ANALYSIS OF STEAM VENT CLEARING LOADS ON THE SUPPRESSION POOL

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SUMMARY

The relief valve piping on a boiling water reactor vents steam to the suppression pool through a submerged discharge. The air in the piping trapped between the pool surface and the valve is injected into the pool on valve actuation in the form of a high pressure bubble. This bubble induces pressures on the walls and bottom of the pool as it oscillates and rises to the surface.

If compressibility effects are neglected and the bubble radius is assumed small the maximum and minimum pressures at the pool boundaries can be found by the solution of the analogous steady state heat conduction problem for the temperatures produced by a point source.

A finite element solution for an axisymmetric pool with an assymmetrically located source is obtained as the sum of Fourier harmonics. The convergence of the solutions is shown to depend on the pool radius and the proximity of the bubble to the boundaries.

Analytical solutions are obtained for the horizontal and vertical force resultants and good agreement with the finite element solutions is demonstrated.

Fluid structure interaction effects due to vertical rigid body motion of the pool are investigated by modeling the pool as a rigid spring supported tank in a rigid building with the building foundation modeled as a two degree of freedom mass spring system.

Results are obtained for a bubble with an oscillation frequency of 10 cps for tank frequencies of about 10 and 16 cps. Building foundation frequencies are 3.4 and 10.4 cps in both cases. Building and tank response spectra neglecting the effects of interaction and considering these effects are presented and it is shown the neglect of interaction effects gives conservative results in both cases. Interaction effects are shown to reduce peak pressures by 30 to 40 percent in the cases considered.

Negative pressure peaks were not reduced in either case. The peak accelerations of the tank were about 1.0 g in both cases and the pressure reductions cited above were based on the assumption the water comprises 35 percent of the total mass of the tank plus its contents. The largest reductions in pressures were found in the case where the tank and bubble were nearly in resonance. The driving force exerted by the bubble which is independent of the fluid mass is in this case substantially reduced by interaction effects. This accounts for the fact that the neglect of interaction leads to an overestimation of structural response.

The finite element method is shown to be suitable for the calculation of pressures and force resultants on the walls and bottom of the suppression pool. The neglect of fluid structural interaction effects in calculating gross structural response is justified. However, it is shown that these effects can substantially reduce peak positive pressures and should therefore be accounted for in the interpretation of test results.
2. Finite Element Analysis of the Pressures on a Rigid Annular Tank
   Due to a Submerged Bubble

2.1 Analysis

The small bubble of high pressure air is assumed to expand adiabatically as a perfect
and to remain spherical as it oscillates in the incompressible fluid. When the bubble
radius is at its maximum or minimum the excess pressures satisfy Laplace's equation outside
the bubble

\[ \nabla^2 p = 0 \]  

(1)

Since the bubble is assumed small the solution in the neighborhood of the bubble is

\[ p = (p_b - p) \frac{r_b}{r} \]  

(2)

where \( p_b, r_b \) are the absolute pressure, radius of the bubble, \( p \) is the static fluid pressure
at the bubble, and \( r \) is distance from its center. Zienkiewicz [1] shows that if a submerged
surface moves in a fluid with a prescribed acceleration and small amplitude of motion that
at the boundaries

\[ \frac{\partial p}{\partial n} = -\rho \frac{a}{n} \]  

(3)

where \( \rho \) is the density of the fluid, and at the free surface

\[ p = 0 \]  

(4)

Since by Green's theorem in this case

\[ \iiint \frac{\partial p}{\partial n} ds = \iiint \nabla^2 p \; dv = 0 \]

we have

\[ \iiint \frac{\partial p}{\partial n} ds = -\iiint \frac{\partial p}{\partial n} ds = -4\pi Q \]  

(5)

with

\[ Q = (p_b - p_s) \frac{r_b}{r} \]  

(6)

as a requirement to be satisfied at the free surface, \( s_f \), which ensures the satisfaction of
the required boundary conditions on the surface of the bubble, \( s_b \), as \( r_b \rightarrow 0 \).

