*Transactions of the 17th International Conference on Structural Mechanics in Reactor Technology (*SMiRT 17)
Prague, Czech Republic, August 17 –22, 2003

Probabilistic assessment of fatigue life including statistical uncertainties in the *S*-*N* curve

Bruno Sudret[1)], Zakoua Guédé[2)], Patrick Hornet[1)], Jean-Michel Stéphan[1)] , Maurice Lemaire[2)]

[1)] Electricité de France – R&D Division – Site des Renardières – 77818 Moret-sur-Loing Cedex, FRANCE
[2)] Institut Français de Mécanique Avancée – LaRAMA, Campus des Cézeaux, BP265, F-63175 Aubière, FRANCE

ABSTRACT

A probabilistic framework is set up to assess the fatigue life of components of nuclear power plants. It intends to incorporate all kinds of uncertainties such as those appearing in the specimen fatigue life, design sub-factor, mechanical model and applied loading. This paper details the first step, which corresponds to the statistical treatment of the fatigue specimen test data. The specimen fatigue life at stress amplitude *S* is represented by a lognormal random variable whose mean and standard deviation depend on *S*. This characterization is then used to compute the *random fatigue life* of a component submitted to a single kind of cycles. Precisely the mean and coefficient of variation of this quantity are studied, as well as the reliability associated with the (deterministic) design value.

KEY WORDS: fatigue life / design curve / probabilistic assessment / statistical treatment / austenitic stainless steel

INTRODUCTION

A number of components in nuclear power plants are submitted to thermal and mechanical loading cycles. The assessment of these components with respect to low cycle and high cycle fatigue is of great importance for the global safety, efficiency and availability of the power plant. The usual design practice for fatigue life prediction of components has been developed in the ASME code in the 1960s. The original reference [1] states that :

The design fatigue curves are based on strain-controlled fatigue tests of small polished specimens. A best-fit to experimental data was obtained by applying the method of least squares to the logarithms of the experimental values. The design stress values were obtained from the best-fit curves by applying a factor of 2 on stress or a factor of 20 on cycles, whichever was more conservative at each point. These factors were intended to cover such effects as environment, size effect and scatter of data.

Different decompositions of the margin factor of 20 have been proposed later on in the literature, as summarized in Refs [2,3]:

- The scatter of experimental data , *i.e.* the fact that the same test carried out with identical experimental conditions on several identical specimens does not provide exactly the same fatigue life *N*, is usually associated with a sub-factor 2 to 2.4.
- The sub-factor for size effect which accounts for the difference between fatigue life expected in full-scale components as compared to laboratory test specimens is 1.4 to 2.5.
- The sub-factor for surface finish is 2 to 3.
- The sub-factor for environmental effects (dissolved oxygen in the water environment, temperature, etc.) is set equal to 3.

The factor of 2 on stress also receive various interpretations (*e.g.* 1.2 for data scatter and 1.66 for the other effects). However none of these sub-factor values seem to be accepted by the whole community. Note that other methods for obtaining design curves from the specimen data have been investigated [4].

The aim of the present paper is to introduce a probabilistic framework in fatigue analysis that will allow to deal rigorously with all kinds of uncertainties appearing in fatigue life assessment, including the ones described above as well as those related to the mechanical model and/or the applied loading.

A new statistical treatment of the laboratory data is proposed to characterize the fatigue life *N* as a random variable depending on the stress amplitude *S* (conventionally defined as the product of the material Young’s modulus by half of the strain range in strain-driven tests). A probabilistic *component fatigue life* is defined and its properties analysed. The probabilistic information is also used to better understand the meaning of the classical (deterministic) design procedure and give a “reliability-based” interpretation of it.

DETERMINISTIC FRAMEWORK FOR FATIGUE ASSESSMENT

The assessment of structural components such as pipes against fatigue requires the following inputs:

- **Description of the loading applied to the component**: it can be thermal loading (evolution in time of the temperature at the pipe inner wall), mechanical loading (evolution in time of the bending moments and normal forces in the pipe line) or a mixture of both.
- **Mechanical model**: it allows to compute the evolution in time of the stresses in each point of the component from the prescribed geometry, material properties and loading. Analytical or numerical (*e.g.* finite element) models can be used. An equivalent stress $\sigma_{eq}(t)$ is then obtained using for instance the Tresca criterion [5].
- **Extraction and counting of the cycles**: from the computed evolution $\sigma_{eq}(t)$, stress cycles are extracted using the Rainflow method [6]. A sequence of stress amplitudes S_i , $i = 1, \cdots N$ can be determined on a time interval $\left[0, T\right]$(each stress amplitude S_i corresponds to half of the difference between the consecutive peaks obtained by the Rainflow method).
- **Choice of a design curve**: as described in the introduction, the design curve $N_d(S)$ is obtained from experimental results that provide the best-fit specimen curve $\overline{N}(S)$, and from margin factors (20 on number of cycles, 2 on stress, whichever is more conservative).
- **Computation of the usage factor**: The Miner's rule is used in order to define the fatigue damage. It postulates that the elementary damage d_i associated with one cycle of amplitude S_i is computed by $d_i = \dfrac{1}{N_d(S_i)}$ (where $N_d(S)$ is the design curve) and that the total damage is obtained by summation.

