*Transactions of the 17th International Conference on Structural Mechanics in Reactor Technology (*SMiRT 17)
Prague, Czech Republic, August 17 –22, 2003
Paper # M04-3

Reliability Assessment of a Reinforced Concrete Beam Subjected to Corrosion

Bruno Capra, Olivier Bernard, Bruno Gerard
Oxand, Avon, France

ABSTRACT

Corrosion is one of the most important phenomenon which alter the durability of reinforced concrete structures. The aim of this article is to present some numerical simulations of a concrete beam subjected to corrosion. The simulation have been made using a finite element code in which reliability procedures have been implemented (FORM, Monte Carlo simulations). In a first part, a three point bending test has been chosen, taking into account some random material parameters, to compare the ultimate load of the beam provided by the French reinforced concrete code, and analytical limit state equation and finite element simulations. In a second part, corrosion effects have been take into account by the use of a random variable related to the loss of steel section. Numerical simulations have been performed and allow to describe the evolution of the safety index with time.

KEYWORDS: reliability, assessment, safety index, finite element, reinforced, concrete, beam, corrosion.

INTRODUCTION

Corrosion is one of the most important phenomenon which alter the durability of reinforced concrete structures. It is difficult to model this degradation accurately because of the numerous factors involved and the still imperfect knowledge of physico and electro-chemical processes of the reactions. Nevertheless, there are a some points that are acknowledged, as for example the role of oxygen and chloride ions on the initiation of corrosion. Global approaches has been made in order to measure the state of a real structure towards corrosion. These methods are based on the measurement of a current of corrosion. The evolution of this current during time allows to estimate the section reduction of steel due to corrosion by the use of Farraday's law.

The aim of this article is to present some numerical simulations of a concrete beam subjected to corrosion. The simulation have been made using a finite element code in which reliability procedures have been implemented. These procedures are at one hand, the FORM method using the safety index of Hasofer-Lind and, on the other hand, Monte Carlo simulations with conditioning procedure.

The corrosion phenomena taken into account in these simulations are simplified compared to reality. Physico-chemical phenomena such as carbonation or chloride diffusion in concrete are not taken into account. The corrosion current, considered constant in this study, is the variable that makes a linkage between physico-chemistry and mechanical behaviour of concrete by the means of rebars section reduction.

Some modelling, as for example those developped by Vu & Stewart [1] and Stewart [2], take into account more precisely chloride penetration inside concrete by diffusion coefficient. These models have been applied to a reinforced concrete bridge designed according to AASHTO LRFD bridge design specifications. The results are obtained in terms of percentage reduction in flexural capacity, shear capacity and cumulative probability of failure (CPF) versus time.

The goal of this article is to compare the French Reinforced Concrete design code [3] with reliability calculus. The simulations compare, at one hand, the ultimate load of a sound beam and, on the other hand, a beam subjected to different degrees of corrosion. The use of reliability approach allows to estimate the evolution of the safety index with time.

SAFETY OF A BEAM AND RELIABILITY ASSESSMENT PROBLEM

Recent reinforced concrete national design code are based on the limit state theory. The principle consists in defining an acceptable probability of failure (P_f) of the structure and to design it consequently. The following figure shows the example of a three points bending beam loaded by a force P. Design codes allow to calculate the maximum admissible value of P such as $P_f=P_{f\,fixed}$ (figure1).

The case of a corroded beam can be divided in two parts. Firstly, the residual reliability assessment of the corroded beam ($P_{f\,corroded}$) provide an information about the security reduction of the beam. The next step is the estimation of the remaining loading capacity of the corroded beam ($P_{corroded}$) that ensure the same probability of failure as initially designed ($P_{f\,corroded}=P_{f\,fixed}$) (figure 1).

