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Simplified Estimation Method for First Excursion Probabiliti of Secondary System with Friction

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ABSTRACT

The secondary system such as pipings, tanks and other mechanical equipment is installed in the primary system such as building. The secondary system has many nonlinear characteristics. The friction characteristic, which is observed in mechanical supports and joints, is one of the most common nonlinear characteristics. In this paper, an estimation method of the first excursion probability of the secondary system with friction, subjected to earthquake excitation is proposed. By using the method, the effect of friction force on the first excursion probability is estimated. It is found that when the tolerance level is normalized by the standard deviation of the response of the secondary system without friction, the first excursion probability of the secondary system is independent of mass ratio of the secondary system to the primary system, the damping ratio and the natural period..

KEY WORDS: vibration, reliability, random vibration, nonlinear vibration, friction, seismic excitation, secondary system, first excursion probability

INTRODUCTION

The piping systems, tanks and other mechanciacl equipment are usually installed on the building and the supporting structure. Those are referred to as the secondary system. The building and the supporting structure are referred to as the primary system. When the secondary systems are subected to seismic excitations, the response is greatly amplified when the natural period of the secondary system is close to that of the primary system[1]. The secondary system has nonlinear characteristics at supports and joints. Firiction characteristic is one of the most important nonlinear characteristics[2]. On the other hand, it is pointed out that seismic safety of the important secondary system is to be evaluated in probabilistic manner[3]. In many types of failure modes, the first excursion failure is one of the most important failure modes[4].

In this paper, an estimation method for the first excursion probability of the secondary system with friction is proposed. This method is based on the equivalent linearization method. As an analytical model, the secondary system and the primary system are modeled as single-degree-of-freedom system respectively. Friction characteristic is assumed to be Coulomb friction. As seismic excitations, nonstationary filtered white noise and artificial time history compatible with the response spectrum are used. The first excursion probability is obtained using the proposed method. In actual aseismic design, response spectrum is used. For the secondary system, the floor response spectrum, the maximum response of the secondary system, is used. In order to consider this point, the tolerance level is normalized by the maximum standard deviation of the response of the secondary system without friction. When the tolerance level is normalized, the first excursion probability of the secondary system is independent of mass ratio of the secondary system to the primary system, the damping ratio and the natural period.

ANALYTICAL MODEL AND EARTHQUAKE MOTION

As an analytical model, two-degree-of freedom model as shown in Fig.1 is used. In this model, the secondary system and the primary system are modeled as single-degree-of-freedom system respectively. m is mass, c is damping coefficient, k is stiffness, x is absolute displacement. Subscript s is used for the secondary system, p is for the primary system. And, y is absolute displacement of ground surface and F_r is friction force.

As input ground motion, two kinds of nonstationary artificial time histories are used. One is nonstationary filtered white noise. Fig.2 shows the envelope function I(t) for nonstationary filtered white noise which represents nonstationary amplitude characteristic of ground motiion. The other is artificial time histories compatible with the response spectrum. Figure 3 shows target response spectrum[5]. S is the response amplification factor, ratio of the maximum response of single-degree-of-freedom system to that of the ground motion. Figure 4 shows envelope function I(t) for artificial time history compatible with the response spectrum which represents nonstationary amplitude characteristic of the ground motion[6].

ESTIMATION METHOD OF FIRST EXCURSION PROBABILITY

Equations of Motion

Equations of motion with respect to relative displacement of the secondary system to the primary system $z_s(x_s\text{-}x_p)$ and that of the primary system to the ground surface $z_p(x_p\text{-}y)$are derived. When the secondary system moves to the primary system, equations of motion is given as:

$$\left.\begin{aligned} &\ddot{z}_s + z\zeta_s\omega_s\dot{z}_s + \omega_s{}^2 z_s + f = -\ddot{y} - \ddot{z}_p \\ &\ddot{z}_p + z\zeta_p\omega_p\dot{z}_p + \omega_p{}^2 z_p - \gamma\left(z\zeta_s\omega_s\dot{z}_s + \omega_s{}^2 z_s + f\right) = -\ddot{y} \end{aligned}\right\} \qquad (1)$$

