CLADDING FAILURE BY LOCAL PLASTIC INSTABILITY

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SUMMARY

Cladding failure is one of the major considerations in analysis of fast-reactor fuel pin behavior during hypothetical accident transients since time, location, and nature of failure govern the early post-failure material motion and reactivity feedback. Out-of-Pile transient burst tests of both irradiated and unirradiated fast-reactor cladding show that local plastic instability, or bulging, often precedes rupture. The time and location of failure is then determined by conditions favorable to bulge formation. Furthermore, the extent of the local instability limits the initial rip length and the initial area available for motion of fuel and fission products from the pin.

To investigate the details of cladding bulging, a perturbation analysis of the equations governing the large deformation of a cylindrical shell has been developed. The overall deformation history is assumed to consist of a small perturbation ε of the radial displacement superimposed on large axisymmetric displacements. The parameter ε, which describes the bulge geometry, is in general a function of the azimuthal and axial coordinates θ and z as well as time t. Furthermore, at the temperature of interest, it is assumed that the internal loads are carried primarily by membrane forces. The only relevant equilibrium equation then comes from the balance of forces perpendicular to the middle surface. Substitution of the deformation kinematics into this equation and into the constitutive equations, and neglecting higher order terms in ε, yields a single linear partial differential equation for the bulge geometry. The dependence of ε on the geometric variables is eliminated by assuming solutions that are trigonometric polynomials in θ and z. This gives a first order differential equation which is easily solved numerically for the bulge amplitude at any time.

Computations have been carried out using high temperature properties of stainless steel in conjunction with various constitutive theories, including a generalization of the Endochronic Theory of Plasticity in which both time-independent and time-dependent plastic strains are modeled. Although the results of the calculations are all qualitatively similar, it appears that modeling of both time-independent and time-dependent plastic strains is necessary to interpret the transient burst test results.

Sources for bulge formation that have been considered include initial geometric imperfections and thermal perturbations due to either eccentric fuel pellets or non-symmetric cooling. Of these, only the first is relevant to out-of-pile burst tests. Here we have found that the most likely imperfection that will grow unstably to failure leads to a bulge around half the circumference with an axial length of 1.1 times the deformed diameter. This is in general agreement with burst test results. For the case of in-reactor fuel pins, we have also found that thermal effects can also significantly effect local instability, particularly if the deformation process is thermally activated with a high activation energy. Although the 60° periodicity associated with coolant perturbations in a hexagonal fuel pin array does not lead to bulge formation, eccentric fuel pellets do. However, subsequent bulge growth adjacent to the hot-spot is slowed by the increase in the thermal gap resistance.

Work supported by the U.S. Energy Research and Development Administration.
1. Introduction

The response of cladding to mechanical loads is one of the major considerations in analysis of fuel behavior during hypothetical accident transients in fast breeder reactors. Failure of cladding under these loads is important in both overpower and undercooling accident sequences. In the overpower accident, mechanical loads lead to cladding failure allowing the initial release of fuel and fission gas from the pin. The release of fuel and its subsequent motion can lead to early neutronic shutdown of the reactor if the location of failure is favorable - such that fuel motion is away from the core midplane – or to an energetic excursion if the location of failure is unfavorable. On the other hand, cladding failure in the high power subassemblies in an undercooling situation usually results from cladding melting. However under some conditions, reactivity feedbacks due to sodium voiding and cladding relocation in undercooled subassemblies can lead to severe power excursions in other subassemblies not yet voided. Behavior of fuel in these subassemblies is expected to be similar to that in the overpower accident.

Loading of the cladding in an overpower accident may result from differential thermal expansion of fuel and cladding, from transient fuel swelling, from molten fuel expansion, or from pressure of fission gas initially in the porosity or released from grains during the transient. The present analysis addresses the response of cladding to those loads that act hydrostatically. This type of loading is thought to be most representative of conditions prior to cladding failure during transient overpower conditions [1]. The analysis is also directly applicable to recent experiments [2] on both fresh and irradiated stainless steel PWR cladding in which gas pressure loading was used. These transient burst experiments have shown that high temperature failure of cladding is often by local plastic instability, or bulging. It is this mode of failure that is the subject of this paper.

