LAGRANGIAN AND EULERIAN FINITE ELEMENT TECHNIQUES FOR TRANSIENT FLUID-STRUCTURE INTERACTION PROBLEMS

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SUMMARY

The paper is concerned with the description of a finite element method developed for the nonlinear transient response of coupled hydrodynamics-structures problems such as encountered in the safety analysis of nuclear reactor components. The choice of the finite element methodology was made by placing the emphasis on the structural response, the final goal of the analysis being to investigate the ability of the structure to maintain its integrity when subjected to transient loads transmitted through the fluid.

Efficient finite element procedures are available for the transient dynamic analysis of solids exhibiting both geometric and material nonlinearities. The particular procedure used here consists in formulating the discrete equations of motion in terms of convected co-ordinates that rotate but do not deform with the elements. Such a technique provides for an economic way of dealing with large-displacement effects provided the strains can be assumed small. The internal resisting forces are evaluated directly in terms of convected stresses and a lumped-explicit scheme is used for time integration of the equations of motion. Elastic-plastic material behaviour is accounted for and isotopic, kinematic and overlay models are available for description of strain hardening. The main characteristics of the structural elements employed are briefly outlined.

It is recognized that the most thoroughly studied techniques for solving compressible flow problems have been the various methods based on finite difference discretization. A survey of recent literature, however, shows an increasing use of the finite element method as a discretization technique for problems in fluid mechanics. It is our opinion that a global use of the finite element methodology for solving coupled hydrodynamics structures problems allows for a simple computer program architecture, permits a straightforward treatment of fluid-solid interfaces and enables the use of arbitrarily shaped elements for modelling both solid and fluid regions.

In this paper, the basic finite element equations for transient compressible fluid flow are presented in a form that allows the elements to be moved with the fluid in normal Langrangian fashion, to be held fixed in a Eulerian manner, or to be moved in some arbitrarily specified way. The co-existence of Lagrangian and Eulerian regions within the finite element mesh will permit to handle greater distortions in the fluid motion than would be allowed by a purely Langrangian method, with more resolution than is afforded by a purely Eulerian method. To achieve a mixed formulation, the conservation statements of mass, momentum and energy are expressed in integral form over a reference volume whose surface may be moving with an arbitrarily prescribed velocity. Direct use can be made of the integral forms of the mass and energy equations to adjust the element density and specific internal energy. The Galerkin process is employed to formulate a variational statement associated with the momentum equation. The difficulties associated with the presence of convective terms in the conservation equations are handled by expressing transports of mass, momentum and energy terms of intermediate velocities derived at each cycle from the previous cycle velocities and accelerations.

The hydrodynamic elements presented are triangles, quadrilaterals with constant pressure and density. The finite element equations associated with these elements are described in the necessary detail. Numerical results are presented based on purely Lagrangian, purely Eulerian and mixed formulations. Simple problems with analytic solution are solved first to show the validity and accuracy of the proposed mixed finite element formulation. Then, practical problems are illustrated in the field of fast reactor safety analysis.

1. Introduction

The licensing of nuclear reactors involves the study of hypothetical accidents in which peak pressures, time scales and material properties may vary over a wide range. In this context, there is a need for numerical techniques and complex computer codes to analyze in the necessary detail the response of reactor components when subjected to anomalous conditions.

In the framework of the nuclear safety programme carried out at JRC-Ispra, a finite element code for fluid-structure interaction problems is under development with the aim of investigating the response of containment structures and core subassemblies when subjected to a wide variety of large, transient loads.

While efficient finite element procedures are available for the nonlinear transient response of complex structural components, there has been only a limited use of the finite element technique in the solution of transient hydrodynamic problems [1][2]. The present paper is therefore particularly concerned with the basic methodology and numerical technique for the treatment of compressible fluid flow in the framework of a two-dimensional hydrodynamic-structural code.

A finite element procedure of solving the time-dependent equations of motion for compressible fluids is presented, which combines the best features of purely Lagrangian and Eulerian approaches. The element nodes can, in fact, be moved with the fluid in normal Lagrangian fashion, be held fixed in an Eulerian manner or be moved in some arbitrary way to give a sort of continuous rezoning capability. The proposed finite element method is thus conceptually similar to the Arbitrary Lagrangian Eulerian finite difference technique ALE [3].

The hydrodynamic elements are triangles and/or quadrilaterals in which pressure, density and specific energies are assumed uniform. The presence of transport terms in the conservation equations of mass, momentum and energy introduces, as usual, inaccuracies and instabilities in the results. To overcome these difficulties, the transport effects are calculated by means of a partial-donor cell technique combined with the use of intermediate values of the velocities and specific energies.

