*Transactions of the 17th International Conference on Structural Mechanics in Reactor Technology (*SMiRT 17)
Prague, Czech Republic, August 17 –22, 2003
Paper # M02-4

Assessment of Structural Safety Under Seismic Hazard by Vulnerability Functionals

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ABSTRACT

In this paper we further develop some of our earlier studies on seismic fragility and vulnerability models. A rather large variety of definitions for the seismic vulnerability concept can be found in the literature of the field. A rather general approach to this type of models was developed by R.T. Duarte in the early 90's, in terms of operators and functionals. We discuss a couple of existing vulnerability functionals / indices from the point of view of this (more mathematically rigorous) concept and propose some ways to reformulate them.

KEY WORDS: seismic vulnerability, seismic fragility, local and global damage indices, mean damage ratio, global damage index, generalized vulnerability models, PGA (peak ground acceleration), Arias intensity.

INTRODUCTION

The seismic fragility models were developed since the early 80's, mainly in connection with the probabilistic seismic safety assessment (PSA) studies of nuclear power plants ; a typical reference, due do a Californian team, is [1]. Basically, the *seismic fragility* is the failure probability of a structure or component conditional on a certain value of a relevant ground motion parameter (like the PGA), also defined as the probability that the expected response of a structure / component will exceed a limit state during an expected level of ground shaking. More recently, they have gained a rather extensive use in the seismic risk assessment of structures of various types, including civil engineering structures in urban areas [2]. Practically, most fragility models accept and use the so-called *double Log-N* format which – besides its rich mathematical properties - allows for using multiplicative composition of quantified influences.

Both vulnerability and (damage-related) fragility models try to give account of the damage levels of a component or structure subjected to seismic excitation. The concept of (generalized) seismic vulnerability of structures is basically expressed [3] in terms of a conditional probability density function of a damage parameter d, conditioned by another parameter q that quantifies the earthquake intensity : $f^{(v)}(d \mid q)$. Parameter d is a measure of the damage state of the structure, e.g. one of the various types of damage indices, and it is often discretized in 3 to 5 damage classes ; q is a measure of the severity of the expected earthquake motions. Instead, the formulations of vulnerability models are mathematically less limited yet conceptually broader and sometimes hazier. We have considered some seismic vulnerability formulations for RC structures, allowing for Bayesian updating, in our paper [4] submitted to the SMiRT 16 Conference.

In this paper we start from some of R.T. Duarte's [5,6] ideas who insists on the mathematical foundations of seismic vulnerability models. Such models operate with *vulnerability functionals* which express the relationship between the essential characteristics of the earthquake action and the essential characteristics of the corresponding structural response. We try to give a couple of extensions and applications of this rather abstract model.

MODELS FOR SAFETY ASSESSMENT OF STRUCTURES UNDER SEISMIC HAZARD

A mathematical (analytic / probabilistic) model is defined [5] as a set of hypotheses which idealize the relevant aspects of nature. In engineering, the models are selected mainly because their usefulness and less because they are "exact". According to D. Veneziano, three aspects can be considered in the evaluation of models: the logical consistency (syntactical aspect), the descriptive power (semantic aspect) and the practical usefulness (pragmatical aspect). More details on this discussion can be found in [5]. As regards the optimality of a model M in a set M of possible models, it may be characterized by means of two functions: a cost functional c that evaluates the cost entailed by using a selected model, and a penalty functional p as the penalty involved because of use of a wrong model. With these functionals, a model M^{o} in M will be optimal if it minimizes $c(M) + p(M)$ over M . The main difficulty here lies in estabilishing these two functionals.

In modeling an engineering process, like the behaviour of a structure under the seismic hazard, it is admitted that (only) one "exact" model theoretically exists in M; it is sometimes called the reference model (RM) and we denote it by M^{r}. It would be characterized by $p(M^{r}) = 0$. The model to be effectively used is almost always a simplified model M^{s}. The relationship between a simplified model and a reference model was mathematically stated by R.G. Muncaster [7]. A set of states P^{r} is assigned to the system under study in the RM (reference model). In fact, P^{r} has to be considered as an open subset of a Banach space B^{r}. An operator E^{r} is defined on space B^{r} whose range is P^{r}. This operator is identified with an equation of motion of the system, expressed in terms of the idependent time (t-) variable:

$$\dot{\mathbf{u}}^{\mathrm{r}}(t) = \mathsf{E}^{\mathrm{r}}(\mathbf{u}^{\mathrm{r}}(t)) \tag{1}$$

with the initial condition $\mathbf{u}^{\mathrm{r}}(0) = \mathbf{p}^{\mathrm{r}} \in \mathsf{P}^{\mathrm{r}}$. It is assumed that Eq. (1) with the specified initial condition always admits a unique solution on some finite interval $\mathsf{T} = [0, T]$. Unlike in [5], we explicitly assume that any element in the space B^{r} is a vector $\mathbf{u}$ describing the motion or – more generally – the behaviour of the system in time, which takes vector values $\mathbf{p}$ in the state space. By the way, the example given by Eq. (2) in [5] is just describing the motion of a discrete model of a structure in terms of a vector $\mathbf{u}^{\mathrm{r}} = [\dot{\mathbf{q}}\ \mathbf{q}]^{t}$ where $\mathbf{q}$ contains the displacements in the degrees of freedom of the model, $\dot{\mathbf{q}}$ gives the corresponding velocities, etc.