where $\zeta\left(= c/\left(2\sqrt{mk}\right)\right)$ is the damping ratio, $\omega\left(= \sqrt{k/m}\right)$ is the natural circular frequency, $\gamma\left(= m_s / m_p\right)$ is mass ratio of the secondary system to the primary system, f is a term of friction characteristic. As friction characteristic, Coulomb friction characteristic as shown in Fig.5 is introduced. $f_r\left(= F_r / m_s\right)$ is acceleration corresponds to friction force. f is given as:

$$f = f_r \frac{\dot{z}_s}{\left|\dot{z}_s\right|} \qquad (2)$$

Equation (1) is given when absolute acceleration of the primary system $\left|\ddot{x}_p\right|$ is greater than f_r. When this condition is not satisfied, the condition where the secondary system does not move to the primary system should be considered. In this case,

$$\dot{z}_s = 0 \text{ and } \left|\ddot{x}_p\right| < f_r \qquad (3)$$

where $\ddot{x}_s$ is absolute acceleration of the secondary system. And,

$$\left.\begin{aligned} \ddot{x}_s &= \ddot{x}_p \\ \dot{z}_s &= 0 \\ z_s &= z_{st} \end{aligned}\right\} \bullet \qquad (4)$$

where z_{st} is displacement when Eq.(3) is satisfied. The secondary system begins to move to the primary system when each of the following equations is satisfied.

$$f = f_r \frac{\dot{z}_s}{\left|\dot{z}_s\right|} \qquad (5)$$

From Eq.(5), when the secondary system is subjected to earthquake motion from the condition $z_s = 0$, the secondary system does not move to the primary system until the condition $\left|\ddot{x}_p\right| > f_r$ is satisfied. Thus, f_r is determined by using the maximum absolute acceleration of the linear primary system without friction characteristic $\left|\ddot{x}_p\right|_{max}$. Since the maximum standard deviation of absolute acceleration of the linear primary system $\left|\sigma_{\ddot{x}_p}\right|_{max}$ corresponds to $\left|\ddot{x}_p\right|_{max}$ [7]. Then, f_r is determined by using $\left|\sigma_{\ddot{x}_p}\right|_{max}$ as follows:

$$f_r = \xi\left|\sigma_{\ddot{x}_p}\right|_{max} \qquad (6)$$

where ξ is normalized friction force. $\left|\sigma_{\ddot{x}_p}\right|_{max}$ is determined by multiplying S given by Fig.2 by the maximum standard deviation of ground acceleration $\left|\sigma_{\ddot{x}_g}\right|_{max}$.

Estimation Equation for First Excursion Probability

It is assumed that failure occurs when absolute value of displacement response of the secondary system z_s first crosses the tolerance level [illegible]. The first excursion probability P_f is obtained by the following equation.

$$P_f(t) = 1 - \exp\left\{-2\int_0^t \nu(t)dt\right\} \qquad (7)$$

When instants at which z_s crosses B_D are statistically independent, $\nu(t)$ is given as:

$$\nu(t) = \frac{1}{2\pi}\frac{D}{\sigma_{z_s}{}^2}\left[\exp\left\{-\frac{B_D{}^2}{2\sigma_{z_s}{}^2}\left(1+\frac{\kappa_{z_s\dot{z}_s}}{D}\right)\right\} + B_D\kappa_{z_s\dot{z}_s}\sqrt{\frac{\pi}{2D\sigma_{z_z}{}^2}}\exp\left(-\frac{B_D{}^2}{2\sigma_{z_s}{}^2}\right)\{1+\mathrm{erf}(C)\}\right] \tag{8}$$

where

$$C = \frac{k_{z_s\dot{z}_s}B_D}{\sqrt{2D\sigma_{z_s}{}^2}},\ D = \sigma_{z_s}{}^2\sigma_{\dot{z}_s}{}^2 - \kappa_{z_s\dot{z}_s}{}^2,\ \mathrm{erf}(u) = \frac{2}{\sqrt{\pi}}\int_0^u \exp(-y^2)dy$$

σ^2 and κ are variance and covariance for variable given by subscripts, respectively.

For nonstationary filtered white noise, power spectral density function of the ground motion is assumed to be given as:

$$G(\omega) = \frac{(2\zeta_g\omega_g\omega)^2 + \omega_g{}^4}{(\omega^2{}_g - \omega^2)^2 + (2\zeta_g\omega_g\omega)^2}G_0 \tag{9}$$

where ζ_g is the damping ratio of the ground model, ω_g is the natural circular frequency of the ground model, G_0 is power spectral density of white noise which is input to the base rock.