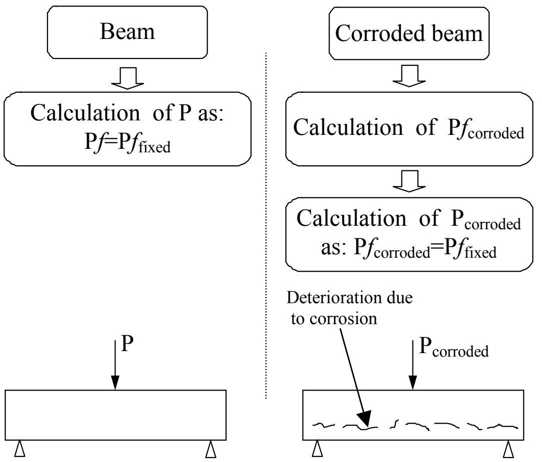


Figure 1. Reliability assessment principle.

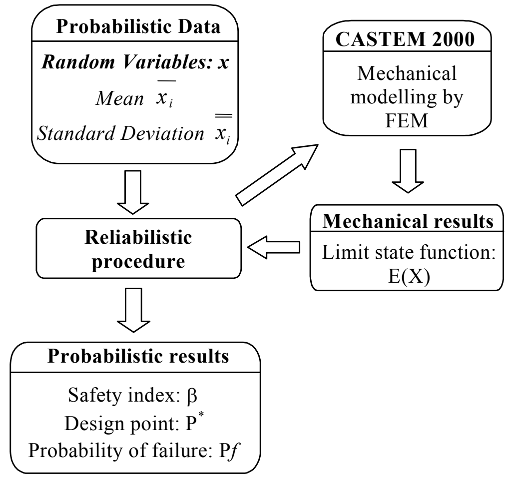


Figure 2. Coupling reliability finite element simulations.

Concrete Beams

The concrete beam simulated in this study is similar those carried out by Castel et al. [4] at Toulouse (France). The experimental study was constituted by two sound beams and two corroded beam subjected to marine environment during 13 years.

Limit State And Random Variables

The limit state considered in this study corresponds to the reach of the ultimate bending moment in the middle section of the beam. The limit state fonction E is expressed by :

$$E(\underline{X}) = M_r(\underline{X}) - M_s(\underline{X}) \qquad (1)$$

where : $\underline{X}$ = vector of random variables, M_r = ultimate resisting bending moment of the section (Nm), M_s = sollicitating bending moment due to loading (Nm).

Random variables used in these simulations are : compressive strength of concrete (f_b), elastic limit of steel (f_e), steel section in tensile zone (A) and applied load on the beam in a three point bending test (P). Distribution laws and coefficients of variation are taken from Sellier [5]. The assumption of uncorrelation between these variables is made.

Table 1. random variables.

Random Variable	Distribution	Mean (μ)	Coefficient of variation (C_α%)	Standard deviation (σ)
A (cm²)	Lognormal	2,26	5	0.113
f_e (MPa)	Normal	560	10,8	60
f_b (MPa)	Normal	45	26,6	12
P (KN)	Normal	20	15	3

PROBABILITY OF FAILURE : LEVEL-2 AND LEVEL-3 METHODS

For a given strucure and a given limit state, the failure domain is denoted D_f. It is defined as the geometrical domain, in the operating random space, where the limit state function $E(\underline{X})$ takes negative or null values. The probability of failure is defined by eq. 2 where $f_{\underline{X}}(.)$ = pdf (probability density function) of the random variables $\underline{X}$ which values are denoted $\underline{x}$.

$$P_f = \text{Prob}(E(\underline{x}) \le 0) = \int_{D_f} f_{\underline{x}}(\underline{X}) d\underline{X} \quad (2)$$