A simplified model (SM) is defined in a similar way as the RM : the equation of motion in this model is of the form (1), with superscript r replaced by s. The corresponding state space / subset and the operator that describes the system motion (or behaviour) are also s-superscripted : $\mathsf{B}^{\mathrm{s}}, \mathsf{P}^{\mathrm{s}}, \mathsf{E}^{\mathrm{s}}$. The relationship between the RM and the SM is established by a mapping $\mathsf{S} : \mathsf{P}^{\mathrm{r}} \to \mathsf{P}^{\mathrm{s}}$ such that a single state of SM corresponds to a state of RM, while the converse does not hold : several states in the reference model may be mapped by S on a state of the SM. In other words, S may be not injective, in general. Thus, the RM can be regarded as a structural idealization while some details may be lost when going from RM to SM through S . Each state $\mathbf{p}^{\mathrm{s}} \in \mathsf{P}^{\mathrm{s}}$ is considered as an initial value of a solution $\mathbf{u}^{\mathrm{s}}(t)$ in the simplified model, assumed to be unique and to correspond to one element $\mathbf{u}^{\mathrm{r}}(\mathbf{p}^{\mathrm{s}}) = \mathbf{u}^{\mathrm{r}}[\mathsf{S}(\mathbf{p}^{\mathrm{r}})] \in \mathsf{B}^{\mathrm{r}}$. This mapping from $\mathsf{P}^{\mathrm{s}} \times \mathsf{T}$ is denoted by U and it induces a parametrization on B^{r} :

$$\mathbf{u}^{\mathrm{r}}(\mathbf{p}^{s}, t) = \mathsf{U}\ [\mathbf{u}^{\mathrm{s}}(t)]. \tag{2}$$

Three conditions have to be imposed on the mapping U :

(*i*) Mapping U is the inverse of mapping S at moment $t = 0$ (what is not mentioned in [4]):

$$\mathsf{S}[\mathbf{u}^{\mathrm{r}}(\mathbf{p}^{s}, 0)] = \mathsf{S}\,[\mathsf{U}\ [\mathbf{u}^{\mathrm{s}}(0)]] = \mathbf{p}^{\mathrm{s}}. \tag{3}$$

(*ii*) Time invariance is preserved between the RM and the SM ; this means that for each $\mathbf{p}^{\mathrm{s}} \in \mathsf{P}^{\mathrm{s}}$ and $t \in \mathsf{T}$ there exists a state $\mathbf{p}' \in \mathsf{P}^{\mathrm{s}}$ such that

$$\mathbf{u}^{\mathrm{r}}(\mathbf{p}^{\mathrm{s}}, t+\tau) = \mathbf{u}^{\mathrm{r}}(\mathbf{p}', t) \tag{4}$$

for all t & τ such that $t + \tau \in \mathsf{T}$.

(iii) Validity of the RM has to be checked by means of the SM ; in other words, the SM should be consistent with the RM. More precisely, the states of the RM defined by mapping U should obey the equation of motion

$$\dot{\mathbf{u}}^{\mathrm{r}}(\mathbf{p}^{\mathrm{s}}, t) = \mathsf{E}^{\mathrm{r}}(\mathbf{u}^{\mathrm{r}}(\mathbf{p}^{\mathrm{s}}, t)) \tag{5}$$

for each $\mathbf{p}^{\mathrm{s}} \in \mathsf{P}^{\mathrm{s}}$ and all $t \in \mathsf{T}$.

The mappings satisfying hypotheses (*i*) & (*ii*) are characterized as follows : let the vector function $\mathbf{u}^{\mathrm{r}}$ and the operator G be defined by

$$\mathbf{u}^{\mathrm{s}}(\mathbf{p}^{\mathrm{s}}, t) = \mathsf{S}\,(\mathbf{u}^{\mathrm{r}}(\mathbf{p}^{\mathrm{s}}, t)),\ \ \mathsf{G}\,(\mathbf{p}^{\mathrm{s}}) = \mathbf{u}^{\mathrm{r}}(\mathbf{p}^{\mathrm{s}}, 0). \tag{6}$$