2. Basic Equations

2.1 Bulge Deformation Kinematics

The cladding is modeled as a thin cylindrical shell of radius \( a_o \) and thickness \( b_o \) subjected to an internal pressure \( P \). The middle surface of the shell in the undeformed state is described by \( r = a_o \), where \( (r, \theta, z) \) is the cylindrical coordinate system shown in Fig. 1a along with the associated unit vectors \( (\hat{\mathbf{r}}, \hat{\mathbf{\theta}}, \hat{\mathbf{z}}) \).

During deformation, at some time \( t \), a material particle which occupied coordinates \( \theta_o, z_o \) on the undeformed middle surface will now occupy the spatial position given by

\[
\mathbf{r} = r \hat{\mathbf{r}} + z \hat{\mathbf{z}}
\]

(1)

where

\[
r, \theta, z = f_1(\theta_o, z_o, t), f_2(\theta_o, z_o, t), f_3(\theta_o, z_o, t)
\]

(2)

With this notation, \( \theta_o \) and \( z_o \) serve both as labels for material particles and as parameters to characterize the deformed middle surface. Once \( f_1, f_2, \) and \( f_3 \) in eqs. (2) are known, properties of the deformation of the shell, such as stretching and curvature change, can be determined.

In order to proceed further, the kinematics are simplified by assuming that the deformation consists of a local perturbation, or bulge, superimposed on an axisymmetric deformation which transforms the undeformed cylinder of radius \( a_o \) into another cylinder of radius \( a(t) \), as shown in Figures 1a and 1b. It is further assumed that the overall deformer...
mation is plane in the axial direction and that the radial displacement is much larger than the tangential displacement. Equations (2) can then be reduced to

\[
\begin{align*}
    r &= a(t) + \epsilon \tilde{r}_o \theta_0, \tilde{z}_o \epsilon(t) \\
    \tilde{\theta} &= \tilde{\theta}_0 \\
    \tilde{z} &= \lambda(t) \tilde{z}_o
\end{align*}
\]  

(3)

where \( \epsilon \) is the perturbation to the cylindrical geometry.

We next consider the geometry of a differential element of the deformed middle surface as shown in Fig. 2. The covariant base vectors \( \tilde{\tilde{r}}_1 \) and \( \tilde{\tilde{r}}_2 \), which lie on the surface along the \( \tilde{\theta}_0 \) and \( \tilde{z}_0 \) surface coordinates, are defined by

\[
\tilde{\tilde{dr}} = \frac{\partial \tilde{r}}{\partial \tilde{\theta}_0} d\tilde{\theta}_0 + \frac{\partial \tilde{r}}{\partial \tilde{z}_0} d\tilde{z}_0 = \tilde{e}_1 d\tilde{\theta}_0 + \tilde{e}_2 d\tilde{z}_0
\]  

(4)

The separation \( d\tilde{s} \) of any two neighboring points is then

\[
d\tilde{s}^2 = d\tilde{r} \cdot d\tilde{r} = E d\tilde{\theta}_0^2 + 2F d\tilde{\theta}_0 d\tilde{z}_0 + G d\tilde{z}_0^2
\]  

(5)

where \( E, F, \) and \( G \) define the first fundamental magnitudes of the surface. Substitution of eqs. (3) and (4) into eq. (5) shows that

\[
\begin{align*}
    E &= a^2 + 2a\epsilon \\
    F &= 0 \\
    G &= \lambda^2
\end{align*}
\]  

(6)

plus higher order terms of the order \( |\epsilon|^2 \).

From eqs. (4) and (5), the surface area \( d\tilde{A} \) and the unit normal \( \tilde{n} \) of the differential element shown in Fig. 2 are given by

\[
d\tilde{A} \tilde{n} = \tilde{e}_1 \times \tilde{e}_2 d\tilde{\theta}_0 d\tilde{z}_0 = \sqrt{E - \tilde{F}^2} d\tilde{\theta}_0 d\tilde{z}_0 \tilde{n}
\]  

(7)

The curvature of the deformed middle surface is described by the second fundamental magnitudes \( L, M, \) and \( N \). Substitution of eqs. (3) into the definitions gives

\[
\begin{align*}
    L &= -\frac{\tilde{e}_1}{\tilde{\theta}_0} \cdot \tilde{n} = a + \epsilon - \frac{a^2}{\tilde{\theta}_0} \epsilon \\
    M &= -\frac{\tilde{e}_1}{\tilde{z}_0} \cdot \tilde{n} = -\frac{\tilde{e}_2}{\tilde{\theta}_0} \cdot \tilde{n} = -\frac{a^2}{\tilde{\theta}_0} \epsilon \\
    N &= -\frac{\tilde{e}_2}{\tilde{z}_0} \cdot \tilde{n} = -\frac{a^2}{\tilde{\theta}_0} \epsilon
\end{align*}
\]  

(8)

plus terms of the order \( |\epsilon|^2 \).