  This leads to compute the usage factor D:

  $$D = \sum_{i=1}^{N} d_i = \sum_{i=1}^{N} \frac{1}{N_d(S_i)} \tag{1}$$

  This usage factor is then compared to 1: if it is smaller than 1, the component is supposed to be safely designed. If it is greater than 1, cracks may appear on the component. Moreover, if D is the usage factor associated with a sequence of cycles which is repeated periodically, the fatigue life T_d of the component may be computed as $T_d = 1/D$ and is interpreted as the number of (periodic) sequences of cycles that the component may undergo before crack initiation.

PROBABILISTIC FRAMEWORK FOR FATIGUE ASSESSMENT

The aim of this paper is to present a probabilistic framework for fatigue assessment that closely follows the classical deterministic approach that has been summarized in the above section. Different kinds of uncertainties will be introduced in a probabilistic way. The end result is the probabilistic definition of the *component fatigue life* $T(\omega)$ which is now a random variable. This result is cast as the probability density function (PDF) of $T(\omega)$, which can be post-processed to get the mean and standard deviation. Moreover, the (deterministic) design fatigue life of the component, denoted by T_d can be positioned onto this PDF, *i.e.* the reliability of the design may be evaluated.

The following types of uncertainties may be taken into account in the analysis :

- **Scatter of the specimen test data**: the specimen fatigue life of a given material will be characterized as a random variable depending on the stress level. It will be denoted by $N\left(S, \omega\right)$, where ω denotes the random nature of the specimen fatigue life at stress amplitude S.
- **Uncertainties in the margin sub-factors** associated with the size effect, surface finish and environmental conditions. These sub-factors may be considered as random variables, whose properties can be taken from the expert judgements reported in the literature. Typically uniform probability density functions over a given interval can be considered, *e.g.* sub-factor for surface polish is $\mathbf{U}[1.8 ; 2.2]$. In this paper, these sub-factors will be given deterministic (fixed) values.
- **Uncertainties in the mechanical model**: the geometrical parameters (*e.g.* inner and outer radius of the pipe) as well as the material properties (Young's modulus, Poisson's ratio, thermal dilatation coefficient, etc.) may be modelled as random variables with appropriate probability density function.

- **Loading**: in case of thermal fatigue, the temperature of the fluid at the inner wall of the pipe may be described as a random process. A "random" counterpart of the Rainflow method for counting the peaks may be introduced.

Once all these random quantities have been characterized, the random fatigue life may be evaluated using probabilistic methods such as Monte Carlo simulation.

The work presented in this paper is now focused on dealing with the first kind of uncertainties, *i.e.* the statistical treatment of data for a probabilistic characterization of fatigue life.

STATISTICAL TREATMENT OF FATIGUE TEST DATA

Experimental data base and principles of the probabilistic characterization of specimen fatigue life

The statistical treatment of fatigue data has been investigated since the 60's and normalized in French Standard A 03-405 [7]. This standard proposes several recommendations. The present work is related to §7 of this document, which is entitled "Statistical treatment of linear S-N or ε-N fatigue curves". However the fitting curves used in the present paper are different from those proposed in the standard in order to be applicable from low-cycle to high-cycle domain as described below. Note that the ESOPE method presented in §8 of the standard could not be applied since it provides equi-probability *S-N* curves but no closed-form expression for the probability density function of the number-of-cycles-to-failure at each stress amplitude.

The experimental data used to develop the statistical method described in the sequel is based on a private database consisting of 304 and 316 austenitic stainless steel specimen. Only those tests with alternating stress (*e.g.* mean value of the cycle equal to zero) are considered. Tests which were not conducted until failure have been discarded in this first approach but should be taken into account in later work.

A set of 325 experimental points has been retained. These points are classified according to the temperature of the test (see Table 1). Since the range of temperature is large, it is likely that various mechanisms of crack initiation may exist, leading to different behaviours in terms of fatigue life. Thus no comparison between the obtained curves in each temperature domain will be done in the sequel.

