from biaxial tension tests with reinforced concrete panels.

Fig. 5 is the fitting graph of the compressive strength of concrete and the cracking stress. This fitting process yielded the following relationship between the square root of the compressive strength and the cracking stress.

\[
f_{cr} = 0.339 \sqrt{f_{ck}} \quad \text{[MPa]}
\]

(6)

Table 2. Test results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( f_{ck} ) [MPa]</th>
<th>( P_{cr} ) [kN]</th>
<th>( f_{cr} ) [MPa]</th>
<th>( \varepsilon_{cr} ) [mm/mm]</th>
<th>( E_c ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-R1-1</td>
<td>44.0</td>
<td>1,064</td>
<td>1.83</td>
<td>0.000115</td>
<td>18,348</td>
</tr>
<tr>
<td>B-R1-1</td>
<td>44.0</td>
<td>889</td>
<td>1.49</td>
<td>0.000098</td>
<td>17,653</td>
</tr>
<tr>
<td>B-R1-2</td>
<td>48.1</td>
<td>966</td>
<td>1.72</td>
<td>0.000077</td>
<td>24,675</td>
</tr>
<tr>
<td>B-R1-3</td>
<td>39.1</td>
<td>1,131</td>
<td>2.04</td>
<td>0.000107</td>
<td>20,467</td>
</tr>
<tr>
<td>U-R2-1</td>
<td>46.2</td>
<td>730</td>
<td>2.10</td>
<td>0.000128</td>
<td>16,484</td>
</tr>
<tr>
<td>B-R2-1</td>
<td>36.6</td>
<td>560</td>
<td>1.47</td>
<td>0.000116</td>
<td>13,966</td>
</tr>
<tr>
<td>B-R2-2</td>
<td>40.0</td>
<td>673</td>
<td>1.76</td>
<td>0.000120</td>
<td>16,167</td>
</tr>
<tr>
<td>B-R2-3</td>
<td>39.1</td>
<td>863</td>
<td>2.31</td>
<td>0.000123</td>
<td>19,837</td>
</tr>
<tr>
<td>U-R3-1</td>
<td>46.2</td>
<td>783</td>
<td>2.17</td>
<td>0.000121</td>
<td>20,331</td>
</tr>
<tr>
<td>B-R3-1</td>
<td>43.4</td>
<td>777</td>
<td>2.06</td>
<td>N.A</td>
<td>N.A</td>
</tr>
<tr>
<td>B-R3-2</td>
<td>36.6</td>
<td>662</td>
<td>1.81</td>
<td>0.000126</td>
<td>16,429</td>
</tr>
<tr>
<td>B-R3-3</td>
<td>39.1</td>
<td>1,111</td>
<td>3.46</td>
<td>N.A</td>
<td>N.A</td>
</tr>
<tr>
<td>B-R3-4*</td>
<td>39.1</td>
<td>1,111</td>
<td>2.07</td>
<td>0.000117</td>
<td>28,803</td>
</tr>
</tbody>
</table>

* Supplementary specimen identical with CL-B-R3-1

*Fig. 5  Relationship between cracking stress and compressive strength of concrete

*Fig. 6  Comparison of strains measured from strain gage and LVDT (0–0.003)*
Prior to the crack occurrence, reinforcements and concrete behave elastically and the applied tensile force is transferred into each material according to its stiffness. When the externally applied load is $P$, tensile stress $f_t$ in the concrete corresponding to tensile strain $\varepsilon$, is calculated as in Eq. (3), using the force equilibrium. Hereby, the relationship prior to the yielding of the reinforcement is as follows:

$$f_t = \frac{P}{A_e} - \rho E_c \varepsilon$$  \hspace{1cm} (7)

Therefore, tensile stress $f_t$, corresponding to tensile strain $\varepsilon$, follows Eq (8) through all of the load steps until the first crack occurs, and this relationship can be drawn as a straight line in Fig. 8.

$$f_t = E_c \varepsilon$$  \hspace{1cm} (8)

where $E_c$ is the modulus of elasticity of the concrete and contemporarily a slope of that graph.

In this test program, as a method for the measurement of strains, the electric resistance strain gages and the linear variable differential transformers (LVDT) were simultaneously used. Introducing an average strain concept to the stress-strain relationship, the LVDTs attached on the specimen surface is able to give better results than the strain gages reflecting the local strain of the reinforcement. Fig. 6 and Fig. 7 show the comparison between those two measurements. The tensile strain from the LVDT was taken and averaged with measurements from four or six LVDTs which were installed at several locations with different gage lengths. In an identical conception, the strain values obtained from many strain gages attached in the same direction were averaged in order to offset the asymmetric behavior of the panel specimen in the lateral and vertical directions.