Following [1] we note that the required solution minimizes the functional

\[ x = \frac{1}{2} \iiint \left( \frac{\partial p}{\partial r} \right)^2 + \left( \frac{\partial p}{\partial \theta} \right)^2 + \left( \frac{\partial p}{\partial \phi} \right)^2 \right) r \; dr \; dz \; d\phi \]  

(7)

and satisfies eqs.(3) on the walls and bottom of the tank as well as eqs.(4) and (5) at its
surface. The axisymmetric pool is divided into triangular ring elements and \( p \) is defined
within each element by the nodal values

\[ p = \sum_{n} \left[ N_i, N_j, N_k \right] \begin{pmatrix} p_i^{(n)} \\ p_j^{(n)} \\ p_k^{(n)} \end{pmatrix} \cos \theta \]  

(8)

where the \( N_i \) are the linear shape functions defined in [1]. Minimization of the contribution of
an element to the functional \( x \) with respect to the nodal values of \( p \) gives the element
'stiffness' matrix for each harmonic, \( H^{(n)} \),

\[ H^{(n)}_{ij} = c_n \iiint \left( \frac{\partial N_i}{\partial r} \frac{\partial N_j}{\partial r} \right) + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} + \frac{\partial N_i}{\partial \phi} \frac{\partial N_j}{\partial \phi} \right) \frac{1}{r^2} N_i N_j \; r \; dr \; dz \]  

(9)
where 
\[ c_0 = 2\pi, \quad c_k = \pi, \quad k \geq 1 \]

The assembled set of equations for each harmonic is then of the form
\[ H(n) \varphi(n) = 0 \] (10)
The value of \( \varphi(n) \) at the source node is the appropriate term of the Fourier series
\[ \varphi_g = c \left[ \frac{1}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \cos n\theta \right] \] (11)
where the value of \( c \) is fixed by the requirement that eq.(5) be satisfied at the pool surface.

2.2 Numerical Results

A finite element computer program implementing the analysis outlined above was used to obtain the radial pressure profile for an annular tank shown in Fig. 1. The resultant forces and overturning moment are also shown. A regular mesh of triangular elements with 18 radial divisions and 18 vertical divisions was used. Fifteen harmonics were used to obtain results with four-figure accuracy. It was found that when the inside radius of the annulus was increased to 50 ft that 30 harmonics were required to obtain comparable convergence. These results show that with a centrally located bubble the pressures on the outer wall are higher than on the inner one. As the inner radius is increased keeping the tank width fixed the pressures decrease and the distribution on the walls becomes more nearly equal.

3. Analytical Determination of Vertical and Horizontal Force Resultants

3.1 Derivation of Formulas

Consider a toroidal tank of rectangular cross section as shown in the sketch below with a pressure source (bubble of vanishingly small radius) at a point with coordinates \( r=c, \quad y=d, \quad \theta=0 \). The pressure field satisfies the equation
\[ V^2 p = 0 \] (1)
the boundary conditions
\[ \frac{3p}{3n} = 0 \quad \text{on} \quad S_2, \quad S_3, \quad \text{and} \quad S_4 \]
\[ p = 0 \quad \text{on} \quad S_5 \]
and
\[ \int_{S_3} \frac{3p}{3n} = -\int_{S_1} \frac{3p}{3n} = -4\pi Q \] (5)
The vertical and horizontal force resultants are
\[ F_v = \int_{S_2} p \, ds \] (12)
\[ F_H = \int_{S_4} p \, \cos \theta \, ds - \int_{S_3} p \, \cos \theta \, ds \] (13)
We make use of the fact that if \( \varphi_1 \) and \( \varphi_2 \) are both harmonic in a given region,
\[ \int_S \frac{3\varphi_2}{3n} = \int_S \frac{3\varphi_1}{3n} \] (14)
To calculate the vertical force we find a harmonic function $\phi_v$, say, which satisfies the conditions

$$\phi_v = 0 \quad \text{on} \quad S_5$$

$$\frac{\partial \phi_v}{\partial n} = 0 \quad \text{on} \quad S_3 \text{ and } S_4$$

$$\frac{\partial \phi_v}{\partial n} = 1 \quad \text{on} \quad S_2$$

we then have

$$F_v = \int_S p \frac{\partial \phi_v}{\partial n} \, ds = \int_{S_2} \phi_v \frac{\partial p}{\partial n} \, ds$$

or

$$F_v = \phi_v(c, d, 0) \, 4\pi \, Q$$

(15)