Remark : It is to be noted that the database has *not* been carefully checked with respect to the testing conditions. It is likely that the data used is rather heterogeneous. Thus the obtained curves should not be taken as is for a real application to fatigue life design of a given component. Only the methodology of statistical treatment is of importance.

Table 1 : Experimental points used in the study

Temperature	Number of points	Mean Young's modulus (MPa)
Room temperature	99	183,547
$T \in [400°C - 550°C]$	70	177,252
$T \in [550°C - 650°C]$	156	161,337

The number-of-cycles-to-failure $N(S,\omega)$ at each strain level is supposed to be a *lognormally* distributed random variable. This may be written as :

$$\ln N(S,\omega) = \lambda(S) + \varepsilon(S,\omega) \quad (2)$$

where $\lambda(S)$ is the mean value of the logarithm of N at stress level S and $\varepsilon(S,\omega)$ are zero-mean Gaussian random variables. It is further assumed that these variables (representing the scatter of $\ln N$ around its mean-value) are *perfectly* correlated. This assumption means that a given sample (which is a "realization" of the material in the probabilistic vocabulary) is good or bad with respect to fatigue life *whatever* the stress level applied. This assumption allows to simplify Eq.(2) into:

$$\ln N(S,\omega) = \lambda(S) + \sigma(S)\, \xi(\omega) \quad (3)$$

where $\xi(\omega)$ is a standard normal random variable. Then the mean $\lambda(S)$ and standard deviation $\sigma(S)$ of $\ln N$ have to be given a particular functional form of *S*. The parameters of these functionals are then determined by regression.

Model for the mean curve of specimen fatigue life $\lambda(S)$

The original curve used in the ASME code was derived after the work by Coffin [8] and Langer [9] in the following form:

$$S = \frac{E}{4\sqrt{N}} \ln \frac{100}{100 - RA} + S_D \tag{4}$$

where E is the Young's modulus, S is the product of E by the strain amplitude, N is the number of cycles to failure, RA is the reduction of area in tensile test (%) and S_D is the endurance limit. This form has been taken up by many authors [7,10] in the literature and in standardizing codes [1,5] in a way such that the number of cycles to failure is proportional to a power of $(S - S_D)$. In this paper the following model is postulated:

$$\lambda(S) = A \ln(S - S_D) + B \tag{5}$$

where parameters A, B and S_D are to be determined.

Model for the standard deviation $\sigma(S)$ and solution to the regression problem

Three different hypotheses have been chosen to represent the standard deviation of the log-number of cycles to failure, *i.e.* the scatter of the data:

- **H1** : the standard deviation is supposed to be *constant* whatever the stress amplitude S (homoscedastic assumption). The values of parameters A, B, S_D are determined using the mean-square minimization method. Let Q denote the number of experimental points, *i.e.* couples $(\ln N_j, S_j)$, $j = 1, \cdots Q$. The sum $\mathbf{\Sigma}$ of the square distances between the observations and the predictions is minimized with respect to A, B, S_D :

$$\mathbf{\Sigma} = \sum_{j=1}^{Q} \left[\ln N_j - A \ln(S_j - S_D) - B \right]^2 \tag{6}$$

  Then the (constant) variance of the sample set is obtained by:

$$\sigma^2 = \frac{1}{Q-3} \sum_{j=1}^{Q} \left[\ln N_j - A \ln(S_j - S_D) - B \right]^2 \tag{7}$$

- **H2** : the standard deviation is supposed to depend on S (heteroscedastic assumption) in a similar manner as in Eq.(5) :

$$\sigma(S) = A' \ln(S - S_D) + B' \tag{8}$$

  In this equation, S_D is the asymptotic limit of $\lambda(S)$ assuming a constant standard deviation of the sample set, *i.e.* computed by minimizing Eq.(6). Parameters A', B' are determined as follows. At each stress level S_k, the empirical standard deviation $\sigma(S_k)$ of the Q_k experimental points corresponding to this stress level are computed. Then the curve in Eq.(8) is fitted with respect to A', B'. In practice, as there are not enough points available at each stress level S_k, the experimental points are grouped into classes and the empirical standard deviation is computed within each class.