2.2 Equilibrium

Mechanical equilibrium of the cladding in the absence of bulging (\( \epsilon = 0 \)) is maintained by membrane forces. We assume here that the situation is not significantly different in the perturbed geometry in that the hydrostatic loading is still carried primarily by membrane forces and it is these forces that determine whether the bulge will grow unstably and lead to local failure. Since bulge growth occurs at high temperatures where signifi-
cant plastic flow is possible, this assumption appears reasonable. The only relevant
equilibrium equation then comes from the summation of forces normal to the middle surface.

The membrane stress \( \sigma \) acting on a differential shell element of thickness \( h \)
whose middle surface is shown in Fig. 2 can be written in dyadic form as
\[
\sigma = \sigma_0 \epsilon_{ij} \epsilon_i \epsilon_j \quad i,j = 1,2
\]
where summation is implied in tensor equations and indexes 1 and 2 refer to \( \theta_o \) and \( z_o \),
respectively. The membrane force resultants (force/unit length) \( \overline{F}_1, \overline{F}_2 \) acting on the faces
of the differential element are then given by
\[
\overline{F}_1 = h \epsilon_1 \frac{\epsilon_i}{|\epsilon_i|} \quad \text{(no sum)}
\]
Here \( \epsilon_1 \) and \( \epsilon_2 \) are the contravariant base vectors defined to be orthonormal to \( \epsilon_2 \) and \( \epsilon_1 \)
as shown in Fig. 2.

Besides the forces \( \overline{F}_1 \) and \( \overline{F}_2 \), the only other net force acting on the shell element
comes from the pressure \( P \). This force \( \overline{F}_n \) is simply
\[
\overline{F}_n = P \epsilon_n
\]
We now sum all of the vector forces shown in Fig. 2 to satisfy equilibrium;
\[
\overline{F}_n + \frac{\partial \overline{F}_1}{\partial \theta_o} d\theta_o + \frac{\partial \overline{F}_2}{\partial z_o} dz_o = 0
\]
The only component of this equation that is of interest is the normal component, which
can be found by taking the inner product of eq. (12) with the unit normal \( \epsilon_n \). Substituting
eqa. (10) and (11) into eq. (12), and making use of eqs. (8) gives
\[
L \epsilon_1^{11} + 2M \epsilon_1^{12} + N \epsilon_2^{22} = \frac{P}{h}
\]
This result can be expressed in more familiar form by introducing the physical
components of the stress tensor \( \sigma^{(1)} \) referred to the unit vectors \( \epsilon_i / |\epsilon_i| \), since it is
these components that have the physical units of stress (force/length\(^2\)). The right physical
components \( \sigma^{(1)} \) are defined in dyadic notation by
\[
\sigma = \sigma^{(1)} \epsilon_i \epsilon_j \frac{|\epsilon_i|}{|\epsilon_j|} \quad i,j = 1,2
\]
If for instance, the bulge parameter \( \epsilon \) is zero, eq. (13) reduces to the familiar result for
the hoop stress in a cylindrical shell
\[
\sigma^{(1)}_{\theta \theta} = \frac{P}{h}
\]
where the subscript "\( \theta \)" is used to denote conditions when there is no perturbation. It is
also noted here that from elementary considerations of a cylindrical shell under internal
pressure
\[
\sigma^{(1)}_{\theta \theta} = 0
\]
If the ends of the cylinder are capped, \( \sigma_{za} \) is just half the hoop stress given by eq. (15).