The structural elements available in the code are rectilinear beams, conical shells and constant strain triangles. The effects of large displacements and elasto-plastic material behaviour are included.

In the final part of the paper, numerical results are presented and discussed for simple problems. More realistic applications in the field of nuclear reactor safety will be available for presentation during the conference.

2. Basic Methodology

The specific procedures used in structural and hydrodynamic elements are discussed

in the next two sections.

2.1 Structural Elements

Because of the two-dimensionality of the code, the analyses are restricted to axisymmetric containment structures or to a horizontal cross-section of a cluster of subassemblies. Rectilinear Euler-Bernoulli beam elements are employed to model the subassembly walls. Conical shell elements and constant strain triangles are available for axisymmetric containment structures.

In all these elements, geometrical nonlinearities caused by large rotations are treated by formulating the equations of motion in terms of convected coordinates, which are coordinates that rotate and translate with the elements. The detailed developments of the convected coordinate procedure may be found in references [4] [5]. In this procedure, the relationships between strains and deformation displacements in the convected coordinate system of each element are linear, provided the strains can be assumed small. The complexity of nodal force computations is thus greatly reduced with respect to other treatments of large displacement effects.

Elastic-plastic relations of the incremental type are used for material description in connection with the Von Mises criterion of yielding. Both isotropic and kinematic hardening models have been implemented in the computer code. Reference [6] gives the detailed developments of the incremental elasto-plastic constitutive laws.

The discrete equations of motion are expressed in the form

$$[M]\ \{\delta^{\cdot\cdot}\} = \{F^{ext}\} - \{F^{int}\} \qquad (1)$$

where $[M]$ is the global mass matrix, $\{\delta^{\cdot\cdot}\}$ lists the nodal components of acceleration, $\{F^{ext}\}$ is the external load vector and $\{F^{int}\}$ represents the internal resisting forces, which are directly evaluated in terms of element stresses. The equations of motion are integrated in time by an explicit procedure combined with a diagonal form of the mass matrix. A central difference technique is employed in which velocities and displacements are computed by

$$\begin{aligned} \{\delta^{\cdot}(t+\Delta t)\} &= \{\delta^{\cdot}(t)\} + \tfrac{1}{2}\Delta t\,[\{\delta^{\cdot\cdot}(t)\} + \{\delta^{\cdot\cdot}(t+\Delta t)\}] \\ \{\delta(t+\Delta t)\} &= \{\delta(t)\} + \Delta t\,[\{\delta^{\cdot}(t)\} + \tfrac{1}{2}\Delta t\,\{\delta^{\cdot\cdot}(t)\}] \end{aligned} \qquad (2)$$

The time step Δt must satisfy the computational stability condition. The sequence of calculations for structural elements is illustrated in the right part of Table I.

2.2 Hydrodynamic Elements

The basic equations are the conservation statements of mass, momentum and total energy expressed with reference to a volume, V, whose surface, S, may be moving with arbitrary velocity $\vec{w}$. The fluid is characterized by a density, ρ, a velocity, $\vec{u}$, an internal specific energy, i, and a pressure, p, defined by the equation of state $p = p(\rho, i)$.

The rate of change of the total mass, M, momentum, Q, and total energy, E, contained

within S at time t is given by [7]

$$\frac{dM}{dt} = \frac{d}{dt}\int_V \rho dV = \int_S \rho(\vec{w}-\vec{u}).\, d\vec{S} \tag{3a}$$

$$\frac{dQ}{dt} = \frac{d}{dt}\int_V \rho\vec{u}\, dV = \int_V \rho\vec{g}\, dV - \int_V \text{grad } p\, dV + \int_S \rho\vec{u}(\vec{w}-\vec{u}).\, d\vec{S} \tag{3b}$$

$$\frac{dE}{dt} = \frac{d}{dt}\int_V \rho e\, dV = \int_V \rho\vec{g}.\vec{u}\, dV - \int_S p\,\vec{u}.\, d\vec{S} + \int_S \rho e\,(\vec{w}-\vec{u}).\, d\vec{S} \tag{3c}$$

where $e = \frac{1}{2}\vec{u}.\vec{u}+i$ and $\vec{g}$ is a body acceleration (like gravity).

When $\vec{w} = 0$, the equations (3) are Eulerian, when $\vec{w} = \vec{u}$ the equations are Lagrangian, when $\vec{w} \neq \vec{u} \neq 0$ a mixed formulation is achieved.