- **H3** : the standard deviation $\sigma(S)$ is supposed to be proportional to the mean value of $\ln N$ (*i.e.* $\lambda(S)$), *i.e.* the coefficient of variation δ of $\ln N(S, \omega)$ is *constant*.

$$\sigma(S) = \delta \cdot \lambda(S) \tag{9}$$

  In this case parameters A, B, S_D and δ are determined in a single shot using the method of maximum likelihood. Assuming that the experimental points $\ln N_i$ follow a Gaussian distribution with mean value and standard deviation given in Eqs.(5),(9) respectively, the likelihood of the observation reads:

$$L_Q = \prod_{j=1}^{Q} \frac{1}{\sqrt{2\pi}\sigma(S_j)} \exp\left[-\frac{1}{2} \left(\frac{\ln N_j - A\ln(S_j - S_D) - B}{\sigma(S_j)} \right)^2 \right] \tag{10}$$

  The determination of parameters A, B, S_D and δ is performed by requiring that they maximise the likelihood, or equivalently minimize the following quantity:

$$-2\ln L_Q = \sum_{j=1}^{Q} \left[\ln \sigma^2(S_j) + \left(\frac{\ln N_j - A\ln(S_j - S_D) - B}{\sigma(S_j)} \right)^2 \right] \tag{11}$$

The minimization problems described above under the various assumptions **H1**, **H2**, **H3** are solved using MathCad routines [11]. The results obtained by the three methods, that is the mean value of $\ln N$ (*i.e.* $\lambda(S)$), and standard deviation $\sigma(S)$ are compared in the sequel based on the 325 points selected from the data base.

Validation of the assumptions and statistical tests

To validate the Gaussian assumption of the sample points $\ln N_i$, the Kolmogorov test has been applied. It consists in computing the maximal discrepancy between the empirical and predicted cumulative distribution functions. The Gaussian assumption is valid with a confidence of 95 % if this discrepancy D_K is smaller than a threshold D_K^c [12]. The test is applied onto the complete sample set under assumption **H1** since the standard deviation is constant in this case. Under assumptions **H2** and **H3**, the test is applied separately onto each class of sample points (corresponding to stress level S_k) which have constant standard deviation. To measure the accuracy of the various models, an empirical correlation coefficient R^2 is also computed after each regression analysis.

Numerical results

The results obtained by the various methods for the three classes of temperature defined in Table 1 are reported in Table 2.

Table 2 : Best-fit curves obtained by the various methods

Temperature	Method	Best-fit curve $\lambda(S)$	Standard deviation $\sigma(S)$	R^2	D_K , D_K^c
$20°C$	H1	$-2.17\ \ln(S-188.39)+23.33$	0.94	0.94	0.11 ; 0.14
	H2	$-2.29\ \ln(S-185.60)+24.10$	$-0.26\ \ln(S-188.39)+2.69$	0.95	—
	H3	$-2.28\ \ln(S-185.80)+24.06$	$0.09\cdot[-2.28\ \ln(S-185.80)+24.06]$	0.95	—
$[400°C-550°C]$	H1	$-2.84\ \ln(S-121.61)+27.69$	0.84	0.91	0.11 ; 0.16
	H2	$-2.83\ \ln(S-121.04)+27.65$	$-0.77\ \ln(S-121.61)+5.88$	0.92	—
	H3	$-2.67\ \ln(S-127.66)+26.52$	$0.07\cdot[-2.67\ \ln(S-127.66)+26.52]$	0.95	—
$[550°C-650°C]$	H1	$-2.09\ \ln(S-190.81)+20.99$	0.70	0.89	0.08 ; 0.10
	H2	$-1.77\ \ln(S-198.24)+18.82$	$-0.47\ \ln(S-190.81)+3.90$	0.96	—
	H3	$-1.95\ \ln(S-195.89)+20.08$	$0.08\cdot[-1.95\ \ln(S-195.89)+20.08]$	0.90	—

The experimental points as well as the best-fit curves $\lambda(S)$ are plotted in Figure 3 in appendix. It can be seen in this figure that all approaches provide similar curves for the two first class of temperature whereas light discrepancies can be seen in the low cycle domain in the third case. It should be emphasized that the obtained endurance limit S_D is almost independent of the regression method. However it depends on the quality and homogeneity of the experimental data points, which has not been carefully checked. Generally speaking, it would depend on the maximal number of cycles considered, *i.e.* from the lowest stress level at which failure specimens have been observed. Note that **H2** and **H3** schemes lead to a standard deviation of ln N that tends to infinity when S tends to S_D. Again it is emphasized that these curves should not be used as is since the data they are based on is not related to a particular type of components.

