Nonlinear Analysis of Reinforced Concrete Shells Subjected to Impact Loads

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ABSTRACT

After the terrorist attacks in USA some studies were performed, to evaluate the structural strength of nuclear power plants for commercial aircraft crash specially the resistance of the reactor building concrete containment. A very important point for this evaluation is the modeling of quasi-brittle materials like concrete, which requires a detailed knowledge of the implemented constitutive modeling used to describe the behavior of these materials. The first difficulty encountered in such problems, is the understanding of the different parameters, which are necessary to perform such a nonlinear finite element analysis. The objective of this paper is to discuss the modeling of reinforced concrete structures subjected to impact loads, such as aircraft impact loads. Two commercial programs are employed ANSYS/LS-DYNA. Therefore, the constitutive model implemented in ANSYS (implicit approach) as well as the constitutive model implemented in LS-DYNA (explicit approach) are discussed.

KEY WORDS: Impact Loading, Aircraft Impact, Dynamic Response, Concrete Shells, Reinforced Concrete, Finite Elements, and Rate Effects.

1- INTRODUCTION

A detailed study was performed to investigate to which extent the external concrete walls of the Reactor Buildings would be affected by the impact of a large commercial airplane. The study was conducted by a structural engineering team of ELETRONUCLEAR, the Company responsible for design and operation of nuclear stations in Brazil. Other issues were also examined by ELETRONUCLEAR in order to evaluate the global safety, such as the influence of siting, the general characteristics of the steel containment and effects on the reactor and its coolant system.

The paper describes an implicit and a explicit approach used in the analysis of the Reactor Building cylindrical walls. A Boeing 767-300, fully loaded, was selected for the study, assuming a normal impact to the surface of the containment. The results reported in this paper correspond to an impact velocity of 100m/s (360 km/h). In companion papers [1],[2] different computer programs and analysis methodologies were employed and discussed. The different methods led to similar results and showed that the concrete wall of the Reactor Building could resist to impact of an airplane at landing or take-off velocity without perforation.

2- CONSTITUTIVE MODEL

The constitutive model applied to concrete modeling available in the commercial finite element code ANSYS/5.6-6.1 [8] characterizes the concrete failure in four distinct domains, which are described below. This model is represented through a failure surface described in terms of the invariants of the stress tensor and encompasses the main characteristics presented in pressure dependent materials (quasi brittle materials). The failure surfaces are dependent of the hydrostatic component of the stress, are curved, smooth and convex having well defined compression and tension meridians. The cracking and crushing behavior are both considered in this model.

The failure criteria [3] for concrete subjected to a multiaxial stress state is represented by the following equation:

\[
\frac{F}{f_{cc}} - S \geq 0
\]  \hspace{2cm} (1)

Where:
- \(F\) – function of the principal stress state;
- \(S\) – failure surface written in terms of principal stress state and through five input parameters;
- \(f_{cc}\) – uniaxial compression strength.
Four domains characterize the concrete failure. Considering that $\sigma_1$ and $\sigma_3$ are respectively the maximum and minimum components of the principal stress state, following that $\sigma_1 \geq \sigma_2 \geq \sigma_3$, the failure domains are described by:

1\textsuperscript{st} domain: $0 \geq \sigma_1 \geq \sigma_2 \geq \sigma_3$ (compression-compression-compression)
2\textsuperscript{nd} domain: $\sigma_1 \geq 0 \geq \sigma_2 \geq \sigma_3$ (tension-compression-compression)
3\textsuperscript{rd} domain: $\sigma_1 \geq \sigma_2 \geq 0 \geq \sigma_3$ (tension-tension-compression)
4\textsuperscript{th} domain: $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0$ (tension-tension-tension)

For each domain independent functions describe $F$ and the failure surface $S$.

1\textsuperscript{st} domain: $0 \geq \sigma_1 \geq \sigma_2 \geq \sigma_3$

For this regime compression-compression-compression, the Willam/Warnke's failure criteria \cite{4} is considered. The function $F$ assumes the following format:

$$F = F_1 = \frac{1}{\sqrt{15}} \left( [\sigma_1 - \sigma_2]^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right)^{\frac{1}{2}}$$

(2)

And the failure surface is defined by:

$$S = S_1 = \frac{2r_2(r_2^2 - r_1^2) \cos \eta + r_2(2\eta - r_2)[4(r_2^2 - r_1^2) \cos^2 \eta + 5r_1^2 - 4r_1 r_2]}{4(r_2^2 - r_1^2) \cos^2 \eta + (r_2 - 2\eta)^2}$$