$\phi_v$ is found by inspection to be $y$. Then

$$F_v = 4\pi \, Q \, \phi_v$$

(17)

To calculate the horizontal force a harmonic function $\phi_H$ is required which satisfies the conditions

$$\phi_H = 0 \quad \text{on} \quad S_5$$

$$\frac{\partial \phi_H}{\partial n} = 0 \quad \text{on} \quad S_2$$

$$\frac{\partial \phi_H}{\partial n} = \cos \theta \quad \text{on} \quad S_4$$

and

$$\frac{\partial \phi_H}{\partial n} = -\cos \theta \quad \text{on} \quad S_3$$

Then, as before,

$$F_H = \int_S p \frac{\partial \phi_H}{\partial n} \, ds = \int_{S_3} \phi_H \frac{\partial p}{\partial n} \, ds$$

or

$$F_H = 4\pi \, Q \, \phi_H(c, d, 0)$$

(19)

$\phi_H$ is found by standard methods to be

$$\phi_H = \sum_{n=1}^{\infty} \left( A_n I_1(k_n \, r) + B_n K_1(k_n \, r) \right) \sin \theta \, \cos \theta$$

(20)

where

$$A_n = C_n \left\{ \frac{K_o(k_n) + \frac{1}{k_n} K_1(k_n) - K_o(k_n) - \frac{1}{k_n} K_1(k_n)}{\Delta} \right\}$$

$$B_n = C_n \left\{ \frac{I_o(k_n) - \frac{1}{k_n} I_1(k_n) - I_o(k_n) + \frac{1}{k_n} I_1(k_n)}{\Delta} \right\}$$

(21)

(22)

with

$$\Delta = \begin{vmatrix} I_o(k_n) - \frac{1}{k_n} I_1(k_n) & \left[ K_o(k_n) + \frac{1}{k_n} K_1(k_n) \right] \\ I_o(k_b) - \frac{1}{k_b} I_1(k_b) & \left[ K_o(k_b) + \frac{1}{k_b} K_1(k_b) \right] \end{vmatrix}$$

(23)
and
\[
C_n = \frac{-2n}{\pi^2 n^2} \quad n=1, 3, 5, 7, ...
\]
\[
C_n = \frac{-4n}{\pi^2 n^2} \quad n=2, 6, 10, 14, ...
\]
\[
C_n = 0 \quad n=4, 8, 12, ...
\]  

(24)

3.2 Numerical Results

Vertical force resultants given by eq. (17) agree to two figures with those given by converged finite element solutions. Horizontal force resultants obtained using five terms in eq. (20) have agreed with finite element results to within 3 percent.

4. Estimation of Fluid Structure Interaction

4.1 Derivation of Equations of Motion

Fluid structure interaction effects due to vertical rigid body motion of the pool are investigated by modeling the pool as a rigid, spring supported tank in a rigid building with the building foundation modeled as a two degree of freedom mass spring system as shown in Fig. 2. A dashpot in parallel with the spring supporting the tank provides a percentage of critical damping \( \zeta_c \) while the two degree of freedom model of the foundation is assumed to have modal damping \( \zeta_a \).