Then mapping U satisfies conditions (*i*) & (*ii*) if and only if $\mathbf{u}^{\mathrm{r}}(\mathbf{p}^{\mathrm{s}}, t) = \mathsf{G}\,(\mathbf{u}^{\mathrm{s}}(\mathbf{p}^{\mathrm{s}}, t))$, and the functions $\mathbf{u} : \mathsf{P}^{\mathrm{s}} \times \mathsf{T} \to \mathsf{B}^{\mathrm{s}}$ with the operator $\mathsf{G} : \mathsf{P}^{\mathrm{s}} \to \mathsf{B}^{\mathrm{s}}$ satisfies the semigroup property

$$\mathbf{u}^{\mathrm{r}}(\mathbf{p}^{\mathrm{s}}, t+\tau) = \mathbf{u}^{\mathrm{r}}(\mathbf{u}^{\mathrm{s}}(\mathbf{p}^{\mathrm{s}}, \tau), t), \tag{7}$$

the initial condition $\mathbf{u}^{\mathrm{s}}(\mathbf{p}^{\mathrm{s}}, 0) = \mathbf{p}^{\mathrm{s}}$, respectively the constraint $\mathsf{S}\,(\mathsf{G}\,(\mathbf{p}^{\mathrm{s}})) = \mathbf{p}^{\mathrm{s}}$ for all $\mathbf{p}^{\mathrm{s}} \in \mathsf{P}^{\mathrm{s}}$, $\tau \in \mathsf{T}$ and $t + \tau \in \mathsf{T}$. The last step in this construction consists in formulating the equation of motion for the simplified model. This can be accomplished by Muncaster's [7] theorem, which assumes that the functions $\mathbf{u}^{\mathrm{s}}(p, t)$ are *t*-differentiable in order that functions $\mathbf{u}^{\mathrm{r}}(p, t)$ are also *t*-differentiable, and the operators G and U are Frechet-differentiable. This theorem states that the function $\mathbf{u}^{\mathrm{r}} : \mathsf{P}^{\mathrm{s}} \times \mathsf{T} \to \mathsf{B}^{\mathrm{s}}$ satisfies hypotheses (*i*), (*ii*), (*iii*) if and only if the operator G is a solution of the system of equations

$$[D\mathcal{G}]([D\mathcal{S}(\mathcal{G}(\mathbf{p}^s))])\mathcal{E}^r(\mathcal{G}(\mathbf{p}^s)) = \mathcal{E}^r(\mathcal{G}(\mathbf{p}^s)), \quad \mathcal{S}(\mathcal{G}(\mathbf{p}^s)) = \mathbf{p}^s, \quad \mathbf{p}^s \in \mathcal{P}^s. \tag{8}$$

In Eq. (8), D denotes the differential operator. In this Eq. (8), for each $\mathbf{p}^s \in \mathcal{P}^s$, $\mathbf{u}^s(\mathbf{p}^s, t)$ is a solution of the initial value problem

$$\begin{cases} \dot{\mathbf{u}}^s(t) = \mathcal{E}^s(\mathbf{u}^s(t)), \\ \mathcal{E}^s : \mathcal{E}^s(\mathbf{p}^s) = [D\mathcal{S}(\mathcal{G}(\mathbf{p}^s))]\mathcal{E}^r(\mathcal{G}(\mathbf{p}^s)). \end{cases} \tag{9}$$

As also mentioned in the Introduction, the concept of vulnerability has been defined in a large variety of ways. It is not our aim to formulate a “better” definition than other existing ones. Instead, we are going to see if and how the general model (just recalled) could be applied to more formalized definitions / quantification of the seismic vulnerability of structural systems.

SEISMIC HAZARD AND STRUCTURAL DAMAGE MODELING

Seismic damage description and quantification

Almost all the studies on the seismic vulnerability assessment of structures involve the notion of damage (state). More precisely, an analytic model is elaborated aiming to provide a damage level measure for a given type of structural system or – more generally – for a class of similar structures. Certainly, the seismic hazard description is quite relevant for predicting the future behaviour of systems in a given area. A large amount of seismological data is used, together with time histories of notorious earthquake motions. A significant problem in working out credible / realistic models for vulnerability analysis consists in the (often) limited amount of damage assessment data due to previous earthquakes. However, such data have been obtained and used for seismically active regions in Italy, Portugal, California, etc. An interesting idea consists in translating some vulnerability curves derived for a given region to another zone, taking into account the differences between the seismological features of the two regions. A typical study of this type is [8].

A large variety of indicators for the damage state evaluation exist in the literature. It is not our aim, in this paper, to give a comprehensive presentation of such damage indices or damage functionals, nor to discuss them in more detail. We try to propose rather general formulations for the events consisting of the attainment by the structure of a certain damage level (including a collapse state), with special attention paid to the damages induced by earthquake motions. Many interesting and rather widely accepted models and methods have been developed within the Structural Reliability Theory. Certainly, the mere consideration of an event consisting in the change of the damage state of a component (or substructure), or of a whole structure, makes not sense if a probability is not assigned to such an event. But this latter problem is just the object of the seismic fragility and vulnerability models, to be approached in the next section. Let us now try to discuss a couple of approaches which are possible in modeling the damage levels experienced by a structure subjected to actions from the environment, including the effects of (major) earthquake events. We thus extend a recent study submitted to 12 ECEE [9].