(3)

Where the terms of this expression are defined by:

$$\cos \eta = \sqrt{2([\sigma_1 - \sigma_2]^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)^{\frac{1}{2}}}$$

(4)

Where $\eta$ is known as similarity angle. From this equation we can observe that for $\sigma_1 = \sigma_2 \geq \sigma_3$ the angle $\eta$ is $60^\circ$. Therefore, the uniaxial compression and biaxial tension laboratory tests are defined for $\eta = 60^\circ$, being the meridian defined by this angle called compression meridian. In the same way for $\sigma_1 \geq \sigma_2 = \sigma_3$ the angle $\eta$ is $0^\circ$, defined as tension meridian where the uniaxial tension and biaxial compression tests are represented.

$$r_1 = a_0 + a_1 \xi + a_2 \xi^2$$
$$r_2 = b_0 + b_1 \xi + b_2 \xi^2$$
$$\xi = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3f_c} = \frac{\sigma_h}{f_c}$$

(5)

Here $\sigma_h$ is the hydrostatic stress. The coefficients $a_0, a_1, a_2, b_0, b_1, b_2$ are determined as a function of the known properties of concrete, like uniaxial tensile strength $f_t$, uniaxial compression strength $f_c$, biaxial compression strength $f_{cb}$. The failure surface is represented in Figure 1. Figure 2 presents the tension and compression meridians.

2\textsuperscript{nd} domain: $\sigma_1 \geq 0 \geq \sigma_2 \geq \sigma_3$

For the tension-compression-compression regime, the failure criteria is defined by the following functions:

$$F = F_2 = \frac{1}{\sqrt{15}} \left( \sigma_2^2 + \sigma_3^2 + (\sigma_2 - \sigma_3)^2 \right)^{\frac{1}{2}}$$

(6)

Moreover, the failure surface $S$ is defined by:

$$S = S_2 = \left( 1 - \frac{\sigma_1}{f_t} \right) \frac{2p_2(p_2^2 - p_1^2) \cos \eta + p_2(2p_1 - p_2)[4(p_2^2 - p_1^2) \cos^2 \eta + 5p_1^2 - 4p_1 p_2]}{4(p_2^2 - p_1^2) \cos^2 \eta + (p_2 - 2p_1)^2}$$

(7)

Where the terms of this expression are defined by:
\[ p_1 = a_0 + a_1\chi + a_2\chi^2 \]
\[ p_2 = b_0 + b_1\chi + b_2\chi^2 \]
\[ \chi = \frac{\sigma_2 + \sigma_3}{3\sigma_c} \]  

Equation (8)

If the failure criteria (1) is satisfied then cracks at the plane normal to the principal stress \(\sigma_1\) occurs.

**3rd domain:** \(\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0\)

For the tension-tension-compression regime, the failure criteria is defined by the following functions:
\[ F = F_3 = \sigma_1 \quad i = 1,2 \]  

And the failure surface is described by:
\[ S = S_3 = f_1 \left( 1 + \frac{\sigma_3}{S_2(\sigma_1,0,\sigma_3)} \right) \quad i = 1,2 \]  

Equation (9)

Equation (10)

If the failure criteria (1) is satisfied for \(i=1,2\), cracks at normal planes to the principal stresses \(\sigma_1\) and \(\sigma_2\) occur. If the failure criteria is only satisfied for \(i=1\), then cracks will only occur at the plane normal to the principal stress \(\sigma_1\).

**4th domain:** \(\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0\)

For tension-tension-tension regime, the failure criteria is defined by the following functions:
\[ F = F_4 = \sigma_i \quad i = 1,2,3 \]  

In addition, the failure surface \(S\) is defined by:
\[ S = S_4 = f_1 \frac{\sigma_i}{\sigma_c} \quad i = 1,2,3 \]  

Equation (11)

Equation (12)

If the failure criteria (1) is satisfied for \(i=1,2,3\), then cracks occur at the planes normal to the principal stresses \(\sigma_1\), \(\sigma_2\) and \(\sigma_3\). Otherwise if (1) is satisfied for \(i=1,2\), then cracks occur at the planes normal to the principal stresses \(\sigma_1\) and \(\sigma_2\). And finally if (1) is only satisfied for \(i=1\), then cracks will only appear at the plane normal to the principal stress \(\sigma_1\).