Level-2 Methods

When operating in a gaussian standardised random space, a measure of the reliability can be given by the well-known Hasofer-Lind safety index β. The initial random space, in which is defined the vector $\underline{X}$, must then be transformed into a gaussian one through Rosenblatt's transformation, in which is defined the corresponding vector $\underline{U}$ which is a vector of independent standardised gaussian random variables. If the limit state surface has a linear form, then a probability of failure P_{fL} can be expressed by:

$$P_f = P_{fL} = \Phi(-\beta) \quad (3)$$

where $\Phi(.)$ = cdf (cumulative distribution function) of the standardised gaussian distribution. This safety index is defined as the distance between the origin of the of the gaussian space and the design point P* which is the nearest point, on the limit state surface, to the origin. Equation 3 defines the operational value of failure probability. Its accuracy depends on the actual form od the limit state surface (the linear form is the ideal one) and the precision of P* location. To calculate its coordinates, minimisation algorithms are required. Their efficency depends on the convexity of the limit state surface. These procedures are part of the level-2 methods.

The determination of β by a level-2 procedure has been implemented in the finite element code CASTEM 2000 developped by the CEA (French Atomic Energy Office). Figure 2 describes the general procedure that allows to calculate the safety index and the probability of failure using finite element (FE) simulations.

The determination of β is an iterative procedure which call the FE procedure and therefore time consuming. Each FE simulation give a value of the limit state function E. A first procedure applies the Rosenblatt's transformation between the intial random space and and the standard space. For each vector of random variables $\underline{X}$ computed, its gaussian space counterpart $\underline{U}$ is calculated and E($\underline{U}$) estimated. In general, the random simulation of $\underline{U}$ does not lead to E($\underline{U}$)=0. Then, the reach of the limit state surface is obtained by finding the scalar *a* which respects E($a\underline{U}$)=0. A relaxation method is then applied to every random variable of $\underline{U}$ until the design point P* is reached.

Level-3 Methods

Usually, the resolution of equation 2 requires non analytical solution. Monte Carlo simulations may then be ran. These procedures are known as level-3 methods.

Monte Carlo simulations are precise methods for estimate the probability of failure but their convergence can be sometimes very slow if the probability of failure is weak. In the case of civil engineering structures, P_f is the order of 10^{-6} and then required at least one million simulations to be representative what is very difficult if a finite element simulation is needed.

In order to increase the convergence, a conditioning procedure [6] has been applied to Monte Carlo simulations and implemented in a finite element code. This procedure needs the previous knowledge of β, so the level-2 method previously detailed must be applied before using this method.

Let us consider a failure domain $B_n(\beta)$ defined such as none realisation in this domain causes failure. It is defined, in the standard gaussian space, by an hypersphere having a radius equals to β. The probability of failure based on conditioning technique is defined as follows :

$$\begin{cases} P_f = P(\underline{U} \in D_f) = P(\underline{u} \notin D_f / (\underline{u} \notin B_n(\beta))).P(\underline{u} \notin B_n(\beta)) = P_1.P_2 \\ \text{where:} \begin{cases} P_1 = P(\underline{u} \in D_f / (\underline{u} \notin B_n(\beta))) \\ P_2 = P(\underline{u} \notin B_n(\beta)) = 1 - \chi_n^2(\beta^2) \end{cases} \end{cases} \quad (4)$$

where $\chi_n^2(.)$ = cdf of Chi-Square distribution with n degrees of freedom. The value of P_1 is calculated by Monte Carlo simulations and expressed by :

$$P_1 = \frac{1}{N_{sim}} \sum_{Nsim} 1_{D_f}(\tilde{\underline{u}} \notin B_n(\beta)) \qquad \text{with: } \begin{cases} 1_{D_f}(\tilde{\underline{u}}) = 1 & \text{if } E=(R-S) \le 0 \\ 1_{D_f}(\tilde{\underline{u}}) = 0 & \text{otherwise} \end{cases} \quad (5)$$

APPLICATION TO A CONCRETE BEAM

In order to test the numerical implementation, two different limit state functions have been compared.