COMPUTATION OF THE RANDOM LIFE TIME OF A COMPONENT

For the sake of simplicity, let us consider a structure which is submitted to a single (equivalent) cyclic stress S. In the low-cycle domain (large values of S), the deterministic design fatigue life is $T_d(S)=N_{bf}(S)/20$. In the high cycle domain (small values of S), the design fatigue life is $T_d(S)=N_{bf}(2S)$. The boundary between these domains is a critical stress level S_{cri} obtained by solving the equation $N_{bf}(S_{cri})/20=N_{bf}(2S_{cri})$. Its value is around 300 MPa for the data points. It is recalled that the factors (20 on N, 2 on S) take into account the material scatter as well as other effects such as surface, size and environmental effects. As a summary, the design fatigue life of the structure can be written as:

$$T_d(S) = \begin{cases} \dfrac{N_{bf}(S)}{20} & \text{if } S > S_{cri} \\ N_{bf}(2S) & \text{if } S_D/2 < S \le S_{cri} \\ \infty & \text{otherwise} \end{cases} \tag{12}$$

In the probabilistic framework, the data scatter is accounted for by the very definition of random variable $N(S,\omega)$. The other effects are gathered into a single “passage” sub-factor which gives, for a given realization of the material, the ratio between the fatigue life of a polished specimen in air to the fatigue life of a component made of the same realization of the material in its own environmental and surface finish conditions. Let us decompose the design factors as follows:

$$\begin{aligned} 20 &= \gamma^N_{scat} \cdot \gamma^N_{passage} \\ 2 &= \gamma^S_{scat} \cdot \gamma^S_{passage} \end{aligned} \tag{13}$$

According to the previous discussion, the random fatigue life of a component is defined by:

$$T(S,\omega) = \begin{cases} \dfrac{N(S,\omega)}{\gamma^N_{passage}} & \text{if } S > S_{cri} \\ N(\gamma^S_{passage} \cdot S, \omega) & \text{if } S_D/\gamma^S_{passage} < S \le S_{cri} \\ \infty & \text{otherwise} \end{cases} \tag{14}$$

which expands into:

$$T(S,\omega) = \begin{cases} \dfrac{N_{bf}(S)}{\gamma^N_{passage}} e^{\sigma(S)\,\xi(\omega)} & \text{if } S > S_{cri} \\ N_{bf}(\gamma^S_{passage} \cdot S)\, e^{\sigma(\gamma^S_{passage} \cdot S)\,\xi(\omega)} & \text{if } S_D/\gamma^S_{passage} < S \le S_{cri} \end{cases} \tag{15}$$

Note that there is a paradox for stress amplitudes S between $S_D/2$ and $S_D/\gamma^s_{passage}$: they do not yield any fatigue damage in the probabilistic framework but do so in the deterministic design. No efficient solution to this problem could be found so far. Outside this particular domain, it is possible to derive the mean value $\mu_{T(S)}$ of the component fatigue life as follows:

$$\mu_{T(S)} = \begin{cases} \dfrac{N_{bf}(S)}{\gamma^N_{passage}} e^{\sigma^2(S)/2} & \text{if } S > S_{cri} \\ N_{bf}(\gamma^S_{passage} \cdot S)\, e^{\sigma^2(\gamma^S_{passage} \cdot S)/2} & \text{if } S_D/\gamma^S_{passage} < S \le S_{cri} \end{cases} \tag{16}$$

The associated coefficient of variation $\delta_{T(S)}$ (*i.e.* the ratio between the standard deviation and the mean value of $T(S,\omega)$) reads:

$$\delta_{T(S)} = \begin{cases} \sqrt{e^{\sigma^2(S)} - 1} & \text{if } S > S_{cri} \\ \sqrt{e^{\sigma^2(\gamma^S_{passage} \cdot S)} - 1} & \text{if } S_D/\gamma^S_{passage} < S \le S_{cri} \end{cases} \tag{17}$$

As an application, the probabilistic characterization of the specimen fatigue life at room temperature under assumption **H3** (see Table 2, row #3) is used. The sub-factor values $\gamma^N_{scat} = 2$; $\gamma^S_{scat} = 1.2$ are selected (see Ref. [3]). The mean value and the coefficient of variation of the component fatigue life are plotted in Figure 1. It is observed that the mean value $\mu_{T(S)}$ is about 2-5 times the design value, which shows the conservatism of the design procedure (this ratio tends to infinity when $\gamma^S_{passage} \cdot S \to S_D$). The coefficient of variation $\delta_{T(S)}$ is exactly that of the specimen fatigue life when $S > S_{cri}$. Its value is 100-200 % which is rather large and explains why deterministic predictions of fatigue life cannot be accurate. When $S \le S_{cri}$ this coefficient is even larger and depends on the choice of $\gamma^S_{passage}$, tending to infinity when $\gamma^S_{passage} \cdot S \to S_D$.