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**3- FINITE ELEMENT MODELS**

The commercial FEM program ANSYS/5.6-6.1 [8] offers the possibility to employ the volumetric solid element (SOLID65) to model the nonlinear behavior of reinforced concrete structures. This element has trilinear form functions, 8 nodes with three degrees of freedom per node. The constitutive model described previously allows cracking in three
orthogonal directions for each integration point of the element. The numerical integration schema uses Gauss integration with 2x2x2 integration points.

Initially, concrete is assumed as being an isotropic material. As the loading is increased, when a crack occurs at a specific point of integration, the crack is accounted for by the modification of the mechanical properties of the material, which means that it is modeled as a distributed crack or a smeared crack. The presence of a crack at an integration point is represented through the introduction of a weak plane at the direction normal to the crack. Additionally the model allows the inclusion of a shear transfer coefficient $\beta_t$. This coefficient represents the shear strength reduction factor for the post-cracking loading, which causes a sliding parallel to the crack plane. This shear transfer coefficient can assume values between 0 – smooth crack with total lost of shear transfer capacity – and 1 – irregular crack without lost of shear transfer capacity.

Through the inclusion of a stress relaxation factor, it is possible to accelerate the solution convergence process when cracking is imminent. This stress relaxation factor does not introduce any modification in the stress-strain relation at the post-cracking regime. After the convergence to the final cracked state, the stiffness normal to the failure plane is equal to zero.

When the material evaluated at an integration point fails in axial, biaxial or triaxial compression, the material is assumed as crushed at this point. For this finite element, crushing is defined as the complete deterioration of the structural integrity of the material, and the stiffness contribution of this integration point for the element is ignored.

The rebars are modeled as a percentage of the finite element volume, being considered as distributed over the elements. Therefore, each element allows the inclusion of four different materials: the concrete as a matrix material and other three independent materials for the reinforcements. The rebars have only axial stiffness at the specified direction.

4- DESCRIPTION OF THE STRUCTURE AND LOADING

The reactor building, which is presented in this paper, is 63m high, with an outer cylindrical shield wall of reinforced concrete, with 36.66m of outer diameter. This cylindrical building has a spherical cover on its top. Its diameter is smaller than the wingspan of the Boeing 767-300 (47.6m). However, the global impact from the entire mass of the aircraft on the structure was conservatively considered.

The steel containment is a further barrier, which was not considered in this analysis.

The concrete was specified for the construction with a f_{ck} of 28 MPa and presented the following average test results: 28 days – 36.2 N/mm²; 90 days - 44.2 N/mm²; nowadays (about 25 years) 54.6 N/mm². The compression and tension strength adopted in the analysis was 45 MPa and 4.5 MPa respectively. The Young Modulus and Poisson ratio adopted were respectively 34 GPa and 0.2. The concrete stress capacity is increased in case of high strain rates, as presented in [6],[7]. Conservatively, a majoring factor of 1.35 was adopted, which is compatible with the loading time function.
The reinforcement steel bars used in the concrete have yield strength of 500 MPa and are distributed in a mesh of 1-inch diameter bars in the vertical and circumferential directions, in the internal and external wall surfaces. The Elasticity Modulus adopted is 210 GPa and a Plastic Modulus of 1050 MPa (0.5% of elastic modulus).

The analysis herein reported was performed for the impact of a Boeing 767-300. Considering that the dimensions of the reactor building are of the same order as those of the airplane and the difficulties reaching the target with precision, it was estimated that a landing velocity would be compatible with this kind of attack. The most unfavorable condition was considered: the horizontal impact in a normal to direction to the cylindrical surface.

The load time function was determined based in [9] and considered the mass and stiffness distributions along the length of the aircraft. Two impact areas were considered: the fuselage region, with a circular diameter of 6 m², and the region of the wings together with the engines, consisting of a total area of 52 m². The load time function presented in [1] shows that the maximum force occurs when the wings and engines of the plane (Area 2) reach the structure, approximately 200 ms after the beginning of the impact, with a peak force of about 100MN. The load model neglects the wing part beyond the engines, considering that this part would not resist and break. However, the corresponding mass was conservatively kept in the analysis.

5- MODELS FOR THE CYLINDRICAL CONCRETE SHELL

Model 1A represents a cylindrical panel of 35 degrees and 20.5 m high. The model consists of three-dimensional finite elements (SOLID65), where only one fourth of the region is modeled considering symmetry conditions. Fig. 4 shows a picture of Model 1A used in the analysis together with its loaded area and boundary conditions. A static pressure load was applied at the impact area previously defined. The non-linear analysis was performed by incremental loading and step by step checking of the displacements and stresses in the materials. Cracks open or concrete crushes when the tensile or compressive strengths are exceeded at any point and direction, leading to a stiffness matrix correction for the next step.