Analytical Limit State Function

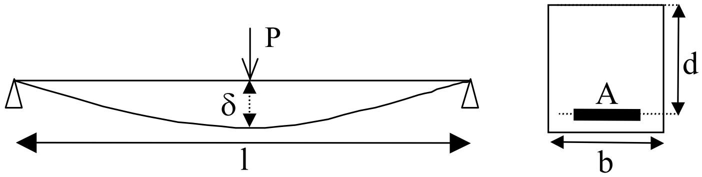
The first limit state function is analytical and based on classical design made by reinforced concrete design codes. The structure under study is the beam previously defined. Both materials (concrete and steel) have perfect elasto-plastic behaviour:

Table 2. Material properties.

Mean value	Concrete	Steel
Young's modulus (MPa)	32000	210000
Poisson's coefficient	0.2	0.3
Compressive strength (MPa)	45	560
Tensile stress (MPa)	0.1	560

Only compressive strength of concrete (f_b) and steel (f_e) are random variables. The other parameters are deterministic (mean value). The tensile strength of concrete is deliberately taken close to 0 in order to compare the results with design codes where tensile strength of concrete is neglected.

The calculus of the resisting $M_r(\underline{X})$, and soliciting $M_s(\underline{X})$, ultimate bending moment in the middle section of the beam leads to the following expressions:



$$\begin{cases} M_r(\underline{X}) = Af_e d - \dfrac{A^2 f_e^2}{2bf_b} \\ M_s(\underline{X}) = \dfrac{Pl}{4} \end{cases} \quad (6)$$

Figure 3. Geometry and bending moments.

The limit state equation $E(\underline{X})$ is defined by: $E(\underline{X}) = M_r(\underline{X}) - M_s(\underline{X})$ with $\underline{X} = (A, f_e, f_b, P)$ (7)

Non Analytical Limit State Function

The same beam is now calculated by finite element simulations. The mesh used is defined on figure 4. The steel plates at the end and in the midle of the beam are modeled to avoid stress concentration near supports and load application point. They have the same mechanical properties as rebars. A plane stress modelling has been done. In order to reduce computation time, only the central part of the beam as an elasto-plastic behaviour (figure 5). The lateral sides of the beam have elastic behaviour.

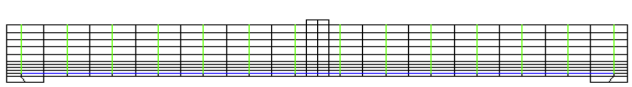


Figure 4. Finite element mesh of the beam.

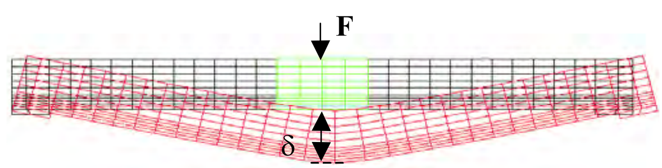


Figure 5. Deformed mesh of the beam.

The conventional ultimate carrying capacity of the beam is obtained when the maximum steel strain reachs 10.10^{-3} as prescribed in the French reinforced concrete design code.

Comparison Between The Different Methods

The previous procedures allow to calculate, for different values of the mean loading force P, the corresponding safety index β. Figure 6 shows the evolution of β versus P mean for the analytical and finite element limit state function. In the case of ultimate limit state, European design code (Eurocode 1) prescribes a safety index of 3.8. Table 3 compares P mean, for analytical and finite element analysis and the corresponding classical reinforced concrete design (BAEL 91). It can be seen that values are comparable. The differences come from the hypothesis taken into account for calculation : perfect elasto-plastic behaviour of materials for simulations whereas design code prescribes "parabola-rectangle" for concrete; beam therory for analytical and design calculation and finite element simulations with support plates. Differences between the methods are acceptable and show that is is possible to design this kind of structure with a defined safety index. The analytical method being conservative in this case.