In a finite element solution, direct use could in principle be made of equations (3a) and (3c) to update the density and the specific total energy. As shown below, great care must nevertheless be taken in the evaluation of the transport terms to ensure accuracy and stability of the numerical solution.

The Galerkin process is here employed to formulate a variational statement associated with the momentum equation (3b). The fluid velocity in each element is approximated by

$$u_i = \sum_I N_I(\vec{X})\, u_{iI}^{(e)} \tag{4}$$

where $N_I(\vec{X})$ are the shape functions expressed in terms of the coordinates at time t = 0 and $u_{iI}^{(e)}$ are the velocity components at node I of element e. Assuming that the velocity $\vec{w}$ can be expressed locally in the form of eq. (4), the application of the Galerkin method yields the following equations at the element level:

$$\frac{d}{dt}\left([m]^e\{u\}^e\right) = \{P\}^e + \{G\}^e + \{T\}^e + \{\bar{F}\}^e \tag{5}$$

In this equation, $[m]^e$ is the element mass matrix. In order to obtain the nodal velocities by an explicit time integration, a diagonal mass matrix is used, defined by equally subdividing the element mass among its nodes. $\{P\}^e$ represents the nodal loads induced by the element pressure field:

$$P_{iI} = \int_{V^e} \frac{\partial N_I}{\partial x_i} p(\vec{x})\, dV^e \tag{6}$$

$\{G\}^e$ accounts for body acceleration $\vec{g}$:

$$G_{iI} = \int_{V^e} \rho N_I g_i dV^e \tag{7}$$

$\{T\}^e$ is associated with the transport of momentum components:

$$T_{iI} = \int_{S^e} \rho N_I (N_J u_{iJ})(n_x N_J u^*_{xJ} + n_y N_J u^*_{yJ})\, dS^e - \int_{V^e} \rho (N_J u_{iJ})\left[N_J u^*_{xJ}\frac{\partial N_I}{\partial x} + N_J u^*_{yJ}\frac{\partial N_I}{\partial y}\right] dV^e \tag{8}$$

In eq. (8), $\vec{u}^* = \vec{w} - \vec{u}$ and n_x, n_y are the components of the outward normal on S^e. Repeated indices indicate a summation. Finally, $\{\overline{F}\}^e$ represents externally applied loads eventually acting on the element boundary.

The fluid domain is here represented by means of quadrilateral and/or triangular elements. A linear velocity variation is prescribed along the element sides and it is assumed that coordinates, mesh velocity $\vec{w}$ and fluid velocity $\vec{u}$ can be interpolated over the element by the shape functions N_I. Referring to the typical case of a quadrilateral element with nodes i, j, k, l (fig. 1), the shape functions are given by [8]:

$$N_i = \frac{1}{4}(1+\xi)(1-\eta) \;;\; N_j = \frac{1}{4}(1+\xi)(1+\eta)$$
$$N_k = \frac{1}{4}(1-\xi)(1+\eta) \;;\; N_\ell = \frac{1}{4}(1-\xi)(1-\eta) \tag{9}$$

where (ξ, η) are normalized coordinates which take values ± 1 along the element sides.

Assuming a uniform pressure over the element, the internal resisting forces, eq. (6), can easily be expressed in terms of the current coordinates of the nodes. For an axisymmetric quadrilateral element with nodes i, j, k, l ordered in anticlockwise order, we find at node i with current coordinates r_i, z_i:

$$P_r^i = \frac{p}{2}\left[\frac{r_i+r_j}{2}\cdot(z_j-z_i) + \frac{r_i+r_\ell}{2}\cdot(z_i-z_\ell)\right] \tag{10a}$$

$$P_z^i = \frac{p}{2}\cdot\bar{r}\cdot(r_\ell - r_j) \tag{10b}$$

where p is the element pressure and $\bar{r}$ the radius of its centroid. Similar formulae are obtained for a triangular element. For plane elements, the mean radii in eqs. (10) should be replaced by unity.