For nonstationary artificial time historiy, the expected value of power spectral density function of artificial time histories used in this study is given by Eq.(9). Values of those parameters are $\zeta_g = 0.5$, $T_g(= 2\pi/\omega_g) = 0.285s$, $G_0 = 1.94x10^{-3}(1/s)$ [8].

The Fokker-Planck equation with respect to relative displacement of the secondary system to the primary system z_s, relative velocity $\dot{z}_s$, relative displacement of the primary system to the ground z_p, relative velocity $\dot{z}_p$, relative displacement of the ground to the base rock z_g, relative velocity $\dot{z}_g$ is expressed as:

$$\begin{aligned}\frac{\partial p}{\partial t} = &-\frac{\partial p}{\partial z_p}\dot{z}_p + 2\zeta_p\omega_p p - \frac{\partial p}{\partial \dot{z}_p}\left\{-2\zeta_p\omega_p\dot{z}_p - \omega_p{}^2 z_p + \gamma\left(2\zeta_s\omega_s\dot{z}_s + \omega_s{}^2 z_s + f\right) + I(t)\left(2\zeta_g\omega_g\dot{z}_g + \omega_g{}^2 z_g\right)\right\}\\ &-\frac{\partial p}{\partial z_s}\dot{z}_s + 2\zeta_s\omega_s(1+\gamma)p - \frac{\partial p}{\partial \dot{z}_s}\left\{-2\zeta_s\omega_s(1+\gamma)\dot{z}_s - \omega_s{}^2(1+\gamma)z_s - (1+\gamma)f + 2\zeta_p\omega_p\dot{z}_p + \omega_p{}^2 z_p\right\}\\ &-\frac{\partial p}{\partial z_g}\dot{z}_g + 2\zeta_g\omega_g p + \frac{\partial p}{\partial \dot{z}_g}\left(2\zeta_g\omega_g\dot{z}_g + \omega_g{}^2 z_g\right) + \frac{\partial^2 p}{\partial \dot{z}_g{}^2}\frac{\pi G_0}{2}\end{aligned} \tag{10}$$

In order to use Eq.(8), the second moments with respect to z_s and $\dot{z}_s$ are to be obtained. Applying partial integral method to Eq.(10), the moment equations with respect to the second moments with respect to z_s, $\dot{z}_s$, , $\dot{z}_p$, z_g and $\dot{z}_g$ are obtained[9]. Solving the moment equations and using Eqs.(8) and (7), the first excursion probability P_f is obtained.

Equivalent Linearization Method

Nonlinear force f is equivalently linearized as follows.

$$f = C_{eq}\dot{z}_s \tag{11}$$

where C_{eq} is the equivalent damping coefficient. It is assumed that friction force is not large and the secondary system moves to the primary system near the main shock. And C_{eq} is approximately obtained by assuming stationary random process since main shock is long. Equivalent damping coefficient for sinusoidal excitation is given as:

$$C_{eq}' = \frac{4f_r}{\pi\omega_i Z_s} \tag{12}$$

where Z_s is amplitude of steady-state response of the secondary system and ω_i is frequency of excitation. It is assumed that probability density function of amplitude is Rayleigh distribution. Expected value of C_{eq}' is obtained as:

$$C_{eq} = \frac{2\sqrt{2}f_r}{\sqrt{\pi}\omega_s\sigma_s} \tag{13}$$

C_{eq} near the main shock is given as Eq.(13). However, from the beginning of excitation to the main shock and after the main shock, Eqs.(3), (4) and (5) should be considered. From Eq.(5), when the system is subjected to excitation from equilibrium position, the secondary system moves to the primary system when $|\ddot{x}_p| > f_r$. Considering this condition, C_{eq} is approximately obtained by using $\sigma_{\ddot{x}_p}$, standard deviation of z_{st}. It is assumed that standard deviation is proportional to the maximum response. First, $|\sigma_{\ddot{x}_p}|_{max}$, the maximum value of standard deviation of linear system $\sigma_{\ddot{x}_p}$, is obtained. Next, it is assumed that when standard deviation of absolute acceleration response of the primary system with friction $\sigma_{\ddot{x}_p}$ is less than $|\sigma_{\ddot{x}_p}|_{max}$, Eqs.(3) and (4) are satisfied with probability p_s. $\xi|\sigma_{\ddot{x}_p}|_{max}$ corresponds to the friction force and the second moments with respect to response of the secondary system is 0 with probability p_s. p_s is approximately given as:

$$p_s = 1 - \frac{\sigma_{\ddot{x}_p}}{\xi|\sigma_{\ddot{x}_p}|_{max}} \tag{14}$$