The equations of motion for a spherical gas bubble in an incompressible perfect fluid contained in a rigid vessel which is vibrated in the vertical direction have been derived by H. H. Bleich [2]. These results are extended here to apply to model shown in Fig. 2. Following Bleich the expression for the kinetic energy of the tank and the contained fluid can be written
\[
T = \frac{1}{2} (m_L + m_f) \dot{x}^2 + \frac{1}{2} m_b (\dot{x} + \dot{z})^2
+ 2\pi \rho \left( \frac{3}{2} \frac{z^2}{r^2} + \frac{1}{6} \frac{z^2}{r^2} - 2r^2 \dot{z}^2 \right)
\]

(25)

where: \( m_L \), \( m_f \), and \( m_b \) are the masses of the tank, fluid, and gas in the bubble, respectively, \( r \) is the bubble radius, \( z \) is the depth of the bubble below the surface of the fluid with density \( \rho \), and \( x \) is the vertical displacement of the tank. Lagrange's equations of motion for the generalized coordinates \( r \), \( z \), and \( x \) are:
\[
\ddot{r} = \frac{1}{r} \left( \frac{p-p_a}{\rho} - \frac{3}{2} \frac{r^2}{z^2} + \frac{1}{z} \ddot{z} + \frac{1}{4} \dot{z}^2 \right)
\]

(26)

\[
\ddot{z} = \frac{1}{g} \left( -\frac{6}{g} \frac{r}{z} - \frac{3}{2} \frac{r^2}{g} - \frac{6}{g} \frac{r}{z} + \frac{3}{2} \frac{r^2}{g} + \frac{1}{2} \frac{6}{g} \frac{r}{z} \right)
\]

(27)

\[
(m_L + m_f + m_g) \ddot{x} - \frac{4 \pi \rho r^2}{3} \left( \frac{6}{r^2} \frac{z^2}{r^2} + \frac{6}{r^2} \frac{z^2}{r^2} + \frac{3}{2} \frac{z^2}{r^2} \right) = 0
\]

(28)

where:
- \( p \) = gas pressure in bubble
- \( p_s = p_a + \rho g z \)
- \( p_a \) = atmospheric pressure
- \( m \) = gravity
- \( m \) = gravity
- \( m_g = \frac{2}{3} \pi \rho r^3 \)
- \( F_d \) = drag force on bubble
- \( F_d = \frac{1}{2} C_d \pi r^2 \rho |\dot{z}| \)
- \( Q \) = force exerted on the tank by the spring and dashpot connecting to the supporting structure
\[ Q_x = -c \left( \dot{x} - \dot{x}_2 \right) - k(\ddot{x} - \ddot{x}_2) \]

\[ x_2 = \text{displacement of structure} \]

Using eqs. (26) and (27) in eq. (28) gives

\[
\begin{align*}
\left( m_l + m_v + m_H \right) \frac{6 \pi}{r^2} \left( \frac{4 - 5 \pi}{1 + m_H} \right) \dddot{x} + \frac{(2 - m_v)}{1 + m_H} m_v g + 4\pi(p - p_s) r z \\
+ \frac{2 - m_v}{1 + m_H} \frac{m_v}{r} (\ddot{z} + c_d \dot{z}) \ddot{z} - \frac{3 m_v}{r} \left[ \frac{z}{r} \left( r^2 + \frac{1}{2} z^2 \right) + 4 \dot{z} \dot{r} \right] = Q_x
\end{align*}
\]

(29)

The force exerted on the tank by the fluid is

\[
P = m_l \dddot{x} - Q_x
\]

(30)

Substitution of eq. (30) into eq. (29) gives

\[
P = 4\pi(p - p_s) r z - \left[ m_v - m_H \left( \frac{6 \pi}{r^2} \frac{4 - 5 \pi}{1 + m_H} \right) \right] \dddot{x} - \frac{2 - m_v}{1 + m_H} \frac{m_v}{r} (\ddot{z} + c_d \dot{z}) \ddot{z}
\]

\[
- \frac{(2 - m_v)}{1 + m_H} \frac{m_v}{r} \frac{3 m_v}{r} \left[ \frac{z}{r} \left( r^2 + \frac{1}{2} z^2 \right) + 4 \dot{z} \dot{r} \right]
\]

(31)

The differential equation governing the pressure in the bubble is derived on the assumption the air in the bubble expands adiabatically as a perfect gas. This equation can be written

\[
\dot{p} = \gamma \frac{1}{v_b} \left( h_o(t) - \gamma - 1 \rho V_b \right)
\]

(32)

where \( v_b = \frac{4}{3} \pi r^3 \)

\( \gamma = \text{gas constant} \)

\( h_o(t) = \text{enthalpy input rate} \)

The properties of the gas bubble are treated by specifying \( \gamma \) and \( h_o(t) \) as well as the initial values of \( p, r, \) and \( r \).