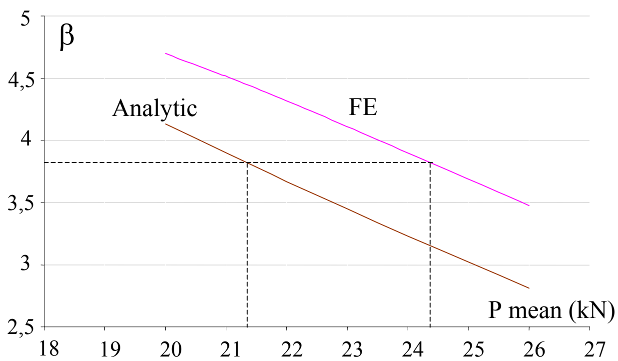


Figure 6. Comparison between analytical and finite element limite state function.

Table 3. Mean loading for a safety index of 3.8

	Reliability analysis method (level II)		French RC code B.A.E.L. 91
Calculation method of E(X)	Finite Element Analysis	Analytical calculation	
Mean value of P (kN)	24,3	21,3	22,1

APPLICATION TO A CORRODED CONCRETE BEAM

Corrosion Mechanisms

Corrosion is an electrochemical mechanism of degradation of steel in concrete. The process requires the presence of a sufficient amount of water otherwise dissolution reactions can not take place. Reactions products (different types of rust) exerts pressure on the surrounding cementitious matrix close to rebars. These pressure are sometimes high enough to crack concrete and the accelerate the penetration of deleterious agents (carbone dioxyde that unprotects steel or chloride ions that increase corrosion processes). Castel [7] has noticed that cracks and maximum loss of mass are mainly located in the tensile zone of concrete beams submitted to chloride environment. This can be explained by the easier penetration of chloride ions through opened cracks due to the application of a bending moment during 14 years.

The conclusions of the experiments show that [4]:

- service state: both steel section reduction and loss of adherence between steel and concrete have an influence on the mechanical behaviour;
- utimate state: only steel section reduction has an effect on the ultimate resistance. A reduction of 20% of the ultimate carrying capacity has been observed but also a reduction of 70% of the beam ductility.

Like in the previous study of the sound beam, we consider here only the ultimate behaviour. So, we take into account the loss of steel section as the only corrosion effect. The distribution of corrosion rate, even if it globally follows the crack pat tern, can be represented by a random variable. The identification of the statistical parameters of this variable is not easy because it is difficult to obtain significant experimental data.

Nevertheless, we postulate the following hypothesis:

- Steel corroded section $A_{corroded}$, follows a lognormal distribution of mean $\mu_{Acorroded}$ and standard deviation $\sigma_{Acorroded}$;
- Mean of corroded steel section $\mu_{Acorroded}$, is related to mean of sound steel section μ_A, by the rate of corrosion τ :

$$\mu_{A_{corroded}} = (1-\tau)\, \mu_A$$

-Standard deviation of corroded section $\sigma_{Acorroded}$ is related to $\mu_{Acorroded}$ by a coefficient of variation C_τ:

$$\sigma_{A_{corroded}} = C_\tau\ \mu_{A_{corroded}}$$

As corrosion is difficult to evaluate, the coefficient of variation C_τ is taken equal to 25% to take into account this uncertainty.

MODELLING AND RELIABILITY ASSESSMENT OF THE CORRODED BEAM

Comparison Between Level-2 And Monte Carlo Simulations

Due to the high number of samplingv required by Monte Carlo simulations and the computing time of finite element determination of safety index, we choose the analytical limit state function in order to compare level-2 and level-3 methods. If the limit state surface of our study is well defined towards level-2 method, then we will be able to omit level-3 method that is more precise but time consuming.

The following figure compares the failure probabilities calculated by level-2 and Monte Carlo simulations with conditioning procedure. Statistical parameters were defined in table 1. Corrosion rate is equal to 40% (τ=0.4) and C_τ=25%.