In general, a vector of damage parameters $\mathbf{d} = (d_1, d_2, \ldots, d_m)$ can be considered, and a damage functional $g(\mathbf{d}, \theta)$ can be defined for expressing the state of a component or structure: an inequality of the form

$$g(\mathbf{d}, \theta) \le 0 \tag{10}$$

can characterize either the loss of service capacity or just the failure ; θ is a vector of relevant structural and load parameters. If it is accepted that some of the components of $\mathbf{d}$ and θ are random variates, Eq (10) represents a random event. With specific probabilistic distributions assumed for these random parameters distributions (possibly characterized only by their expectations / medians and standard deviations), the ultimate damage state, involving the collapse of the structure, can be expressed as a conditional probability

$$P[g(\mathbf{d}, \theta) \le 0 \,|\, Q = q] \tag{11}$$

where Q is an earthquake intensity (or level) parameter, like the PGA. The components of the parameter vector θ are estimated from the design information and/or statistical evidence. If the model is applied to a structural component, the event in Eqs (10) and (11) represents the attainment or exceedance of the limit state by that component. It is clear that such a model is based on <u>two</u> possible states only : *failure* | *non-failure*, repectively *exceedance* | *non-exceedance of the limit state*. In the evaluation of the damage state of a structure or component, several damage levels are used. They are expressed in terms of a synthetic damage functional D that can take values in an interval $[0, \delta^*]$. This interval is divided in several subintervals, each of them corresponding to a damage severity class (or category). An example of a scale for qualitative degrees of damage is

$$[\text{ no damage } | \text{ minor damages } | \text{ moderate damages } | \text{ severe damages } | \text{ collapse }] . \tag{12}$$

Such a scale was considered in Park & Ang [10], and also used in Singhal and Kiremidjian [11]. In this latter reference, the interval for the damage index is normalized to $[0,1]$, and the subintervals corresponding to the five damage severity classes in list (12) are (respectively): $[0,0.1)$, $[0.1,0.2)$, $[0.2,0.5)$, $[0.5,1)$, ≥ 1. It follows that a damage functional would have to be analytically expressed as a function of the damage state vector $\mathbf{d}$, that is

$$D = \varphi(\mathbf{d}) \tag{13}$$

where $\varphi : \Delta \to [0,1]$ is a function from the space of (the possible values of) the damage parameters in the vector $\mathbf{d}$, $\mathbf{u} = (a_1, \ldots, a_i, \ldots, a_m)$ to the unit interval. As regards the function g that occurs in Eqs. (10) & (11), it can be stated a connection between it and φ of Eq. (13), for instance $\varphi(\mathbf{d}) = 1 - cg(\mathbf{d}, \theta)$, where $c > 0$ is a normalizing constant. The structural / loading parameters in θ are implicitly included in φ but is also possible for them to appear explicitly in its functional expression. This formulation is quite general, so far. Let us now see a couple of examples of damage indicators or functionals considered in some references dealing with seismic damage modeling. The cinematic ductility is defined in Cosenza and Manfredi [12], for a SDOF structure with EPP (elastic – perfectly plastic) behavior, by

$$d = \mu_s = \frac{x_{\max}}{x_y} \tag{14}$$

where $x_{\max}$ is the maximum plastic excursion and x_y is the yielding displacement. The value of this damage parameter is equal to 1 at yielding (i.e., it is denoted $d_y = 1$) while the value at collapse is d_u = the maximum allowable value of ductility $\mu_{u,\mathrm{mon}} = x_{u,\mathrm{mon}} / x_y$ where $x_{u,\mathrm{mon}}$ is the maximum displacement determined by monotonic tests. If $x_{\max}$ is taken in absolute value then Eq. (14) gives the cyclic ductility. An analytic form for a normalized damage functional is proposed in Cosenza and Manfredi [12] by

$$D = \begin{cases} 0 & \text{for } d \leq d_y , \\ \left(\dfrac{d - d_y}{d_u - d_y} \right)^{\alpha} & \text{for } d_y < d \leq d_u , \alpha > 0 . \end{cases} \tag{15}$$

It is easy to see that this damage functional takes values in the interval [0,1], and it effectively depends on a single damage parameter d given by Eq. (14), but also on the parameters d_y, d_u and α. Hence, this simple example shows that a more realistic formulation for the function φ would need to extend its argument(s) so as to include some parameters : $D = \varphi(\mathbf{d}, \theta)$ with $\upsilon = (a_y, a_u, \alpha)$. The first two of them are the characteristic values of d at yielding / collapse while α is a parameter involved in the analytic form of function φ. The domain of φ is $\Delta = [0, d_u]$. The corresponding form of the normalized damage functional in terms of kinematic or cyclic ductility is $D_{\mu} = (\mu - 1) / (\mu_{\mathrm{u,mon}} - 1)$. The P-A damage functional of Park and Ang [10] is defined as a linear combination of the maximum displacement and the plastic dissipated energy ; its expression is given by

$$D_{\mathrm{PA}} = \frac{x_{\max}}{x_{\mathrm{u,mon}}} + \beta \frac{E_h}{F_y x_{\mathrm{u,mon}}} = \frac{\mu_s + \beta(\mu_{\mathrm{e}} - 1)}{\mu_{\mathrm{u,mon}}} . \tag{16}$$