Consider now the equilibrium equation (13) for the perturbed geometry. The stress \( \bar{\sigma} \) will differ from the unperturbed stress \( \sigma_a \) by an amount \( \sigma_e \), which must tend to zero as \( \varepsilon \) tends to zero, or

\[
\bar{\sigma} = \sigma_a + \sigma_e
\]

Substituting these stress perturbations and \( L, M, \) and \( N \) from eqs. (8) into eq. (13) and neglecting terms of the order \( |\varepsilon|^2 \), gives

\[
\sigma_e^{(1)} = \frac{p \varepsilon}{h} + \frac{P h \varepsilon}{h^2} + \frac{F}{h} \frac{\partial^2 \varepsilon}{\partial z^2} + \frac{\sigma_a}{h} \frac{\partial^2 \varepsilon}{\partial z^2}
\]

(18)

where the zeroth order approximations have been eliminated by using eqs. (15) and (16). The subscript \( \varepsilon \) is used here to denote the first order approximations. It is worth noting that only the hoop-stress perturbation gives a first order contribution to the equilibrium of the shell. The first two terms on the right-hand side of eq. (18) represent a hoop-stress perturbation due to the change in radius and wall thickness, while the second two terms are the contributions due to the curvature changes.

2.3 Strain-Displacement Relationships

The Lagrangian strain tensor \( E_{k\ell} \) serves as a finite strain measure of the relative deformation of neighboring material particles. Considering only membrane strains, \( E_{k\ell} \) is defined by

\[
ds^2 - ds_o^2 = 2E_{k\ell} \frac{dx^k}{dx^l}, \quad k, \ell = 1, 2;
\]

(19)

where \( ds \) and \( ds_o \) are the separation of neighboring material particles in the deformed and undeformed state, and \( (x^1, x^2) \) represent the convected coordinates \( (\varrho^0, z^0) \). Substituting eqs. (5) and (6) into eq. (19) and neglecting terms of the order \( |\varepsilon|^2 \) gives

\[
2E_{11} = \varepsilon^2 - \varepsilon_0^2 + 2ae
\]

\[
2E_{12} = 2E_{21} = 0
\]

\[
2E_{22} = \lambda^2 - 1
\]

(20)

Equations (20) show that to the first order of approximation, there is no shear distortion of the middle surface. It is therefore possible to introduce the more familiar engineering large strain measure \( \varepsilon^{(1)} \) - the so-called "true" strain - defined as the natural logarithm of the stretches \( ds/ds_o \) along the coordinate directions. These ratios can also be determined from eqs. (5) and (6). Again neglecting terms of the order \( |\varepsilon|^2 \),

\[
\varepsilon^{(1)} = \varepsilon_\varrho_0 = \ln \left( \frac{\varrho^2}{\varrho_0^2} \right) + \frac{\varepsilon}{\varrho} - \frac{\varepsilon_\varrho}{\varrho_0}
\]

\[
\varepsilon^{(2)} = \varepsilon_{z} = \ln \lambda
\]

(21)
The third principal strain \( \varepsilon^{(3)} \) perpendicular to the middle surface is simply

\[
\varepsilon^{(3)} = \ln \left( \frac{h'_{a}}{h_{o}} \right) + \frac{h_{c}}{h_{a}} - \frac{h_{o}}{h_{o}}
\]  

(22)

Here the subscript "o" refers to the stress-free undeformed state and the terms involving \( \varepsilon_{o}(0, 0, z_{o}) \) and \( h_{o}(0, 0, z_{o}) \) allow for initial imperfections of the geometry.

A further simplification to the strain-displacement relationships can be made if the deformation conserves volume. Equations (6) and (7) show that in this case

\[
\begin{align*}
    h_{a} &= \frac{h_{a}}{\delta^{2}} \\
    h_{c} &= -\frac{h_{a}}{\delta^{2}}
\end{align*}
\]

(23)

2.4 Constitutive Equations - An Example

In this section, the perturbation analysis is applied to cladding whose constitutive equation is assumed to have a particular form, although many of the results will be of a more general nature. The analysis for a different set of constitutive equations is a straightforward exercise. In fact, since the perturbation equations are always linear, the most difficult part of the problem is the determination of the zeroth-order approximation for the unperturbed stresses and strains in a perfect cylindrical shell.

It is assumed here that the cladding material is incompressible and obeys the Prandtl-Reuss flow equations, and that the elastic contributions to the strains are negligible. The differential strain increment \( \Delta \varepsilon^{(1)}(j) \) is then

\[
\Delta \varepsilon^{(1)}(j) = \frac{3}{2} S^{(1)}(j) \frac{\Delta \sigma}{\sigma} \quad i, j = 1, 2, 3
\]

(24)

where \( S^{(1)}(j) \) is the deviatoric component of the stress and \( \Delta \sigma \) and \( \Delta \varepsilon \) are the equivalent stress and equivalent strain increment. (See, for instance, [3]).