The results of the analysis are presented in Figs. 5 to 7. The figure 5, presents the total displacement occurred on the panel and the stress intensity (difference between the principal stress) at a load level near the failure. The finite element SOLID65 allows the inclusion of rebars in three arbitrary directions inside the element. The reinforcement is defined through the ratio between its volume and the corresponding element, therefore being considered smeared and not discrete. For the panel analyzed in this paper, the reinforcement was considered only at the circumferential and longitudinal directions, as shown in Fig. 4. Shear reinforcement could also be considered including the rebar in the third possible directions.
The figure 7 shows the stress occurring at the circumferential and longitudinal rebars at inner surface. The model has adopted a mesh with four elements through the thickness. These elements have different thickness with the objective to simulate as close as possible the correct reinforcement position, remembering that the reinforcement is considered smeared through the elements.

Cracking in concrete is represented graphically by ANSYS through circles. These circles are drawn at the normal plane to the direction of the stress that produced the crack. Once the stresses are evaluated at the integration points, each integration point can suffer up to three cracks at different planes, each crack is represented with a different color.

The figure 7 represents the cracks at the internal, external and central finite elements, for a load level near the failure load.

Model 1B, presented at figure 8, represents the complete external shield of the building as a shell model, and was used for the determination of the dynamic behavior and amplification factors in a linear-dynamic elastic analysis. It was also useful for the evaluation of the influence of the size of Model 1A on the results. The relation between the maximum static and the maximum dynamic displacements gives the amplification factor of 1.13 for the dynamic effect.

Model 1C, presented at figure 8, representing part of the shell (15m high, circular sector of 112 degrees) in a linear-dynamic elastic analysis, with the same boundary conditions as indicated in Fig. 4 for Model 1A, led to a higher amplification factor equal to 1.41. Taking this last dynamic factor for multiplying the static force (Peak from the load time function), the ultimate load capacity is not reached.
A static pressure load was applied at the impact area previously defined. The non-linear analysis was performed by incremental loading and step by step checking of the displacements and stresses in the materials. The cracks open or the concrete crushes when the tension or compression limits are exceeded at any point and direction, leading to a stiffness matrix correction for the next step.

6- SOME CONSIDERATIONS ABOUT THE CONCRETE CONSTITUTIVE MODEL IMPLEMENTED AT ANSYS/LS-DYNA

The solver LS-DYNA incorporated to the program ANSYS, is a finite element explicit solver applicable to dynamic transient problems, whish uses a lumped mass formulation. The lumped mass formulation leads to a diagonal mass matrix $M$, which enables that the following equation be trivially solved for each load step.

$$Ma_{n+1} = f^{ext} - f^{int}$$  \hspace{1cm} (13)

Where $f^{ext}$ represents the external loads and $f^{int}$ the internal loads at the elements. The accelerations $a$ at step $n+1$ are then easily obtained, the velocities are then updated and the new positions for the nodes coordinates are determined through the use of central finite differences.

For problems related to short duration dynamical loading, like impact, the LS-DYNA is highly recommended. An extensive constitutive model library is available for different materials. For concrete the constitutive model #72 –
“Concrete Damage model” is available for the ANSYS/LS-DYNA version. A detailed explanation of this model can be found in [3]. This model includes several characteristics of quasi-brittle materials. The disadvantage of this model is the big amount of input parameters necessary to describe the material.

![Figure 8 – Complete FE MODEL 1B, Partial FE MODEL 1C.](image)

7- CONCLUSIONS

The present paper showed the modeling of reinforced concrete shell structure subjected to impact loads normal to the shell’s surface, through the use an implicit finite element formulation and the constitutive model implemented at ANSYS 5.6-6.1 is employed. Further investigations are being made, where the influence of the angle of impact and the point of impact are taken into account together with higher plane velocities.

For short duration loads the program LS-DYNA can be successfully employed. But as mentioned before, a sound knowledge of the constitutive model is necessary to avoid misunderstandings and errors that can be introduced through the improper choice of input parameters. The evaluation of structural response proceeding from an explicit dynamic analysis is under development.). In companion papers [1],[2] different computer programs and analysis methodologies were employed and discussed. The different methods led to similar results and showed that the concrete wall of the Reactor Building could resist to impact of an airplane at landing or take-off velocity without perforation.

8- REFERENCES