The physical interpretation of the Park-Ang damage functional is based on the assumption that, under plastic dissipation of energy, the collapse does not occur when the the kinematic / cyclic ductility reaches the ultimate value of the monotonic test $\mu_{\mathrm{u,mon}}$ but the fraction β of the hysteretic ductility μ_{e} must be added to the former ductility. This factor β may be considered as a free deterioration parameter that characterizes the structural elements. Since $\mu_{\mathrm{u,mon}}$ is experimentally determined for a certain structure or component and it does not depend on the maximum EPP displacement $x_{\max}$ we may consider it as a structural parameter rather than a damage indicator ; if we denote it by η the analytic expression of the function φ corresponding to the Park-Ang damage functional of Eq. (16) is

$$D_{\mathrm{PA}} = \varphi(d_1, d_2; \beta, \eta) = \frac{d_1 + \beta d_2 / 2}{\eta}. \tag{17}$$

The damage parameters d_1 and d_2 have been previously defined. Several damage functionals, including that of Eq. (16), are discussed and compared in Cosenza and Manfredi [12].

A rather recent proposal for more general damage functionals has been formulated in [13] by Y. Bozorgnia and V.V. Bertero. They are called damage spectra and they are provide continuous generalizations of previous damage indices, which can be obtained for the maximum / minimum values of the coefficients involved in the respective convex linear combinations. The expressions of these improved damage indices are

$$DI_1 = (1 - \alpha_1) NPD + \alpha_1 NHE, \tag{18}$$

$$DI_2 = (1 - \alpha_2) NPD + \alpha_2 NHV, \tag{19}$$

where :

$$NPD = (\mu - \mu_{\mathrm{e}}) / (\mu_{\mathrm{mon}} - 1) = \text{ the Normalized Plastic Deformation,} \tag{20}$$

$$NHE = E_{\mathrm{H}} / E_{\mathrm{mon}} = \text{ the Normalized Hysteretic Energy,} \tag{21}$$

$$NHV = [E_{\mathrm{H}} / E_{\mathrm{mon}}]^{1/2} = \text{ the Normalized Hysteretic Velocity.} \tag{22}$$

Expressions in Eqs. (18) & (19), with $\alpha_1, \alpha_2 \in [0,1]$, allow the two damage indices to reduced to “classical” ones. For instance $\alpha_1 = \alpha_2 = 0$ reduce the two DI’s in Eqs. (18) & (19) to special forms of damage indices based on displacement ductility only, while $\alpha_1 = 1$ will result in the normalized hysteretic energy as a damage index. These improved indices take values between 0 and 1 (what is not the case for D_{PA}).

Fragility and vulnerability models in seismic damage analysis

It should be here raised the problem of the nature of the selected damage parameters from the probabilistic point of view. If at least one of them has a random nature then the damage functional itself becomes random. Clearly, all of the damage parameters may be random with specific distributional assumptions accepted for modeling their random behavior. Let us denote by F_i the *cdf* (cumulative distribution function) of the random parameter d_i . Then the *cdf* of the damage functional D will be defined by

$$F_D(\delta) = P(D < \delta) = P(\varphi(d_1, \ldots, d_i, \ldots, d_m) < \delta). \tag{23}$$

The evaluation of the probability in Eq. (23) will clearly depend not only on the distributions assumed for the random components of **d** (i.e., for the arguments of φ) but also on the analytic nature of this function. This problem is examined (in more detail) in the next section.

We have already given an informal characterization of the notion of fragility. In the classical fragility models, the event taken into consideration is the failure of a component or a structure as the effect of a (strong) ground motion. But just the term of failure may get various meanings. If the damage state of a component / structure is the object of the analysis or prediction, the event of “failure” should be replaced by “reaching a certain damage state”. Eq (11) could give a rather general analytical formulation of a fragility model. Correspondingly, the conditional event $[g(\mathbf{d}, \theta) \le 0 \,|\, Q = q]$ should be replaced by an event of the form

$$[\delta_i \le \varphi(\mathbf{d}, \theta) < \delta_{i+1} \,|\, q_j \le Q < q_{j+1}] \tag{24}$$

where δ_i's are the limits of the subintervals corresponding to the damage levels in List (12), for instance ; q_j's are the limits of the subintervals that identify classes (or levels) of the ground motion severity (MMI / MSK intensity, Arias intensity, magnitude or others). The probability of the event in Eq. (24) would give the fragility of the component or structure under study in terms of the damage level i induced by an earthquake of severity j . The resulting expression is

$$P[\delta_i \le \varphi(\mathbf{d},\theta) < \delta_{i+1} | q_j \le Q < q_{j+1}] = F_\varphi(\delta_{i+1} | \overline{q}_j) - F_\varphi(\delta_i + 0 | \overline{q}_j), \tag{25}$$

where F_φ is the *cdf* of $\varphi(\mathbf{d},\theta)$ that occurs in Eqs. (24) & (25), and $\overline{q}_j$ is a central value in the interval $[q_j, q_{j+1}]$; $\delta_i + 0$ shows that the limit at right of δ_i is taken for the composite *cdf* F_φ. The problem of determining an analytical expression for the *cdf* F_φ may be a not very simple question. It essentially depends of the nature of the function φ and also on the *cdf* 's of the random components of $\mathbf{d} = (d_1, d_2, \ldots, d_m)$.