It is further assumed that the temperature of the cladding is sufficiently high for time-dependent plastic strains to be dominant, and that these strains can be represented by a power-law function of the form

\[
\frac{\Delta \varepsilon}{\Delta t} = C \sigma^{n}
\]

(25)

Here \( C \) and \( n \) are material constants, of which only \( C \) is assumed to have a strong temperature dependence. Substitution of eq. (25) into eq. (24) gives

\[
\Delta \varepsilon^{(1)} \frac{\Delta t}{\Delta t} = \frac{3}{2} C \sigma^{-n-1} S^{(1)}(j) \quad i, j = 1, 2, 3
\]

(26)

The constitutive equation (26) can be reduced to a zeroth-order approximation and a first-order perturbation in the same manner that the equations in the preceding sections were reduced. The zeroth-order equation is obviously
\[
\frac{de_a^{(1)}}{dt} = \frac{3}{2} C_a \frac{n-1}{s_a^{(1)}} s_a^{(j)} \quad 1, j = 1, 2, 3 \tag{27}
\]

where again the subscript "a" refers to quantities when \( e = 0 \). Substituting the zeroth-order stresses from eqs. (15) and (16), and the zeroth-order contributions to the strain from eqs. (21), (22), and (23) into eq. (27) yields the following two nonlinear differential equations,

\[
\frac{1}{a} \frac{ds}{dt} = C \left[ \frac{Pa^2}{a_o b_o} - \frac{1}{2} \sigma_a \right] \cdot \frac{n-1}{s_a^{(1)}} \tag{28}
\]

\[
\frac{1}{\lambda} \frac{d\lambda}{dt} = C \left[ \sigma_a - \frac{1}{2} \frac{Pa^2}{a_o b_o} \right] \cdot \frac{n-1}{\sigma_a} \tag{29}
\]

The zeroth-order displacements \( s(t) \) and \( \lambda(t) \) can be determined from these equations by numerical integration.

We now turn to the analysis of the stresses and strains in the perturbed geometry. The constitutive relationship between the strain perturbation \( \varepsilon_a^{(1)} \) and the stress perturbation \( S_a^{(1)} \) can be found by substituting the stress from eq. (17) into eq. (26). Neglecting higher order terms leads to

\[
\frac{de_a^{(1)}}{dt} = \frac{3}{2} C \left[ \frac{1}{s_a^{(1)}} \left( \frac{n-1}{s_a^{(1)}} S_a^{(j)} \sigma_a^{(j)} + \frac{n-1}{s_a^{(1)}} S_a^{(j)} \varepsilon_a^{(j)} \right) \right] + \frac{3}{2} C \frac{1}{\sigma_a^{(1)}} \frac{n-1}{s_a^{(1)}} S_a^{(j)} \varepsilon_a^{(j)} \quad 1, j = 1, 2, 3 \tag{30}
\]

where

\[
\frac{\sigma_a}{\varepsilon_a} = \frac{3}{2} \frac{1}{\sigma_a^{(1)}} \left[ \frac{S_a^{(1)} \sigma_a^{(1)} e^{(1)} + S_a^{(2)} \sigma_a^{(2)} e^{(2)}}{e^{(1)}} \right] \tag{31}
\]

and \( C_a \) is the perturbation in \( C \) due to small nonuniformities in the temperature.

Substituting the first order contributions from eqs. (21), (22), and (23) for the strain perturbations in eq. (30), and combining the results, gives an equation for the bulge displacement \( e/a \) of the following form:

\[
\frac{d}{dt} \left( \frac{e}{a} \right) = \frac{3}{2} C \left[ T_{1a} \sigma_a^{(1)} e^{(1)} + \frac{3}{2} C \right] \tag{32}
\]

Here \( T_{1a} \) and \( T_{2a} \) are rather lengthy expressions involving the unperturbed stress which can easily be found from eq. (27), once eqs. (28) and (29) have been solved.