When $\sigma_{\ddot{x}_p} / \xi|\sigma_{\ddot{x}_p}|_{max}$ is greater than 1, p_s is assumed to be 0. Figure 6 shows relation between p_s and $\sigma_{\ddot{x}_p}$. The second moments with respect to response of the secondary system, $\sigma_{z_s}{}^2$, $\sigma_{\dot{z}_s}{}^2$ and $\kappa_{z_s\dot{z}_s}$, are obtained by multiplying those obtained from the moment equations by $\left\{\sigma_{\ddot{x}_p} / \xi|\sigma_{\ddot{x}_p}|_{max}\right\}^2$. The second moments with respect to the secondary system and the primary system or the ground, for example $\kappa_{z_s z_p}$, are obtained by multiplying those obtained from moment equations by $\sigma_{\ddot{x}_p} / \xi|\sigma_{\ddot{x}_p}|_{max}$.

NUMERICAL RESULTS

Friction Force and Tolerance Level

Friction force is determined as Eq.(6). The tolerance level B_D is normalized by the maximum standard deviation of relative displacement response of the secondary system without friction $|\sigma_{s\ell}|_{max}$ as:

$$\delta_t = B_D / |\sigma_{s\ell}|_{max} \tag{15}$$

$|\sigma_{s\ell}|_{max}$ is determined as:

$$|\sigma_{s\ell}|_{max} = R|\sigma_{\ddot{x}_p}|_{max} / \omega_s{}^2 \tag{16}$$

where R is ratio of the maximum response of the secondary system without friction to that of the primary system. Some values of parameters are fixed as γ=0, ζ_s=0.01 and ζ_p=0.05. For the natural period, the least feasible condition, that is T_s=T_p, is selected. In this case, response of the secondary system is greatly amplified. Values of the natural period are selected between 0.3s and 0.8s. In this condition, R in Eq.(16) is about 10. P_f is function of time. The important secondary system should maintain their function after earthquake excitation. In this paper, P_f at the end of earthquake excitation is obtained.

Obtained Results

From Table 1 to Table 4, results for the system subjected to filtered white noise are shown. From Table 1 to Table 3, ξ defined as Eq.(6) is fixed as 0.05. Table 1 shows the first excursion probability P_f for different values of the damping ratio of the secondary system ζ_s. P_f decrerases slightly with the increase of ζ_s. However, variation of P_f is very small. Table 2 shows P_f for different values of mass ratio γ. Variation of P_f is very small. Table 3 shows P_f for defferent values of the natural period T_s and T_p. P_f decreases as the natural period becomes longer. However, variation P_f is very small, especially for relatively long natural period. Thus, when the tolerance level is normalized as Eq.(15), P_f is independent of the damping ratio, mass ratio and the natural period.

Table 4 shows P_f for different values of ζ_s. In this table, ξ is 0.1. As Table 1, P_f decrerases slightly with the increase of ζ_s. However, variation of P_f is very small.

In Table 5, results for the system subjected to artificial time history are shown. This table shows P_f for different values of ζ_s. In this case, P_f increrases slightly with the increase of ζ_s. However, variation of P_f is very small.

From these tables, it is cocluded that when the tolerance level is normalized by the maximum standard deviation of relative displacement response of the secondary system without friction as Eq.(15), the first excursion probaility is independent of mass ratio of the secondary system to the primary system, the damping ratio and the natural period. The maximum standard deviation of the response of the scondary system without friction corresponds to the maximum reposponse of the secondary system, that is, floor response specrum. For the secondary system, the floor response spectrum, the maximum response of the secondary system, is used. Thus, the first excursion probability is practically estimated by using the floor response spectrum.