Fig. 6 shows the comparisons between strain gage and LVDT outputs within a strain range of 0.003. While this graph indicates some discrepancies, on the whole it shows a good agreement. On the other hand, magnifying the strain range from zero to 0.0002 where the first crack appeared, strains obtained from strain gages are much higher than those from LVDTs (Refer to Fig. 7). Assuming that the specimen shows perfect elastic behavior up to the first crack occurrence, these two outputs should theoretically be in accord. This phenomenon is due to the fact that tensile stress is distributed nonlinearly over the concrete cover depth until the first crack appears on the specimen surface, and strains from strain gages reflect relatively more local strain of the reinforced concrete panel than those from LVDTs.

Analyzing the stress-strain curves obtained from LVDTs, strains slightly increase until the first crack occurrence but suddenly increase simultaneously with the crack occurrence. Within the range before the first crack occurrence, therefore, strains measured by using strain gages seem to be more acceptable. Hereby, Fig. 8 is plotted with strain measurements obtained from strain gages.

In Fig. 8 and Table 2, the average cracking strain of concrete is 0.000113, and Eq. (8) can be normalized by dividing $f_t$ and $\varepsilon$, by $f_{\varepsilon}$, and 0.000113, respectively. The following equation is for a normalized tensile stress-strain relationship until the first crack occurrence.

$$f_t / f_{\varepsilon} = \varepsilon / 0.000113$$
\[ \frac{f_c}{f_{cr}} = \frac{\varepsilon}{0.000113} \]  

(9)

The elasticity modulus of concrete can be obtained by substituting Eq. (6) and Eq. (8) for Eq. (9).

\[ E_c = 3,000\sqrt{f_{ck}} \]  

[MPa]  

(10)

The relationship between \( f_{ck} \) and \( E_c \), calculated by using experimental measurements, is plotted in Fig. 10 and is compared with Eq. (10). In Fig. 10, a curve with a coefficient of 2,855 is the result of a regression analysis of the test data, and the two curves show a good agreement.

\[ E_c = 2,855 \sqrt{f_{ck}} \]  

Fig. 9  Relationship between modulus of elasticity and compressive strength of concrete

Fig. 10  Force, strain, normal stress, and bond stress distribution in cracked reinforced concrete member

After Cracking

As shown in Fig. 10, when two cracks appear on the reinforced concrete member subjected to tension force, reinforcements at the location of the cracks sustain a total external force with the stress \( \sigma_{so} \). At an arbitrary location between those two cracks, however, the tensile stress of the reinforcement is smaller than \( \sigma_{so} \), and its difference is transferred by the concrete. From the force equilibrium, therefore, the following relationship is valid at an arbitrary location between two cracks.

\[ P = A_s \sigma_{so} = A_s \sigma_s(x) + A_s \sigma_s(x) \]  

(11)

Eq. (11) can be re-written in terms of stress as follows:

\[ \sigma_{so} = \sigma_s(x) + \frac{1}{\rho} \sigma_s(x) \]  

(12)

Denoting the measured strain over the several cracks by \( \varepsilon_s \) to obtain the relationship between stress and strain in the direction \( x \),

\[ \varepsilon_s = \frac{1}{L} \int_0^L \varepsilon_s(x) \, dx \]  

(13)

where, \( \varepsilon_s(x) \) is the local strain of reinforcement, and the relationship \( \varepsilon_s(x) = \sigma_s(x)/E_s \) is valid for arbitrary cross-section \( x \) before the first yielding of the reinforcement. Substituting the above relationship into Eq. (13),
\[ \varepsilon_s = \frac{1}{E_s} \left(1 - \frac{1}{L} \int_0^L \sigma_s(x) \, dx \right) \]  

(14)

The part in parenthesis can be defined as an average stress of reinforcement \( \sigma_s \).

\[ \sigma_s = \frac{1}{L} \int_0^L \sigma_s(x) \, dx \]  

(15)

Then, Eq. (14) becomes,

\[ \varepsilon_s = \frac{\sigma_s}{E_s} \]  

(16)