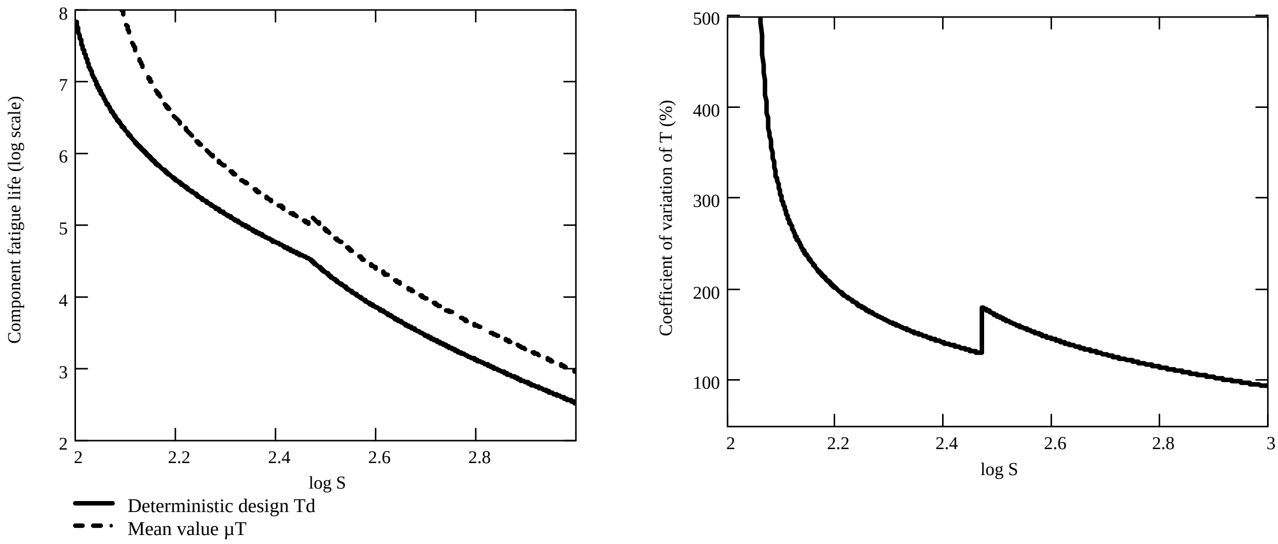


Fig.1 : Mean value and coefficient of variation of the component fatigue life $T(S,\omega)$ submitted to a single type of cycles of amplitude *S*

It is now possible to compute the reliability of the design procedure with respect to the uncertainty in the specimen fatigue life. Precisely let us denote by P_f the probability that the (random) component fatigue life $T(S,\omega)$ is smaller than the design fatigue life $T_d(S)$. Using Eqs.(12),(15) yields:

$$P_f = P\left[T(S,\omega) \le T_d(S)\right] = \left| \begin{array}{ll} \Phi\left(-\dfrac{\ln \gamma^N_{scat}}{\sigma(S)}\right) & \text{if } S > S_{cri} \\ \Phi\left(\dfrac{\lambda(2S) - \lambda(\gamma^S_{passage} \cdot S)}{\sigma(\gamma^S_{passage} \cdot S)}\right) & \text{if } S_D / \gamma^S_{passage} < S \le S_{cri} \end{array} \right. \qquad (18)$$

where Φ is the standard normal cumulative distribution function.

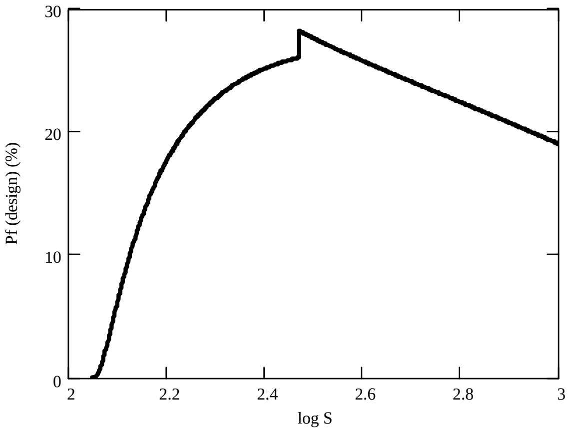


Fig.2 : Probability of failure of the design component fatigue life vs. stress amplitude *S*

This quantity is plotted in Figure 2. It appears that the reliability of the design is rather dependent of the stress amplitude *S*. P_f is about 15-28 % in the low-cycle domain (large values of *S*) and the design is all the safer since *S* is large. In the high-cycle domain (small values of *S*), the values of P_f vary within 0-25%. It appears that the design is all the safer since *S* is close to the endurance limit S_D or below. This behaviour is explained by the fact that both $\lambda(\gamma^S_{passage} \cdot S)$ and $\sigma(\gamma^S_{passage} \cdot S)$ tend to infinity when $\gamma^S_{passage} \cdot S \to S_D$, their ratio being constant.