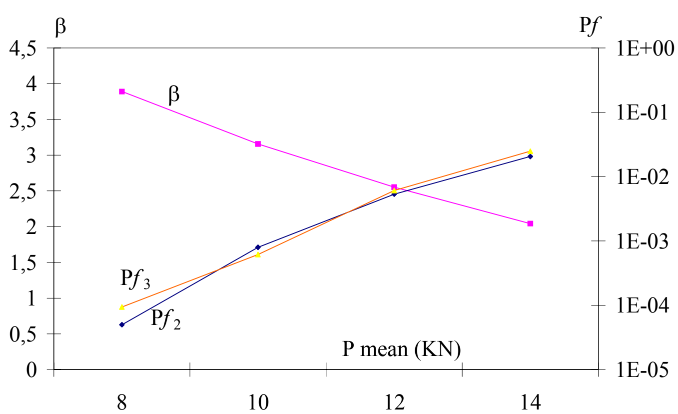
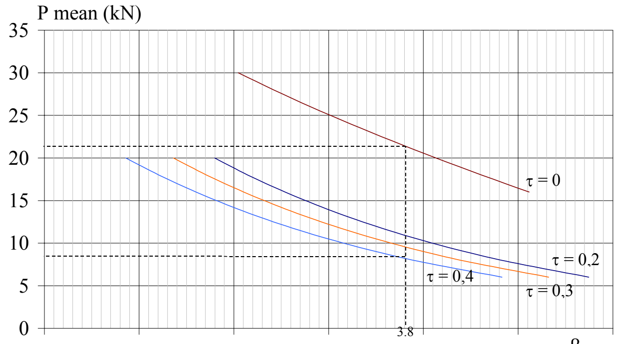


Figure 7. Comparison between level-2 (P_{f2}) and level-3 (P_{f3}) methods with conditioning procedure for a corroded beam.

We can see that the two methods give the same response, in the range of P browsed here. With regard to computation time, level-2 method is much more efficient (factor 5 to 10). So, for the next simulations of corrosion effects, only level-2 method will be used.

Reliability Assessment Of A Corroded Beam

As for the sound beam, the simulations are made for the analytical and finite element limit state function. Figure 8 shows the evolution of P mean for different safety indexes and corrosion rates. It can be seen that for a design safety index of 3.8, the corresponding value of P mean decreases, when the corrosion rate is 40%, from 21.5 kN to 8.5 kN that represents 60.5%. Compared to a deterministic design, the reduction of P mean is more important than the reduction of section. This consequence is due to the standard deviation on corrosion effects and shows the importance of such a design method when degradation processes are not well known. The same kind of simulations was made with the finite element procedure. The comparison between the two methods, for a corrosion rate of 20%, is represented on figure 9. Once again, the results are comparable and the analytical simulation is more conservative than the finite element one. Figure 10 depicts, for a constant safety index of 3.8, the evolution of ultimate load P mean for different corrosion rates.



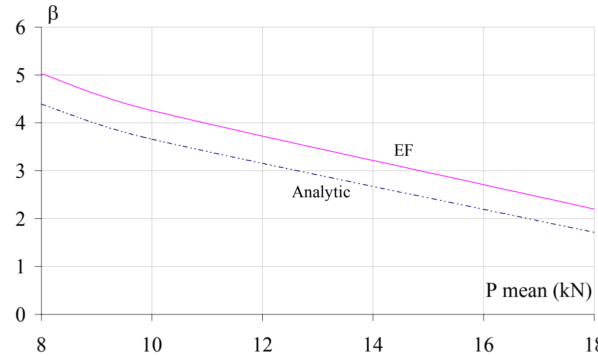
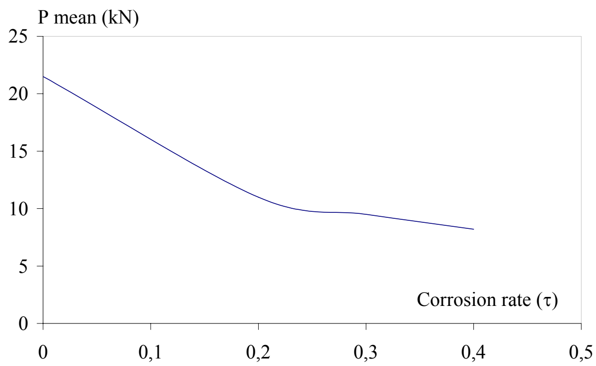


Figure 8. Reliability assessment curves of a corroded beam, analytical simulation ($C\tau$=25%, C_P=15%).