In most of the typical fragility models the seismic input parameter is the PGA capacity A, that is the peak ground acceleration corresponding to the failure of the system. In the general case, the random variable M is represented as

$$M = \widetilde{M}\, \varepsilon_R\, \varepsilon_U \quad \text{with} \tag{26}$$

$$\text{Med}[M] = \widetilde{M}, \ \text{Med}[\varepsilon_R] = \text{Med}[\varepsilon_U] = 1, \ \sqrt{\text{Var}[\varepsilon_R]} = \beta_R, \ \sqrt{\text{Var}[\varepsilon_U]} = \beta_U. \tag{27}$$

These three parameters in Eqs. (27) are sufficient for expressing the seismic fragility of the structure in the double log-normal format by the *cdf* defining the failure probability conditional on the median seismic capacity C, and by the *pdf* that accounts for the random variability of C around its median :

$$P_f(m) = \Phi\left[\frac{\ln(m/C)}{\beta_R}\right] \quad \text{with} \quad f_C(c) = \frac{1}{\sqrt{2\pi}\,\beta_U\, c} \exp\left[-\frac{1}{2}\left(\frac{\ln(c/\widetilde{C})}{\beta_U}\right)^2\right] \tag{28}$$

where Φ is the standard normal *cdf*, while the variabilities due to randomness (of the response) and to uncertainty (in the theoretical model for C) are accounted for by the standard deviations in Eqs. (27) & (28). Fragility curves for a global damage index, defined as a weighted sum of element damage indices, were derived for RC structures of lower / higher rise by Singhal et al. [11]. The damage functional there used was an equivalent form of Park & Ang damage index (similar to that of Eq. (11)). Its analytic expression is

$$D = \frac{\theta_m}{\theta_u} + \frac{\beta}{M_y\,\theta_u} \int dE \tag{29}$$

Using the general form of a damage functional, we may write it as $D = \varphi\ (dE\ ;\ \theta_m, \theta_u, \beta, M_y)$ where θ_m = the maximum positive or negative plasting hinge rotation, θ_u = ultimate hinge rotation capacity under monotonic loading, β is a model parameter (evaluated to be = 0.15), Q_y = the calculated yield strength, and $d\,E$ = the incremental dissipated hysteretic energy. A set of four typical fragility curves, obtained by fitting LogN distribution functions to the simulation results are shown in Figure 1 below.

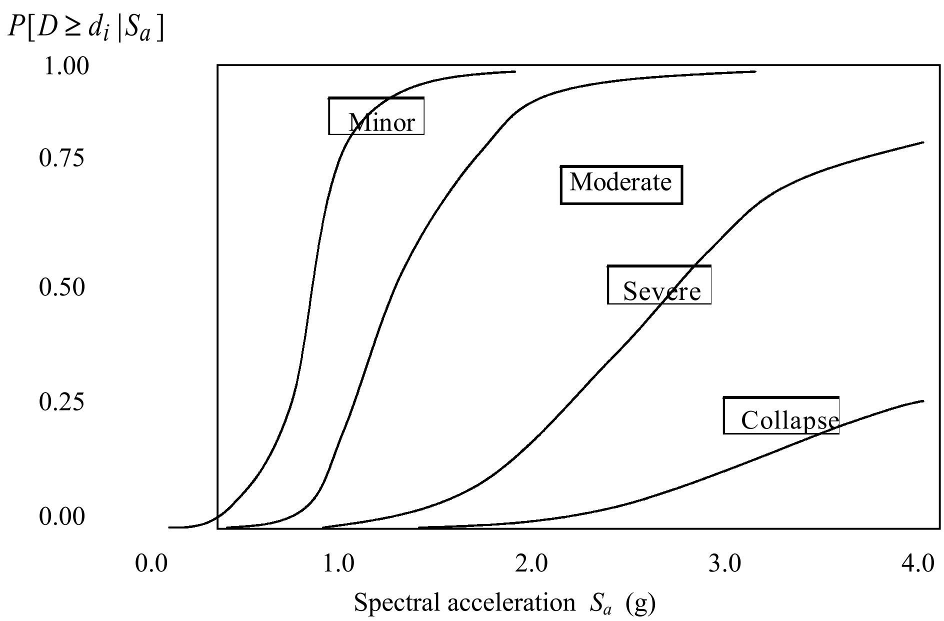

Fig. 1 Fragility curves for a mid-rise RC structure

The *seismic vulnerability* of structures is a term used in various acceptions. There exists a similarity between the notion of seismic fragility and seismic vulnerability of components and structures. However, a comparison or a connection cannot be discussed before stating what we actually mean by vulnerability. An expression of a damage index similar to that of Eq (5) is mentioned as an example. Both fragility and vulnerability give probabilistic measures of the likelihood of a possible change of state of a structure as a consequence of a forthcoming major earthquake. In the case of the "classical" fragility, the change is from non-failure to failure ; in the fragility models for seismically induced damages, the change is from a lower degree of damage to a higher damage level. As we mentioned, in almost all of fragility models, the lognormal distributions are accepted (see Eqs. (26-28)). Such an assumption is not specific to the vulnerability models. Instead, the analysis of damage is essentially involved in the studies of vulnerabilty.