Similar computations were carried out using the other probabilistic characterizations summarized in Table 2. The trends in these results are identical to those presented above in terms of mean and coefficient of variation of the fatigue life as well as reliability of the design, although the numerical values are slightly different.

CONCLUSION

A probabilistic framework has been set up to assess the fatigue life of components of nuclear power plants. It intends to incorporate all kinds of uncertainties such as those appearing in the specimen fatigue life, design sub-factor, mechanical model and applied loading. This paper details the first step, which is the statistical treatment of the specimen test data. The specimen fatigue life at stress amplitude *S* is represented by a lognormal random variable whose mean and standard deviation depend on *S*. The correlation between fatigue life at different level is supposed to be perfect.

This characterization is then used to compute the *random fatigue life of a component* submitted to a single kind of cycles. Precisely the mean and coefficient of variation of this quantity are studied, as well as the reliability associated with the (deterministic) design value. The non homogeneity of the reliability of the design is emphasized. Extension to a design under multiple types of cycles is trivial due to the linearity in the Miner's rule. However numerical methods such as Monte Carlo simulation should be used in this case.

The probabilistic characterization presented in this paper could also be straightforwardly used to characterize the *random usage factor*, *i.e.* its probability density function, mean and standard deviation, all quantities being computed as a function of time.

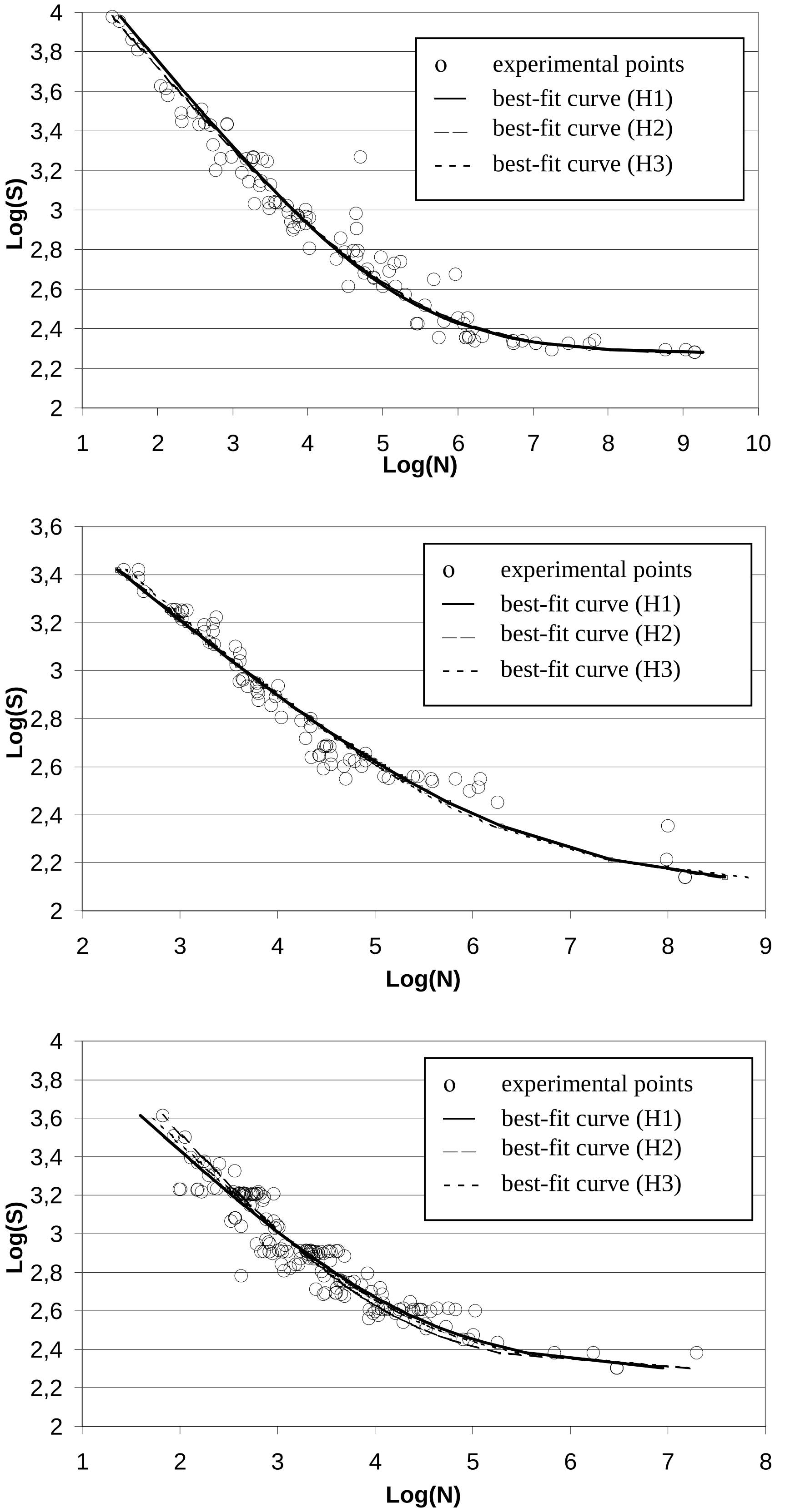
The application of the complete framework including loading requires the characterization of the latter in terms of random processes. This work is currently in progress.