Figure 9. Comparison between analytical and finite element simulations for a corrosion rate of 20%.



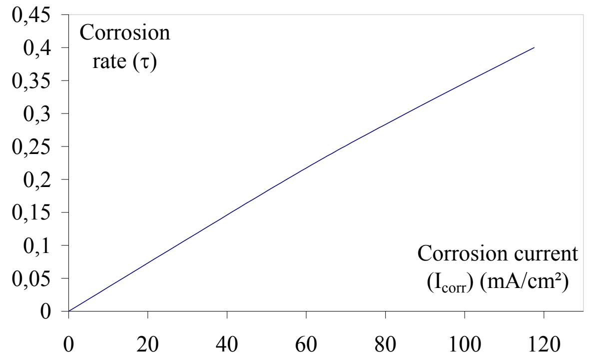
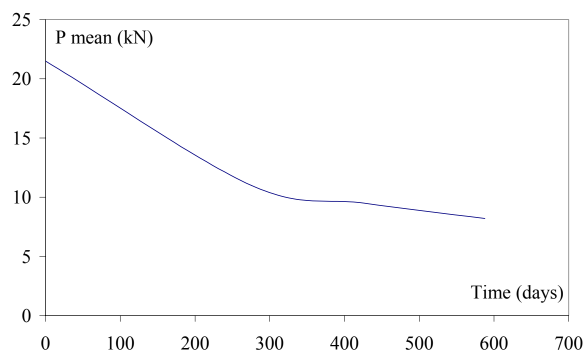


Figure 10. Evolution of P_{mean} versus corrosion rate (β=3.8).

Figure 11. Corrosion rate versus corrosion current.



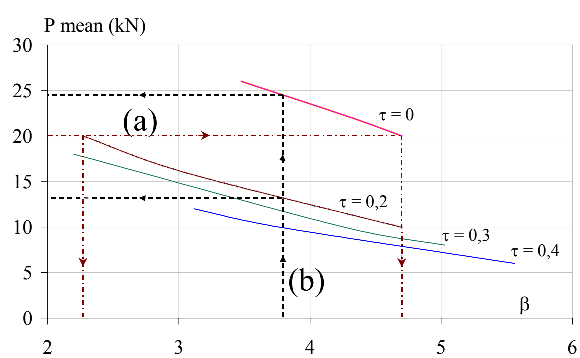


Figure 12. Evolution of carrying capacity of the corroded beam with time for b=3.8.

Figure 13. (a) Evolution of safety, (b) reliability assessment problem.

Experimental techniques of corrosion quantification are based on the measure of a corrosion current I_{corr}. In the case of an uniform corrosion, the following relation has been proposed by Petre-Lazar [8] for the reduction of steel diameter:

$$D(t)=\sqrt{(1-\tau)\frac{4A}{\pi}} \qquad \text{with : } D(t)=D_0-0.023\int_0^t I_{corr}\,dt \qquad (8)$$

where: I_{coor} is the corrosion current (A/m^2), D_0 the initial diameter of rebar and D(t) the diameter of rebar at time t.

If experimental data are available, the mesure of I_{corr} then gives the evolution of corrosion rate τ (figure 11) using eq. 8. As it exists a relation between corrosion rate and P mean (figure 10), the measure of I_{corr} allows to determine the mean carrying capacity of the beam, for a designed safety index (figure 12).

The measure of the corrosion current describes the mass loss of steel during time. The value of I_{corr} is not constant, it depends on the weather (temperature, humidity) and on the intrinsic variability of corrosion mechanisms. A characteristic value can be estimated for a structure, or part of structure, subjected to identical conditions [9].