Theoretical and applicative approaches to the notion of seismic vulnerability have been the object of many research programs developed at the INCERC (Building Research Institute) of Bucharest, and led by H. Sandi. References [3,14] are only a couple of recent papers among the many reported results due to this group. The "classical" (or reference) vulnerability models are those in which the structure under analysis is assumed to be in a "no damage" state beforre a possible seismic event ; in the "generalized" models, it is admitted that the structure has undergone certain pre-event damages. In both types of models, the following items have to be identified, quantified or calibrated : **1º** the category or class of structures under study, denoted by $S(i)$; **2º** the severity of the seismic action, quantified by a parameter q or m ; **3º** the severity of damage, both in the pre-event or post-event case, quantified by a damage measure / index d ; **4º** the probabilistic distribution of the damage D. The subscript j identifies the earthquake severity class, while the subscript k corresponds to one of the five damage states : $k = 0$ for *no damage*, $k = 1$ for *minor* damages, $k = 2$ for *moderate* damages, $k = 3$ for *severe* damages, and $k = 4$ for *collapse*. In order to make difference between pre-event and post-event damages, this subscript k will appear as k' in the former case and k'' in the latter one.

The probabilistic (seismically induced) damage distribution is expressed in terms of either a *damage probability matrix* (DPM) or of a *conditional probability density function* (*cpdf*). The entries of a DPM in the classical / generalized vulnerability models are probabilities as those presented in Table 1 that follows ; the corresponding forms for the two types of *cpdf*'s are also included.

Table 1. Damage probabilities and vulnerability distributions
in the classical and generalized models

Discrete / continuous parameters q , d	Classical Vulnerability	Generalized Vulnerability
1) Continuous d & q	$f^{(v)}(d\|q)$	$f^{(v)}(d''\|q,d')$
2) Discrete d & q	$p^{(v)}_{k\|j}$	$p^{(v)}_{k''\|j,k'}$
3) Discrete d, continuous q	$p^{(v)}_{k}(q)$	$p^{(v)}_{k''\|k'}(q)$
4) Continuous d, discrete q	$f^{(v)}(d\|j)$	$f^{(v)}(d''\|j,d')$

Several logical-probabilistic conditions on the probabilities that occur in the cases 2-G) and 3-G) are stated in Sandi [3]. We recall only a couple of them :

$$\sum_{k''} p^{(v)}_{k''|j,k'} = 1 \quad \text{for any} \quad j, k' \; ; \tag{30}$$

$$p^{(v)}_{k''|k'}(q) = 0 \qquad \text{for} \quad k'' < k' \; ; \tag{31}$$

$$\bar{d}^{(v)}_{k''|j_2,k'} \ge \bar{d}^{(v)}_{k''|j_1,k'} \quad \text{for} \quad j_2 > j_1 \quad \text{and} \quad \bar{d}^{(v)}_{k''|j,k'_2} \ge \bar{d}^{(v)}_{k''|j,k'_1} \quad \text{for} \quad k'_2 > k'_1 , \tag{32}$$

where $\bar{d}_{j,k'} = \sum_{k''} k'' \, p^{(v)}_{k''|j,k'}$ is a conditional *expected* damage severity.

Let us also mention that the fourth line in Table 1 has been introduced by us. Case 4-C) would correspond to the fragility models, while 4-G) to the updated fragilities.

Using models based on operator theory for updated / translated vulnerability functionals

As we have earlier seen, two problems connected with modified vulnerability functions are: (*i*) quantification of generalized vulnerability measures, taking into account the possibility of damage occurrences induced by previous earth-quake events; and (*ii*) translation of earthquake vulnerability functions from a region to another.

Regarding problem (*i*), it is relevant for specified classes of structures in a given region, whose behaviour (e.g. strength) properties might change after a series of earthquake motions occurred after the last evaluation of the seismic capacity of the structures. Thus, the conditional probabilities that occur in Table 1 and Eq. (30) could be obtained using a model based on operators (under R.T. Durate's approach). Concerning problem (*ii*), we have already cited Ref. [8] where a methodology is presented for shifting a reference curve describing the MDR (mean damage ratio) vs. the MMI (or other type of seismic) intensity. This translation is composed with a rotation of the curve arround the point where the structural response becomes inelastic. Both these transformations can be mathematically formulated in terms of the models presented in the second section (after Introduction). Adequate expressions have to be found for the operator S, and the "simplified" model will now be the updated model for (*i*) , respectively the target model for (*ii*).