The main difficulties in an Eulerian or mixed calculation are associated with the evaluation of the transport terms in the conservation equations for mass, momentum and energy. Usually, the calculations are divided into two steps within each time interval [9]. A first, quasi-Lagrangian, step yields intermediate values of the velocities and specific total energies, which are then used in a second step to evaluate the transport effects. Here, it was found possible to avoid the first step by evaluating the intermediate velocities by extrapolation of the previous time step velocities:

$$\{\tilde{u}(t+\Delta t)\} = \{u(t)\} + \frac{\Delta t}{2}\{\dot{u}(t)\} \tag{11a}$$

$$\{\tilde{w}(t+\Delta t)\} = \{w(t)\} + \frac{\Delta t}{2}\{\dot{w}(t)\} \tag{11b}$$

The intermediate specific total energy $\tilde{e}$ is then evaluated in each element by

$$\tilde{e} = e(t) - \frac{\Delta t}{M(t)}\int_{S^e} \bar{p}(t)\,\vec{\tilde{u}}\cdot\vec{dS}^e \tag{12}$$

where M(t) is the element mass at time t and $\bar{p}(t)$ a weighted average of the pressures in the elements on either side of boundary S^e:

$$\bar{p}(t) = \frac{M^{(1)}(t)\cdot p^{(1)}(t) + M^{(2)}(t)\cdot p^{(2)}(t)}{M^{(1)}(t) + M^{(2)}(t)} \tag{13}$$

By means of the intermediate quantities, eqs. (11)(12), the transport terms in the mass, momentum and energy equations can be calculated. For example, the mass transport across side i-j of the quadrilateral element in fig. 1 is computed from eq. (3a) as

$$\int_{i\text{-}j} \rho\,(\vec{w}-\vec{u})\cdot d\vec{S} = \frac{r_i+r_j}{2}\cdot\frac{\bar{\rho}_{ij}}{2}\left[(\tilde{u}_{ri}-\tilde{w}_{ri}+\tilde{u}_{rj}-\tilde{w}_{rj})(z_j-z_i) + (\tilde{u}_{zi}-\tilde{w}_{zi}+\tilde{u}_{zj}-\tilde{w}_{zj})(r_i-r_j)\right] \tag{14}$$

where $\bar{\rho}_{ij}$ is a weighted average of the densities in the elements on either side of boundary segment i-j:

$$\bar{\rho}_{ij} = \frac{1}{2}\left[(1-\alpha_{ij})\rho^{(1)} + (1+\alpha_{ij})\rho^{(2)}\right] \tag{15}$$

The coefficient α_{ij} is proportional to the velocity flux through side i-j and evaluated as indicated in reference [10].

Total energy transport, eq. (3c), is evaluated in the same way using the intermediate specific total energies given in eq. (12).

The nodal loads, eq. (8), accounting for transport of momentum components consist of a surface term and a volume term. Using the shape functions, eq. (9), the surface term can be evaluated by analytical integration along the element sides. The intermediate velocities eq. (11), and the weighted density, eq. (15), are used in this calculation. The volume term in expression (8) is evaluated using a 2x2 Gaussian quadrature formula.

For problems involving shock waves, it is necessary to add to the hydrostatic pressure p an artificial viscous pressure q. As in the purely Lagrangian scheme [2], a quadratic pseudoviscosity is employed, which is given by

$$q = \begin{cases} -\rho\, C_Q^2\, d\,(\nabla\cdot\vec{u})^2 & \text{if } \nabla\cdot\vec{u} < 0 \\ 0 & \text{if } \nabla\cdot\vec{u} \geqslant 0 \end{cases} \tag{16}$$

In this expression, d = element area/max($\Delta r, \Delta z$) and C_Q^2 is a numerical coefficient that in our tests assumed values between 2 and 5.

Once the global nodal loads in momentum equation (5) have been obtained by assembly of the element contributions, the nodal components of fluid velocity can be advanced in time by the central difference scheme:

$$\{u(t+\Delta t)\} = [M(t+\Delta t)]^{-1}\left\{[M(t)]\,\{u(t)\} + \frac{\Delta t}{2}\left(\{F(t)\} + \{F(t+\Delta t)\}\right)\right\} \tag{17}$$

where $[M]$ is the global diagonal mass matrix and $\{F\}$ the total nodal load vector. To ensure numerical stability the time step must satisfy the condition [11]:

$$\Delta t < \frac{\text{Area of element}}{|u_r|\Delta z + |u_z|\Delta r + cD + 4\,\text{Area}|\nabla\cdot\vec{u}|} \tag{18}$$

where c is the sound speed and D is $\max(\Delta r, \Delta z)$.

The sequence of calculations for purely Lagrangian, purely Eulerian and mixed hydrodynamic elements is illustrated in the left part of Table I.

3. Numerical Results

In this section we consider two examples designed to test the accuracy of the finite element procedure described in the paper. More realistic applications in the field of nuclear reactor safety will be available for presentation during the conference.