The remaining equation needed to calculate the bulge displacement comes from the equilibrium equation. Substituting eq. (23) into eq. (18) gives

\[
\sigma_a^{(1)} e^{(1)} = \frac{2P_a}{a_o b_o} e/a + \frac{P_a}{a_o b_o} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a
\]

\[
\sigma_a^{(1)} e^{(1)} = \frac{2P_a}{a_o b_o} e/a + \frac{P_a}{a_o b_o} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a
\]

\[
\sigma_a^{(1)} e^{(1)} = \frac{2P_a}{a_o b_o} e/a + \frac{P_a}{a_o b_o} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a
\]

\[
\sigma_a^{(1)} e^{(1)} = \frac{2P_a}{a_o b_o} e/a + \frac{P_a}{a_o b_o} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a
\]

\[
\sigma_a^{(1)} e^{(1)} = \frac{2P_a}{a_o b_o} e/a + \frac{P_a}{a_o b_o} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a
\]

\[
\sigma_a^{(1)} e^{(1)} = \frac{2P_a}{a_o b_o} e/a + \frac{P_a}{a_o b_o} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a
\]

\[
\sigma_a^{(1)} e^{(1)} = \frac{2P_a}{a_o b_o} e/a + \frac{P_a}{a_o b_o} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a
\]

\[
\sigma_a^{(1)} e^{(1)} = \frac{2P_a}{a_o b_o} e/a + \frac{P_a}{a_o b_o} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a + \frac{\lambda}{2} \frac{\lambda}{2} e/a
\]
Elimination of $\sigma_{(1)}^t$ between eqs. (32) and (33) yields a single linear differential equation for $e/a$.

3. Results and Discussion

3.1 Material Properties

In order to illustrate certain features of the bulging analysis presented in the preceding sections, it is desirable to choose a representative set of material properties. In general, cladding conditions for which the constitutive equation (25) is applicable are those of high temperature and moderate strain rates. We have chosen here a set of data from D. F. Atkins' work [4] on the stress-rupture behavior of stainless steel tubes under internal pressure. The particular results we have used are for unirradiated type 316 stainless steel with 10 to 15% cold work. Fitting the two material constants $C$ and $n$ in eq. (25) to Atkins' curves of stress versus strain rate leads to the constants given in Table I. The data base for these constants covers temperatures from 538 to 760°C and strain rates from $10^{-6}$ to $10^{-3}$ hrs$^{-1}$. Although these strain rates are very low for application to accident analysis, comparison of Atkins' data with high strain rate tensile test data of J. M. Steinchen [5] shows that Atkins' results can probably be extrapolated up to $10^{-2}$ sec$^{-1}$ (36 hrs$^{-1}$) at the highest temperature (760°C). Beyond this point, the mechanical behavior rapidly becomes independent of strain rate as work-hardening processes begin to dominate. Of course for cladding temperatures above 760°C, the strain-rate regime for which eq. (25) is applicable is correspondingly greater.

In the present analysis, the onset of unstable bulge growth is coincident with the onset of unstable growth of either of the unperturbed parameters $a$ or $\lambda$. If only the time to failure is of interest, and if the failure strains are large, eqs. (28) and (29) can be integrated over the loading history to the point where either $a$ or $\lambda$ tends to arbitrarily large values. Due to the extremely rapid growth of strains near failure, the resulting time to failure is insensitive to the actual value of $a$ or $\lambda$ chosen. For the case of constant-pressure loading of closed tubes at constant temperature, eqs. (28) and (29) can easily be solved. Integrating these equations shows that the axial deformation $\lambda$ remains constant—equal to $1$, while the radius $a(t)$ tends to infinity when time $t$ reaches

$$t = \frac{a^n}{3^n \frac{n+2}{2} \left[ \frac{P_a}{P_{o}} \left( \frac{a}{b_0} \right)^n \right]}$$

This equation, with the material constants from Table I, predicts times to rupture for Atkins' data (symbols) as shown by the curves in Fig. 3. Although the comparison is surprisingly good, it must be pointed out that the parameters in Table I are based on strain rates averaged over the life of each specimen. It is therefore impossible to distinguish between behavior governed by eq. (25) and a more general constitutive equation of a form where the right-hand side of eq. (25) is multiplied by a material function $G(t)$. It is interesting to note, however, that equations of this form are often employed to describe creep behavior where both primary creep and steady-state creep are important. It should also be noted that the deviation of the high temperature data from the curve in Fig. 3 is real, as shown in other results from Atkins' report. This deviation probably represents a change from failure by plastic instability to failure by grain-boundary sliding at
longer times.