The vulnerability functionals can be obtained as follows. It is assumed that the earthquake action has s components and a finite duration T_1 while the duration of interest T_2 for the structural response is assumed possibily larger : $T_2 > T_1$. The expected earthquake actions $\mathbf{a}$ are supposed to belong to the space L_s of vector-valued functions being absolutely Lebesgue-integrable over T_2 . The response of the structure is also characterized in terms of a vector function $\mathbf{r}$ with r components, belonging to a space L_r. If E denotes an operator from the range space of all the acceleration time histories $\mathrm{F}(\mathbf{h})$ with peak value $\mathbf{h}$ and c is a control variable, then a general *vulnerability functional* is defined by

$$\mathbf{V}(\mathbf{h}) = \mathrm{c}\,[\mathrm{E}\,(\mathrm{F}\,(\mathbf{h})]\,. \tag{33}$$

In general, F ($\mathbf{h}$) has to include a large number of time histories $\mathbf{x}$ (t), but a single action is taken as a representative of the whole class. In most cases, it is only needed to know some functionals of the response like, e.g., the mean peak value:

$$MPV(\mathbf{r}) = \int_{\mathsf{R}_r} \left\| \mathbf{r}\,(t_2) \right\|^{\infty} d\mu_r = \int_{\mathsf{A}_s} \left\| \mathsf{E}(\mathbf{a}(t_1)) \right\|^{\infty} d\mu_a \tag{34}$$

where R_r & A_s are the response and action spaces in the stochatsic model (respectively), while μ_r & μ_s are corresponding probability measures.

CONCLUSIONS

Certain problems met in the seismic vulnerability studies can be approached in terms of a rather general theory that models the structural behaviour under seismic actions. We have discussed several ways to define and analyse seismically induced damages and proposed the use of models based on operators for obtaining modified vulnerability functionals.

REFERENCES

1. Kennedy, R.P., Cornell, C.A., Campbell, R.D., Kaplan, S. and Perla, H.F., Probabilistic Seismic Safety Study of an Existing Nuclear Power Plant. In: Jour. Nuclear Engineering and Design, Vol. 59 (1980), pp. 315-338.
2. Shibata, A., Prediction of Earthquake Damage to Building Groups for Urban Disaster Estimation. Proc. 11th European Conference on Earthquake Engineering, Paris, France, September 1998, A.A. Balkema, Rotterdam-Brookfield (on CD-ROM).
3. Sandi, H., A Format for Vulnerability Characteristics of Damaged Structures, Proc. 11th European Conf. on Earthquake Engineering, Paris, France, September 1998, T7, A.A. Balkema, Rotterdam-Brookfield (on CD-ROM).
4. Robu, C., Vulpe, A. and Carausu, A., Seismic Vulnerability and Fragility Formats in Damage Risk Assessment of RC Structures. Transactions SMiRT 16, Washington DC, August 2001, Division M (edited on CD-ROM).
5. Duarte, R.T., The Mathematical Foundations of Structural Safety Assessment for Earthquake Hazards. Proc. 9th European Conference on Earthquake Engineering, Moscow, 1990, Vol. 1, pp. 152-161.
6. Duarte, R.T., Optimal Values for the Earthquake Reliability of Building Structures. Proc. 9th European Conference on Earthquake Engineering, Moscow, 1990, Vol. 2, pp. 13-22.
7. Muncaster, R.G., Invariant Manifolds in Mechanics I: The General Construction of Coarse Theories from Fine Theories. Archive for Rational Mechanics and Analysis, Springer Verlag, Berlin, Vol. 84, pp. 353-373.
8. Huo, J.-R., Lawson, R.S. and Kishi, N.G., Translation of Earthquake Vulnerability Functions. Proc. 11th European Conf. on Earthquake Engineering, Paris, 1998, T7, A.A. Balkema, Rotterdam, 1998, pp. 1-10.

9. Vulpe, A., Carausu, A., and Vulpe G.E., Bayesian Estimation of Seismic Vulnerability for Damaged Structures. Proc. 12th European Conf. on Earthquake Engineering, London, September 9-13, 2002, Paper 711, Elsevier Sci. Publishers Ltd., 2002, pp. 1-10.
10. Park, I.J., and Ang, A.H.-S., Mechanistic Seismic Damage Model for Reinforced Concrete. ASCE Jour. of Engineering, 111 (1985), pp. 722/739.
11. Singhal, A. and Kiremidjian, A.S., Bayesian Updating of Fragilities with Application to RC Frames. J. of Structural Engineering, 124 (1998), pp. 922-929.
12. Cosenza, E. and Manfredi, G., Seismic Analysis of Degrading Models by Means of Damage Functions Concept. Proc. Workshop on Nonlinear Seismic Analysis of RC Buildings, Bled, Slovenija, July 1992.
13. Bozorgnia, Y. and Bertero, V.V., Estimation of Damage Potential of Recorded Earthquake Ground Motion Using Structural Damage Indices. Proc. 12th European Conf. on Earthquake Engineering, London, September 9-13, 2002, Paper 475, Elsevier Sci. Publishers Ltd., 2002, pp. 1-10.
14. Sandi, H. and Vacareanu, R., Parametric Analysis of the Cumulative Damage. Proc. 11th European Conf. on Earthquake Engineering, Paris, 1998, T7, A.A. Balkema, Rotterdam, 1998, pp. 1-10.