3.1 Shock Tube Problem

A simple but useful problem to test the arbitrary Lagrangian-Eulerian finite element procedure for compressible fluid flow is a shock tube in which a long straight cylinder is divided into two compartments by a central diaphragm.

On one side of the diaphragm there is a gas of density $\rho_1 = 0.0581$ gm/cm^3 and internal energy $e_1 = 4.303\ 10^9$ ergs/gm, while on the other side the gas has density $\rho_2 = \frac{1}{2}\rho_1$ at the same energy. We assume γ-law gases with $\gamma = 1.4$.

Quadrilateral elements were employed with $\Delta z = 1$ cm; the time step was $\Delta t = 2.5\ 10^{-6}$ sec and quadratic viscous pressure terms used with $c_Q^2 = 2.56$ in eq. (16). Three numerical solutions were performed respectively based on a purely Lagrangian, a purely Eulerian and a mixed description with $\vec{w} = \frac{1}{2}\vec{u}$ in eqs. (3).

Figs. 2 and 3 show the pressure and velocity profiles at time $t = 750\ \mu$sec for the purely Lagrangian and purely Eulerian solutions. The comparison with the theoretical predictions is excellent in both cases, the Lagrangian solution being slightly less accurate. The numerical results obtained using the mixed description are not reported here; they were found to be very similar to the Eulerian results.

3.2 Elasto-plastic Shock Cylinder

As shown in fig. 4 this problem corresponds to a radial shock tube in which an elasto-plastic cylindrical diaphragm separates two γ-law gases with $\gamma = 1.4$.

The elasto-plastic external envelope is submitted to a permanent pressure equal to the initial pressure of the outer gas. Two plane strain solutions of this problem were performed using 28 quadrilateral elements to model the two gases and 2 conical shell elements to represent the elasto-plastic walls. The first solution is based on a purely Lagrangian description, while the second is purely Eulerian, except for the fluid elements in contact with the structural parts. These elements are forced to follow the adjacent structural elements.

Figs. 5 and 6 indicate that the two solutions give nearly identical velocity profiles along the radius of the shock cylinder.

4. Conclusions

An arbitrary Lagrangian-Eulerian finite element procedure has been described for the study of transient fluid-structure interaction problems. One of the major problems in the

numerical analysis of the nonlinear transient response of fluid-structure configurations is to avoid excessive computational time requirements.

In the present paper we have tried to combine several sources of computational improvement, so that large scale problems can be solved at a reasonable cost.

An explicit integration scheme is used in combination with a lumped mass matrix, so that the usual problems related to bandwidth or problem size are eliminated. The use of a convected coordinate procedure for dealing with large displacement effects in structural elements considerably simplifies the computations of nodal forces. A constant pressure has been assumed within each hydrodynamic element, so that only one evaluation of the equation of state is required per element. Finally, the evaluation of transport effects in the conservation statements of mass, momentum and energy is based on intermediate velocities and specific energies which are obtained by straightforward extrapolation of the previous time step results.

The test calculations presented indicate the accuracy and stability of the proposed calculational scheme.