CONCLUSIONS

An estimation method for the first excursion probability of the secondary system with friction is examined. A theoretical method for obtaining the first excursion probability is shown. This method is based on the equivalent linearization method. Friction characteristic is assumed to be Coulomb friction. As seismic excitations, nonstationary filtered white noise and artificial time history compatible with the response spectrum are used. The first excursion probability is obtained using the proposed method, When the tolerance level is normalized by the maximum standard deviation of the response of the secondary system without friction, the first excursion probability of the secondary system is independent of mass ratio of the secondary system to the primary system, the damping ratio and the natural period. The maximum standard deviation of the response of the secondary system without friction corresponds to the maximum response of the secondary system. The maximum response of the secondary system is floor response spectrum. For the secondary system, the floor response spectrum, the maximum response of the secondary system, is used. Consider this point, the first excursion probability of the secondary system with friction is practically estimated by the proposed method.

REFERENCES

1. Suzuki,K. And Aoki,S., Convnetional Estimating Method of Earthquake Response of Mechancical Appendage System Installed in the Nuclear Structural Facilities. Transactions of the Sixth International Conference on Structural Mechanics in Reactor Technology. K10/3 (1981), pp.1-8
2. Lin,C.W., Seismic Evaluation of System and Components. Transactions of ASME, Journal of Pressure Vessel Technology. 113-2 (1991), pp.273-283.
3. Lin,Y.K., Probabilistic Theory of Structural Dynamics, Krieger. New York. (1967), pp.293-338.
4. Lin,Y.K. and Cai,G.Q., Probabilistic Structural Dynamics. McGraw-Hill. Singapore. (1995) pp.363-404
5. Japanese Ministry of International Trade and Industry, Guidelines for Aseismic Design of High Pressure Gas Facilities. (1983).
6. Jennings,P.C., Housner,G.W. and Tsai,N.C., Simulated Earthquake Motions. Earthquake Engineering Research Laboratory. California Institute of Technology. (1968), pp.7-8
7. Tajimi,H., A Statistical Mehtod of Determining the Maximum Response of a Building Structure during an Earthquake. Proceedings of the 2nd World Conference on Earthquake Engineering. II (1960), pp.781-798.
8. Aoki,S. and Suzuki,K., First Excursion Probability Estimation of Mechanical Appendage System Subjected to Nonsatationary Earthquake Excitations. Proceedings of the 4th International Conference on Structural Safety and Reliability. I (1985), pp.201-210.
9. Aoki,S., First Excursion Probability of Secondary System Subjected to Earthquake Excitations. Report of Tokyo Metropolitan College of Technology. 23 (1988), pp.1-8.