3.2 Bulge Behavior

The behavior of the bulge displacement \( \varepsilon \) is governed by eqs. (32) and (33). For the moment we consider only the case where the temperature is uniform so that \( C_e = 0 \). The spatial dependency of \( \varepsilon \) can be eliminated from the equations by assuming a solution of the form

\[
\varepsilon/a = B(t) \cos(m \theta \pi \theta / L_o) \cos \left( \frac{m \pi \theta}{L_o} \right), \quad m = 0,1,2, \ldots
\]  

(35)

where \( L_o \) is the length of the bulge as shown in Fig. 1b. Equation (35) can be viewed as either representing the growth of a single imperfection in the geometry with initial amplitude \( B(0) = B_o \), or as a Fourier component of a more general imperfection. Since the governing equations for \( \varepsilon/a \) are linear, solutions of the form of eq. (35) can then be superimposed.

Substituting eq. (35) into eqs. (32) and (33) gives

\[
\frac{dB}{dt} = 3C T_{1a} (t) \left[ \frac{Pa^2}{\sigma_o h_o} \right] \left[ 1 - \frac{1}{2} \frac{a_o^2 \sigma_o}{2P a^2 \sigma} + \frac{a_o^2 \sigma}{\lambda^2 \sigma^2 / \alpha} \right] B
\]  

(36)

Equation (36) determines the ratio of the amplitude of the bulge to the radius \( a(t) \) of the uniform sections of the deformed cladding. Although this equation was derived for a particular constitutive equation, it should be noted that the term inside the square brackets will appear regardless of the constitutive equation since it comes from the equilibrium conditions eq. (33). In the present analysis, the sign of the term governs whether \( dB/dt \) in eq. (36) is positive or negative - that is, whether or not the amplitude of an initial geometric imperfection will grow or shrink as the cladding deforms.

For pressure loading of a closed tube, \( \sigma_a \) is just \( 1/2 \) the nominal hoop stress given by eqs. (15) and (23). In this case \( \lambda = \text{constant} = 1 \). Substituting these results into eq. (36) shows that the only values of \( m \) that will lead to bulge growth are \( 0 \) and \( 1 \). The \( m = 0 \) case represents a bulge around the entire circumference while the \( m = 1 \) case represents a bulge around half of the circumference. If it is assumed that the smallest geometric imperfection that will grow provides the most likely nucleus for the bulge, eq. (36) gives \( m = 1 \) and \( L_o/a \approx 2.22 \). This corresponds to a bulge on one side of the cladding with a length of 1.11 times the deformed diameter. It is encouraging to note that this shape is consistent with the bulge formation that has been reported for burst tests of type 316 stainless-steel cladding [2].

We have written a small computer program to solve eqs. (28), (29), and (36) numerically. Figures 4, 5, and 6 show results for dimensionless radial, axial, and bulge displacements using the material properties for cladding at 760°C from Table I. \( \lambda \) is defined by eq. (33) while \( \varepsilon \) and \( F \) are defined by

\[
\dot{\varepsilon} = a(t)/a_o, \quad \varepsilon = B(t)/B_o
\]  

(37)

All three figures use \( a_o = 2.73 \) mm, \( h_o = .381 \) mm, \( P = 19.26 \) MPa and assume \( m = 1 \).
Figure 4 shows the behavior of a closed tube for \( L_o = 10.0 \text{ mm} \). \( \lambda \) remains constant while the unperturbed radius grows unstably at \( t = 38.9 \text{ hrs} \). This is the same time as given by eq. (34). The bulge displacement is also seen to grow. Figure 5 shows the behavior under similar conditions except that the initial imperfection is assumed to have a length \( L_o = 5.0 \text{ mm} \). The relative amplitude of this imperfection is seen to shrink in this case as the tube deforms.

Figure 6 shows the deformation behavior when the axial stress \( \sigma_{2A} \) is zero. The other conditions are the same as in Fig. 5. This type of loading, which may be more typical of cladding loading during overpower transients, considerably reduces the predicted failure time.