The building foundation is modeled by a two degree of freedom mass-spring system as shown in Fig. 2. The frequencies of the normal modes of vibration of this system isolated from the building are

\[
\omega_1, 2 = \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{2a}}
\]

in which

\[ a = m_1 m_2 \quad b = -m_1 k_2 + m_2(k_1 + k_2) \quad c = k_1 k_2 \]

The corresponding mode shapes can be written

\[
y^{(1)} = \begin{bmatrix} x_1^{(1)} \ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} 1 - \omega_1^2 m_2 / k_2 \ \omega_1 \end{bmatrix}
\]

\[
y^{(2)} = \begin{bmatrix} x_1^{(2)} \ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} 1 - \omega_2^2 m_2 / k_2 \ \omega_2 \end{bmatrix}
\]

The corresponding generalized masses are

\[
M_1 = m_1 \left( \frac{1 - \omega_1^2 m_2 / k_2}{\omega_1^2} \right)^2 + m_2
\]

\[
M_2 = m_1 \left( \frac{1 - \omega_2^2 m_2 / k_2}{\omega_2^2} \right)^2 + m_2
\]
The equations of motion for the foundation with the building attached now become

\[ \ddot{\phi}_1 + 2 \tau \omega_1 \dot{\phi}_1 + \omega_1^2 \phi_1 = \frac{1}{M_1} \left( c(x - x_2) + k(x - x_2) - m_b \ddot{x}_2 \right) \]

\[ \ddot{\phi}_2 + 2 \tau \omega_2 \dot{\phi}_2 + \omega_2^2 \phi_2 = \frac{1}{M_2} \left( c(x - x_2) + k(x - x_2) - m_b \ddot{x}_2 \right) \]

where

\[ x_2 = \phi_1 + \phi_2 \]

The equations for the modal amplitudes \( \phi_1 \) and \( \phi_2 \) are then

\[ \ddot{\phi}_1 = \left( C_1 (2 \tau \omega_1 \dot{\phi}_1 + \omega_1 \phi_1) + C_2 (2 \tau \omega_2 \dot{\phi}_2 + \omega_2 \phi_2) - M_2 Q \right) / D \]

\[ \ddot{\phi}_2 = \left( C_3 (2 \tau \omega_2 \dot{\phi}_2 + \omega_2 \phi_2) + C_4 (2 \tau \omega_1 \dot{\phi}_1 + \omega_1 \phi_1) - M_1 Q \right) / D \]

in which

\[ C_1 = -M_2 + m_b \omega_1 \]

\[ C_2 = m_b M_2 \omega_2 \]

\[ C_3 = -M_1 + m_b \omega_2 \]

\[ C_4 = m_b M_1 \omega_1 \]

\[ D = M_1 M_2 + m_b (M_1 + M_2) \]

The system of eqs. (26), (27), (29), (32), (33), and (34) can be integrated step by step in time to obtain the motion of the bubble, tank, and building.

4.2 Numerical Results

Numerical results were obtained for two cases with the parameters listed in Table I. The same bubble parameters were used in both cases to give a bubble frequency of about 10 cps. When the effects of fluid structure interaction are neglected, the peak differential bubble pressure is 97.36 psi at a bubble radius of 0.744 ft and a depth below the surface of 16.15 ft. The corresponding vertical force from eq. (31) is 2.23 x 10^6 lb. This is about 5 percent greater than the value given by eq. (17) for the same values of pressure, radius, and depth. The normalized vertical force versus time curve with interaction neglected is shown in Fig. 3.