Using this average concept, integration of stresses in the \( x \)-direction, and dividing it by length \( L \) can take an average of the right-side stresses of Eq. (12). Therefore, Eq. (12) can be expressed as

\[ \sigma_s = \frac{1}{L} \int_0^L \sigma_s(x) \, dx + \frac{1}{\rho} \left( \frac{1}{L} \int_0^L \sigma_s(x) \, dx \right) \]  

(17)

where the first term of the right side of Eq. (17) is the average stress of the reinforcement defined in Eq. (15). But the expression in parenthesis of the second term is not a common concept, so it can be defined as

\[ \sigma_s = \frac{1}{L} \int_0^L \sigma_s(x) \, dx \]  

(18)

This stress \( \sigma_s \) can be assumed as the average stress of concrete in tension; accordingly, Eq. (17) is expressed as

\[ \sigma_s = \sigma_s + \frac{1}{\rho} \sigma_s \]  

(19)

Substituting the relationship \( \sigma_s = E_s \varepsilon_s \) from Eq. (16) into Eq. (15), the relationship between the average stress and the average strain of the concrete is derived as

\[ \sigma_s = \rho (\sigma_m - E_s \varepsilon_s) = \rho \left( \frac{P}{A_s} - E_s \varepsilon_s \right) = \frac{P}{A_s} - \rho E_s \varepsilon_s \]  

(20)

Eq. (20) for the range after cracking is exactly identical with Eq. (7) for the range before cracking. This indicates that the equation for the calculation of the average stress of concrete is valid until reinforcements yield.

Consequently, the average stress of concrete \( \varepsilon_s (= \varepsilon_c) \) corresponding to the measured average strain \( \varepsilon_s (= f_s) \) can be calculated from Eq. (20).

![Fig. 11 Average stress-strain relationship of concrete after cracking (Using the data measured from strain gauge)](image1)

![Fig. 12 Average stress-strain relationship of concrete after cracking](image2)
Fig. 11 and Fig. 12 show average tensile stress-strain curves plotted by using strains measured by strain gages and LVDTs, respectively. There were large differences between the measured strain from strain gages and LVDTs before cracking, but after cracking those differences become small.

As shown in Fig. 11, a linearly fitted line (Eq. (21)) is almost the same as an averaged curve.

\[
\frac{f_{t,c}}{f_{c,t}} = 1 - \frac{\varepsilon_t - \varepsilon_{c,t}}{0.001323 - \varepsilon_{c,t}} = 1 - \frac{\varepsilon_t - 0.000113}{0.001323 - 0.000113}
\]

In Fig. 12, obtained from LVDTs, a line of Eq. (21), an averaged curve, and an exponentially fitted curve (Eq. (22)) are compared.

\[
f_t = f_{c,t} \left( \frac{\varepsilon_{c,t}}{\varepsilon_t} \right)^c = f_{c,t} \left( \frac{0.000113}{\varepsilon_t} \right)^{0.44}
\]

The above type of stress-strain relationship after cracking was originally proposed by Tamai[11], and Belarbi verified its validity.[2,4] After taking the above results into consideration, Eq. (22), obtained from LVDT measurements, better reflects the global behavior of the cracked reinforced concrete member than Eq. (21) from the strain gages.

CONCLUSIONS

This study dealt with the procedure and results to obtain a failure envelope and stress-strain relationship for concrete in the tension-tension region. For that purpose biaxial tension tests using reinforced concrete panel members were performed. These results will be of help in the reasonable analysis and design of shell structures subjected to biaxial tension forces, like a nuclear power plant containment. The following are several outstanding results of this study.

1) As main test variables to draw the stress-strain relationship of concrete, reinforcement ratio and tension load ratio in two orthogonal directions were taken into account.
2) A failure envelope of concrete in the tension-tension region was presented by using the cracking stress calculated from the cracking load. These stresses were smaller than those obtained from common tests with plain concrete.
3) Using the behavior and theory of reinforced concrete members subjected to uniaxial tension, the stress-strain relationship of concrete in biaxial tension was derived.
4) The ascending part of the tensile stress-strain curve, that is, the region before cracking, was modeled as a straight line expressed by elasticity modulus and cracking stress.
5) The descending part of the tensile stress-strain curve, that is, the region after cracking, was modeled as an exponential function. Also, this model was obtained using the strains measured from LVDTs, which reflected well the global behavior of reinforced concrete panel members.

ACKNOWLEDGEMENT

The work presented in this paper was supported by National Mid- and Long-term Atomic Energy R&D Program and the National Research Laboratory Program from the Ministry of Science and Technology (MOST) in Korea.

REFERENCES