### 3.3 Temperature Perturbations

The effect of temperature perturbations on the behavior of local plastic instability enters the analysis through the perturbation \( C_p \) of the material constant \( C \). The differences in the values of \( C \) in table 1 suggest that very small temperature variations can lead to a significant value for \( C_p \). In fact, it is usually assumed that \( C \) can be represented by an exponential Arrhenius-type equation

\[
C = C_o \exp\left(-\frac{T^*}{T}\right)
\]  

(38)

with an activation temperature \( T^* \) approximately equal to the activation temperature for self-diffusion. Unfortunately, since the parameter \( n \) in table 1 also varies somewhat with temperature, it is impossible to determine a meaningful value for \( T^* \) from the data. We choose instead to assume that \( T^* \) is equal to the activation temperature for self-diffusion - 33,700 Kelvin. For small temperature variations \( T_c(\theta, \phi, \sigma_0, \zeta) \) about the mean temperature \( T_a \), eq. (38) shows that

\[
C_p = C \frac{T^*}{T - T^*} \frac{T^2}{T_a^2}
\]

(39)

Two sources for cladding temperature perturbations that have been considered are those due to variations in the fuel-cladding gap and those due to local perturbations in the sodium cooling. Two-dimensional heat transfer calculations have shown that the temperature perturbations due to variations in the fuel-cladding gap are expected to range from 1 to 5°C around the circumference. Although results are not presented here, it is noted that initial fuel eccentricity provides a source for bulge growth through \( C_p \) in eq. (32). As the bulge grows away from the fuel, however, the resulting decrease in local cladding temperature tends to stabilize the growth.

Larger temperature perturbations in the cladding are caused by local perturbations in the sodium cooling. Numerical calculations by Chuang, et al [6] have shown that temperatures of cladding hot spots due to the hexagonal geometry vary from 1 to 5°C, while those due to spacers wires are typically 10 to 20°C. These temperature perturbations also act as a bulging source that behaves similar to initial geometric imperfections.

Our calculations show that temperature perturbations with 60° hexagonal periodicity cause initial bulge growth, but do not lead to unstable growth. This is because for \( m = 6 \), the term in the square brackets in eq. (36) is negative. On the other hand, temperature perturbations due to features such as wire spacers have Fourier components with \( m \) equal to
0 and 1. Figure 7 shows an example of the bulge behavior for the same conditions as Fig. 1 but with a temperature perturbation of $10^\circ C (T = 10 \cos(\theta_0)\cos(\pi x_0))$. In both figures $B_0$ is 0.02 mm. It is seen from the different scales in these figures that, for the parameters assumed, the effect of the temperature perturbation is much greater than that due to the initial geometric imperfection.

4. Concluding Remarks

A perturbation theory for cladding failure by local plastic instability has been developed here and results have been given for a power-law constitutive equation. However, qualitatively similar results have been found using a form of the more general endochronic theory of plasticity [7], where both time-dependent and time-independent plasticity were modeled. Since both overpower accident analysis and the experimental transient burst experiments may involve high strain rates, it appears that the inclusion of time-independent plastic effects is important in a quantitative analysis of bulge behavior under these conditions.

References


Table I
Stainless Steel Properties
used in Equation (25)

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>n</th>
<th>C  (MPa)⁻ⁿ Hr⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>538</td>
<td>16.37</td>
<td>1.36 x 10⁻⁴⁵</td>
</tr>
<tr>
<td>649</td>
<td>8.59</td>
<td>6.70 x 10⁻²⁴</td>
</tr>
<tr>
<td>760</td>
<td>5.37</td>
<td>1.94 x 10⁻¹⁴</td>
</tr>
</tbody>
</table>

Fig. 1 Cladding geometry; (A) Undeformed; (B) Deformed.
Fig. 2 Differential element of deformed middle surface.

Fig. 3 Biaxial stress-rupture strength of 10-15% cold-worked stainless steel. Data (symbols) from D. F. Atkins [4].
Fig. 4 Dimensionless radial ($\hat{a}$), axial ($\lambda$), and bulge ($\hat{\varepsilon}$) displacements of a closed tube for an initial 0.02 mm amplitude bulge of length 10.0 mm and no temperature perturbation.

Fig. 5 Same as Fig. 4 except with an initial bulge length of 5.0 mm.
Fig. 6  Same as Fig. 4 except for an open tube with no axial force.

Fig. 7  Same as Fig. 4 except with a 10°C temperature perturbation.