In the first of the interaction cases the tank frequency with the base fixed, 10.87 cps, is close to the bubble frequency. Interaction effects reduce the peak differential bubble pressure to 59.2 psi at a bubble radius of 0.817 ft and depth of 16.15 ft. If the weight of the fluid is assumed to be 35 percent of the total weight of the tank plus fluid the peak force exerted by the fluid on the tank given by eq. (31) is 1.33 x 10^6 lb, about 65 percent of the peak neglecting interaction. The normalized vertical force exerted by the fluid on the tank for Case 1 is also shown in Fig. 3. The acceleration response spectra for the tank and building showing the effects of interaction are given in Fig. 4. Peak spectral values are substantially reduced when interaction effects are included.

In Case 2 the fixed base tank frequency is increased 15.92 cps. The peak differential bubble pressure is 87.17 psi at a bubble radius of 0.762 ft and depth of 16.15 ft. The peak vertical force from eq. (31) is 1.57 x 10^6 lb assuming the fluid weight is 35 percent of the total. The normalized vertical force versus time curve is shown in Fig. 3 for comparison with the case where interaction is neglected. The acceleration response spectra in Fig. 5 show interaction effects are negligible in this case.

5. Conclusions

The finite element method is useful in finding the peak maximum and minimum pressures on the suppression pool boundaries when interaction effects can be neglected. A decoupled analysis in which these pressures are used to calculate the driving force gives conservative results even at resonance. Interaction affects both bubble pressures and pressures on the pool boundaries.
6. References


<table>
<thead>
<tr>
<th>Table I. Data for Cases 1 and 2</th>
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<tbody>
<tr>
<td><strong>Bubble Parameters</strong></td>
</tr>
<tr>
<td>$p_o = 65.68$ psia</td>
</tr>
<tr>
<td>$p_g = 21.85$ psia</td>
</tr>
<tr>
<td>$r_o = 0.497$ ft</td>
</tr>
<tr>
<td>$v_o = 26.46$ ft/sec</td>
</tr>
<tr>
<td>$h_o = 6300$ Btu/sec,</td>
</tr>
<tr>
<td>$b_o = 0, 0 &lt; t &lt; 0.01587$ sec</td>
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<tr>
<td>$z_o = 16.497$ ft</td>
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<tr>
<td>$Y = 1.4$</td>
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<tr>
<td>Weight of air in bubble = 4 lb</td>
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<tr>
<td>Weight of fluid = 62.4 lb/ft³</td>
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<tr>
<td>$C_d = 0.5$</td>
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<tr>
<td><strong>Model Parameters</strong></td>
</tr>
<tr>
<td>Weight of tank plus fluid =</td>
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<tr>
<td>${3.026 \times 10^6 \text{ lb},$</td>
</tr>
<tr>
<td>$2.500 \times 10^6 \text{ lb}}$</td>
</tr>
<tr>
<td>Vibrational frequency =</td>
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<tr>
<td>$10.87 \text{ cps}$</td>
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<td>Weight of building =</td>
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<td>$7.50 \times 10^6 \text{ lb}$</td>
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<td>Weights of soil masses $W_i =$</td>
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<td>${7.880 \times 10^6 \text{ lb},$</td>
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<tr>
<td>$8.000 \times 10^6 \text{ lb}}$</td>
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<tr>
<td>$W_2 = 3.310 \times 10^6 \text{ lb}$</td>
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<td>$\tau_t = 0.04$</td>
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<td>$\tau_s = 0.07$</td>
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</table>
\[ F_H = 869.25^K \]
\[ F_V = 3516.2^K \]
\[ M = 51746^K \]
\[ Q = 100 \text{ psi-ft} \]

**Fig. 1. Pressure Profile - Radial Section**
**Fig. 2.** Model for Interaction Analysis

**Fig. 3.** Normalized Vertical Force Exerted by the Fluid on the Tank
Fig. 4. Acceleration Response Spectra - Case 1

Fig. 5. Acceleration Response Spectra - Case 2