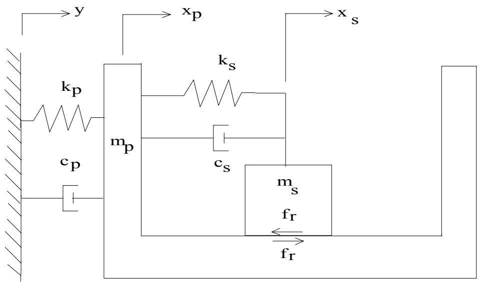

Fig.1 Analytical model of the secondary system with friction

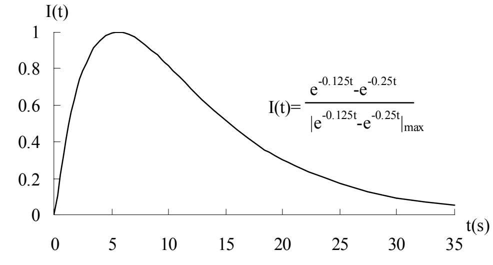

Fig.2 Envelope function for filtered white noise

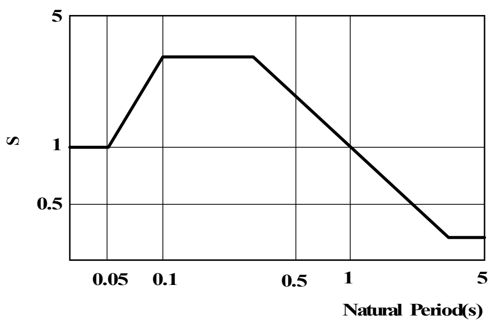

Fig.3 Target response spectrum for 5% damping ratio of critical

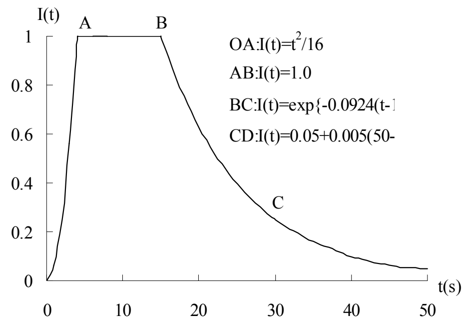

Fig.4 Envelope funcyion for artificial time history

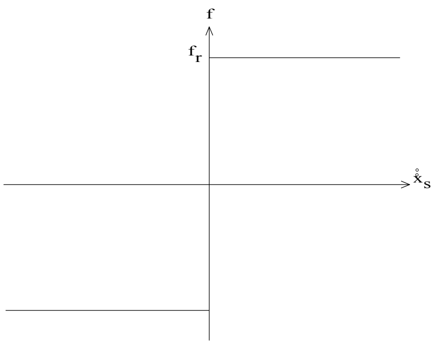

Fig.5 Coulomb friction characteristic

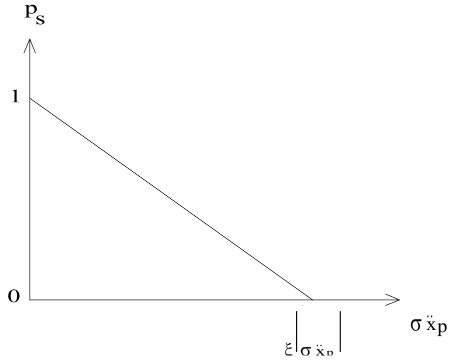

Fig.6 Relation between p_s and $\sigma_{\ddot{x}_p}$

Table 1 First excursion probability for filtered white noise (γ =0, ζ_p =0.05, T_s=T_p=0.5s, ζ_g =0.4, T_g0.5s, ξ=0.05)

δ_t	ζ_s			
	0.01	0.02	0.05	0.10
1.5	0.999	0.998	0.996	0.996
2.0	0.917	0.877	0.847	0.841
2.5	0.490	0.433	0.399	0.394
3.0	0.136	0.116	0.105	0.104

Table 2 First excursion probability for filtered white noise (C_{eq}=0.01, ζ_p =0.05, T_s=T_p=0.5s, ζ_g =0.4, T_g0.5s, ξ=0.05)

δ_t	γ			
	0	0.01	0.02	0.05
1.5	0.999	0.996	0.996	0.997
2.0	0.917	0.843	0.849	0.856
2.5	0.490	0.395	0.403	0.413
3.0	0.136	0.104	0.105	0.112

Table 3 First excursion probability for filtered white noise ([illegible]=0, ζ_s =0.01, ζ_p =0.05, ζ_g =0.4, T_g0.5s, ξ=0.05)

δ_t	Ts=Tp(s)			
	0.3	0.5	0.8	1.0
1.5	1.000	0.999	0.997	0.997
2.0	0.976	0.917	0.853	0.826
2.5	0.645	0.490	0.405	0.377
3.0	0.207	0.136	0.107	0.098

Table 4 First excursion probability for filtered white noise (γ =0, ζ_p =0.05, T_s=T_p=0.5s, ζ_g =0.4, T_g0.5s, ξ=0.1)

δ_t	ζ_s			
	0.01	0.02	0.05	0.10
1.5	1.000	0.998	0.997	0.996
2.0	0.925	0.886	0.858	0.855
2.5	0.509	0.449	0.415	0.413
3.0	0.146	0.124	0.112	0.112

Table 5 First excursion probability for artificial time history compatible with response spectrum (γ =0, ζ_p =0.05, T_s=T_p=0.5s, ζ_g =0.5, T_g0.285s, S_0=9.7, ξ=0.05)

δ_t	ζ_s			
	0.01	0.02	0.05	0.10
1.5	1.000	1.000	1.000	1.000
2.0	0.958	0.959	0.966	0.968
2.5	0.577	0.594	0.632	0.644
3.0	0.170	0.183	0.209	0.218