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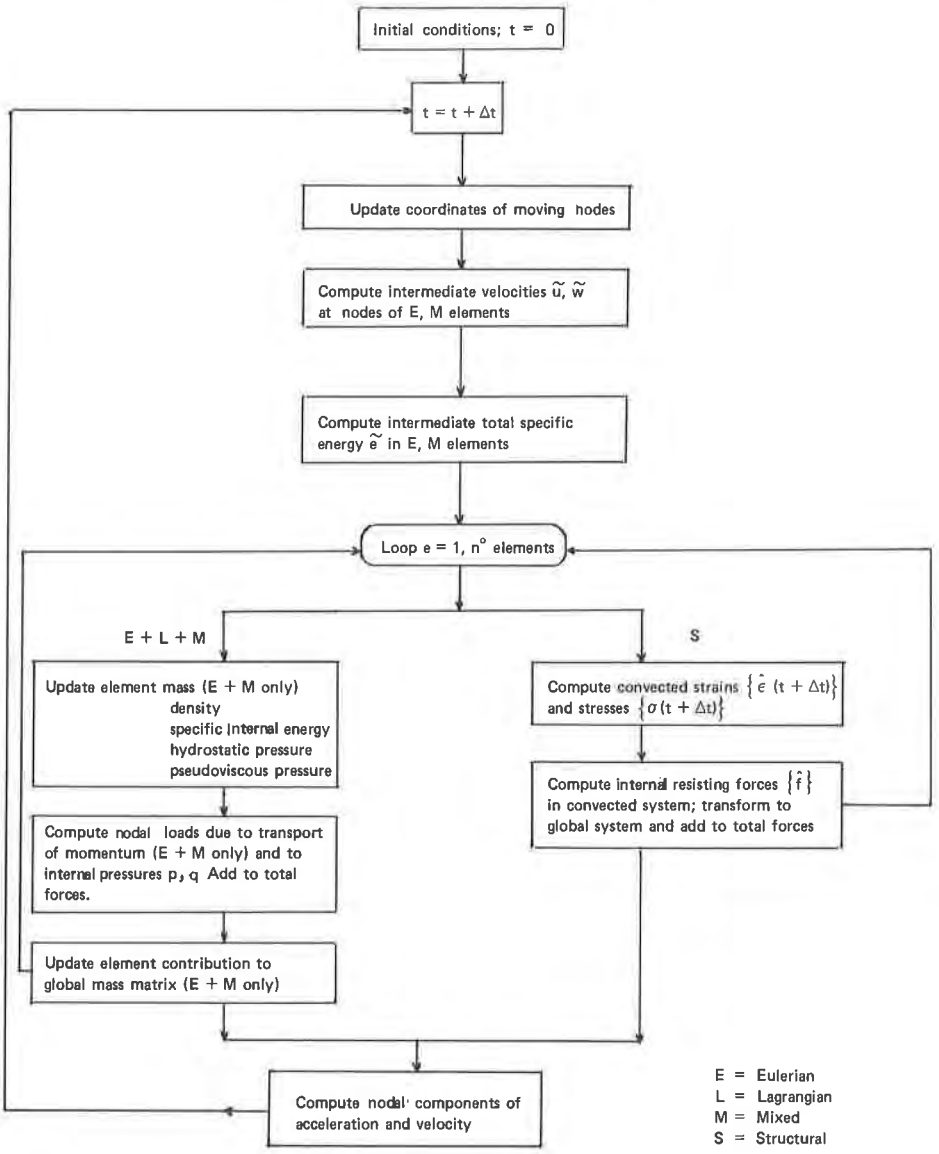
Table I : Flowchart of Computational procedure



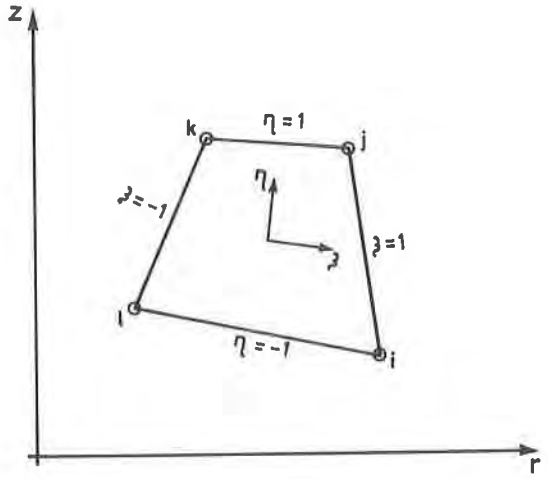


Fig. 1 : Typical quadrilateral hydrodynamic element.