NOMENCLATURE

$N(S,\omega)$	Random specimen fatigue life at stress amplitude *S* (lognormal law)
$\lambda(S)\,,\sigma(S)$	Parameters of the lognormal law of random variable $N(S,\omega)$
A, B, S_D	Parameters of the regression for $\lambda(S)$
$N_{bf}(S)$	Best-fit deterministic curve
$T(\omega)$	Random fatigue life of a component
$T(S,\omega)$	Random fatigue life of a component submitted to a single type of cycles at stress amplitude *S*
$T_d(S)$	Design fatigue life of a component submitted to a single type of cycles at stress amplitude *S*
$\mu_{T(S)}\,,\delta_{T(S)}$	Mean value and coefficient of variation of random variable $T(S,\omega)$
$\gamma^N_{scat}, \gamma^N_{passage}$	Sub-factors for data scatter and for passage from specimen to component in the low-cycle domain
$\gamma^S_{scat}, \gamma^S_{passage}$	Sub-factors for data scatter and for passage from specimen to component in the high-cycle domain

REFERENCES

1. *Criteria of the ASME Boiler and Pressure Vessel Code for Design and Analysis in Sections III and VIII, Division 2,* The American Society of Mechanical Engineers, New York, 1969.
2. Rosinski, S., *Evaluation of fatigue data including reactor water environmental effects – Materials Reliability Project (MRP-54),* EPRI report n° 1003079, December 2001.
3. *Re-evaluation of Fatigue Analysis Criteria*, FRAMATOME report n°EE/S 98.317 (Final report to CEC-DG XI contract B4-3070/95/000876/MAR/C2), 1998.
4. Shen, C.L., Wirsching, P.H., Cashman, G.T., *Design curve to characterize fatigue strength*, J. Eng. Mat. Tech., Trans. ASME, **118**, pp.535-514, 1996.
5. *Règles de Conception et de Construction des Matériels des Ilots Nucléaires,* AFCEN, Paris, Juin 2000.
6. Socie, D. and Dowing, S., *Simple Rainflow counting algorithms,* Int. J. of Fatigue, pp. 31-40, 1982.
7. AFNOR, *Produits sidérurgiques, "essais mécaniques, partie 2 : fatigue, mécanique de la rupture », tome 9, norme A 03-405*, 1992.
8. Coffin, L.F., *A study of the effects of cyclic thermal stresses on a ductile metal*, Trans. ASME, **67**, 1954.
9. Langer, B.F, *Design of pressure vessels for low-cycle fatigue,* J. of Basic Eng., Trans. ASME, pp. 389-402, 1962.
10. Keisler, J.M., Chopra, O.K., Shack, W.J., *Statistical models for estimating fatigue strain-life behavior of pressure boundary materials in light water reactor environments,* Nucl. Eng. Design, **167**, pp. 129-154, 1996.
11. *MathCad User's manual*, MathSoft Inc., 2001.
12. Saporta, G., *Probabilités, analyse de données et statistique,* Editions Technip, 1990.



(a) $T = 20°C$

(99 points)

(b) $T \in [400°C - 550°C]$

(70 points)

(b) $T \in [550°C - 650°C]$

(156 points)

Fig.3 : Experimental points and best-fit curves under assumptions H1,H2, H3 for each class of temperature