To illustrate our subject, as experimental data are not available for the studied beam, we postulate that a constant corrosion current of 0.2 $\mu A/cm^2$ (mean value taken from [10]), is representative of reality. Then, it is possible to predict the evolution of the mean loading force P during time which respects a prescribed safety index. In this case, after 600 days, the admissible force which respects the condition is divided by two compared to the initial designed loading (figure 12).

Figure 12 is representative of the reliability assessment schema presented in this article. Corrosion processes effects that alter the durability of concrete can be regarded following two points of view. At one hand, the determination of the mechanical influence of corrosion allows to determine the residual safety of the structure and therefore to take remedial measures. On the other hand, the reliability assessment approach presented here allows to determine the residual carrying capacity of the structure that ensure a determined security. These two approaches are summarized on figure 13:

- (a) evolution of safety is function of corrosion rate for a constant applied load;
- (b) reliability assessment, evolution of carrying capacity with corrosion at constant safety.

CONCLUSION

Corrosion consequences on the structural behaviour of a structure are not easy to take into account due to the complexity of the numerous phenomena involved. The aim of this article was to present the interest of using reliability methods in this case. For the particular exemple of the corroded beamp simulated, level-2 (FORM approach) and level-3 (Monte Carlo simulations with conditioning procedure) give approximately the same response but the computing cost makes FORM approach more efficient. In our exemple, it was possible to compare analytical and finite element estimation of design point. Obviously, the resolution of the analytical case is faster than the finite element solving. Nevertheless, the validation of the finite element approach is important because, in certain cases, it is the only usable method. Indeed, we only studied the ultimate behaviour of the corroded beam, where steel diameter reduction is the major parameter of corrosion effects. In the case of service behaviour, it has been seen that interface conditions between steel and concrete are very important and requires a more complex treatment. Another point that requires finte element simulations is the variability of corrosion along the beam. In this case, the whole structure has to be computed with a random distribution of corrosion rate along the beam. The last important point is the interest to better estimate the coefficient of variation associated to corrosion rate because its a parameter of great influence towards numerical simulations.

REFERENCES

1. Vu, K. & Stewart M.G. 2000. Structural reliability of concrete bridges including improved chloride-induced corrosion models. *Structural safety* 22: 313-333.
2. Stewart, M.G. 2001. Reliability-based assessment of ageing bridges using risk ranking and life cycle cost decision analyses. *Reliability Engineering and System Safety* 74: 263-273.
3. BAEL 91. *Béton Armé aux Etats Limites*. Paris: Eyrolles 1991.
4. Castel, A., François, R. & Arliguie, G. 2000. Mechanical behaviour of a reinforced concrete beams damaged by reinforcement steel corrosion – Part1: Experimental study of corroded beams. *Mater. Struct*. 33 (233): 539-544.
5. Sellier, A., 1995. *Modélisations probabilistes du comportement des matériaux et structures en génie civil*. PhD thesis, France, ENS Cachan.
6. Mébarki, A. & Sellier, A. 1995. Importance zone and importance sampling in reliability analysis of civil structures. *Int. J. Pres. Ves. & Piping* 61 :513-526.
7. Castel, A., François, R. & Arliguie, G. 1999. Effect of loading on carbonation penetration in reinforced concrete elements. *Cement and Concrete Research* 29: 561-565.
8. Petre-Lazar, I. 2000. *Evaluation du comportement en service des ouvrages en béton armé soumis à la corrosion des aciers*. PhD thesis, Québec, Laval University.
9. Raharinaivo, A., Arliguie, G., Chaussadent, G., Pollet, V. & Taché, G. 1998. *La corrosion et la protection des aciers dans le béton*. Paris: Presses de l'Ecole Nationale des Ponts et Chaussées.
10. Andrade, C., Alonso, C. & Sarria J., 1998. Natural wheathering influence on the rebar corrosion rate, *Proceedings of ICCRRCS'98, Orlando, USA*.