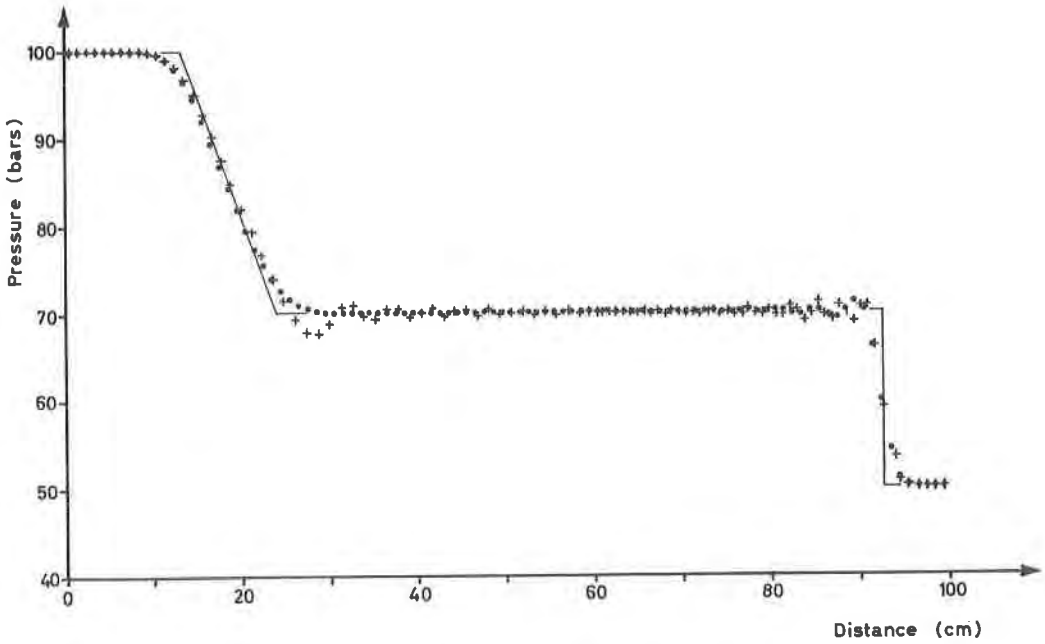


Fig. 2 : Pressure profiles at t = 750 μsec for Lagrangian (+) and Eulerian (·) calculations of the shock tube problem. Solid lines are theoretical predictions.

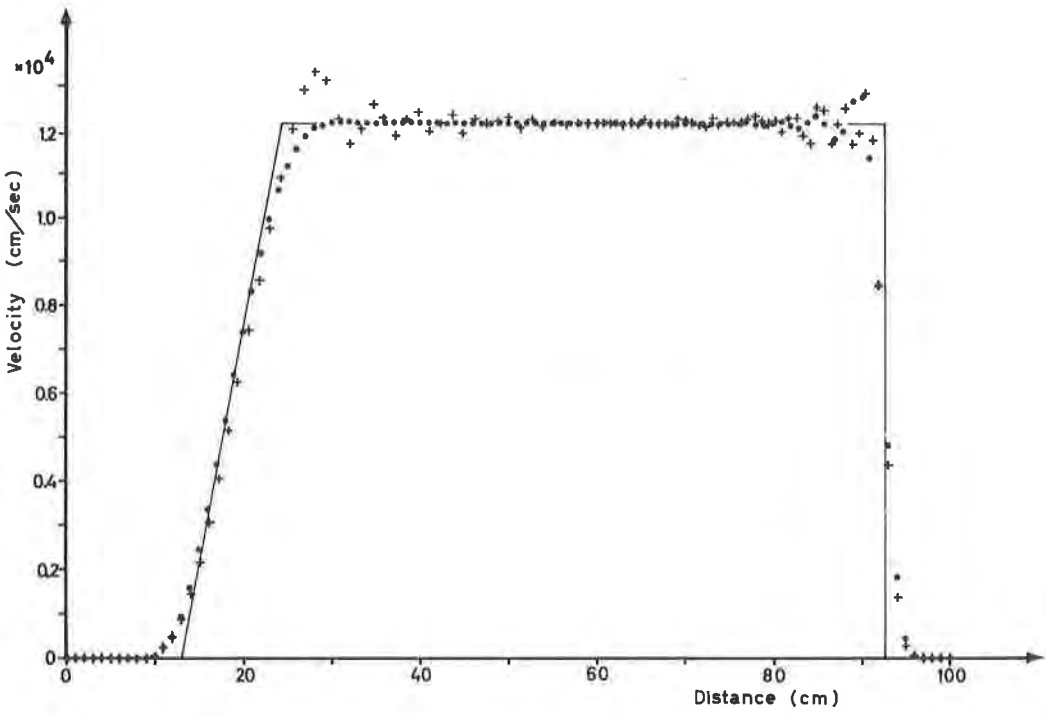


Fig. 3 : Velocity profiles at t = 750 μsec for Lagrangian (+) and Eulerian (•) calculations of the shock tube problem. Solid lines are theoretical predictions.

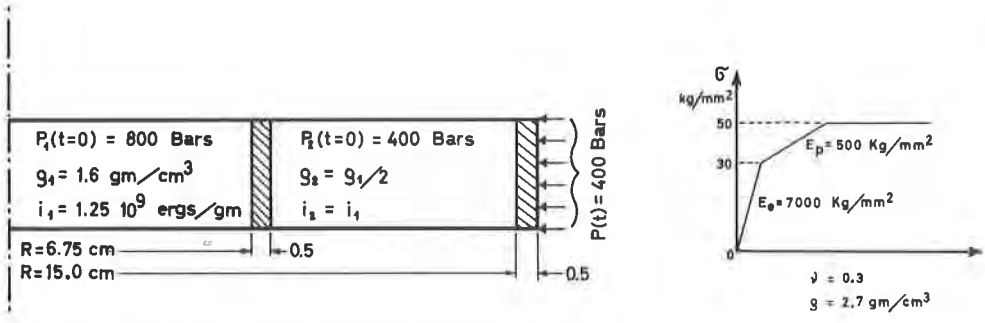


Fig. 4 : Elasto-plastic shock cylinder.

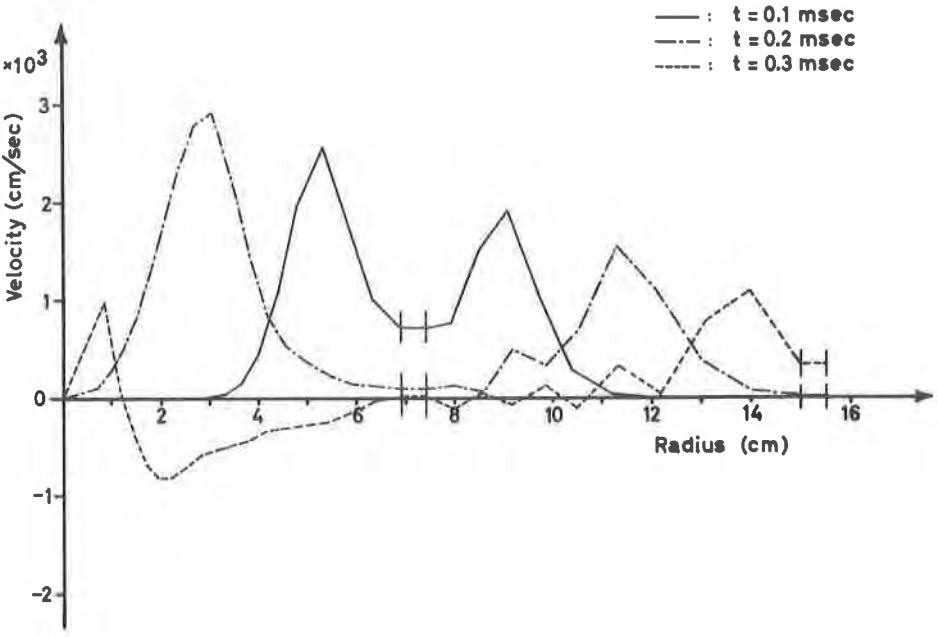


Fig. 5 : Velocity profiles for a Lagrangian calculation of the shock cylinder.

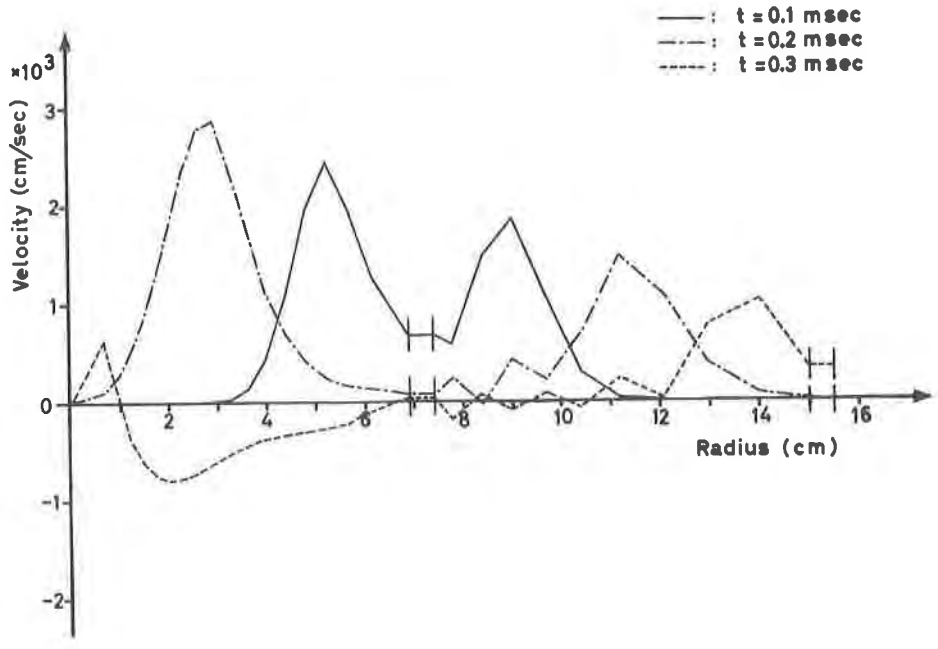


Fig. 6 : Velocity profiles for an Eulerian calculation of the shock cylinder.