STRESS-INTENSITY FACTORS FOR IRRADIATION-EMBRITTLED HEXAGONAL SUBASSEMBLY DUCTS

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SUMMARY

The cumulative effects of fast neutron fluxes on the mechanical properties of stainless steel are complex, but under certain combinations of irradiation and test temperatures, can include the following: (1) the yield strength is increased, (2) the ductility is decreased, and (3) the fracture toughness is decreased. Thus, although the hexagonal subassembly ducts (hexcans) of LMFBR designs are not likely to experience unstable crack propagation in their virgin state, the integrity of a highly-irradiated, cracked hexcan may be threatened under accident loading conditions. In order to be able to analyze the effects of the presence of a crack in a hexcan, plane-strain stress intensity factors have been calculated by a variety of techniques for several crack locations and loading conditions.

In earlier work the case of a crack in a corner of a uniformly pressurized hexcan was studied using curved-beam theory and superposition techniques, and bounds on the stress intensity factor were obtained. In the present work, the results of a finite-element analysis of the same problem are reported, and they support the applicability of beam theory to the hexcan problem. In particular, modelling the hexcan corner as a cracked strip whose crack faces are loaded with the hexcan stress distribution has been found to give very good results for the particular hexcan studied.

The finite-element results for uniformly pressurized hexcans have been used as the basis for computing stress intensity factors for cracks in non-uniformly loaded hexcans, under loading conditions that might simulate those of fission gas released from failed fuel pins or other non-uniform accident loading conditions. The weight function technique employed can be used to study a variety of conditions in a quick and efficient manner, for all that is required is knowledge of the stress intensity factor of the reference problem of the uniformly-loaded hexcan and the stress distribution through the thickness of an unflawed hexcan. This information is used in conjunction with a simple representation for the crack shape, and it is shown how one may thereby compute stress intensity factors for other loading conditions by the evaluation of simple integrals. The technique is illustrated by the problem of a hexcan loaded by concentrated forces at opposite midflats.

A finite-element analysis also has been carried out for the cracked hexcan loaded by midflat forces. The results of the different techniques are compared, and the simple model of a cracked strip loaded with the hexcan stress distribution is found again to agree closely with the finite-element results for the hexcan considered. However, the weight function technique is expected to apply to geometries for which the strip model is inadequate and to hold over a wider range of crack sizes.

The stress-intensity factor calibrations presented in this paper, together with fracture toughness properties of irradiated subassembly material, will enable one to analyze the integrity of cracked hexagonal ducts.

1. Introduction

Nicks, scratches, and other flaws that are inadvertently introduced into otherwise sound structural components may cause significantly higher stresses to develop than otherwise anticipated in the vicinity of the geometrical irregularity, which is often modelled as a crack. If the material possesses sufficient ductility and toughness, local yielding will relieve these high stresses, and the component may continue to function. If, on the other hand, sufficient local yielding cannot occur, because of geometric constraint or material properties, unstable propagation of the crack can occur at otherwise tolerable load levels.

The hexagonal subassembly ducts (hexcans) of LMFBR designs, like all structural elements, are expected to have flaws (cracks) introduced during fabrication, assembly, and handling, and it is natural to ask if the flaws that can reasonably be expected to exist in these components might threaten the integrity of a hexcan under abnormal loading conditions. Although the stainless steel (20% cold-worked Type 316) of which LMFBR hexcans are made is normally a very tough material, the prolonged effects of fast neutron irradiation are expected to include a considerable degradation of a hexcan's ability to resist brittle fracture. Thus irradiation embrittlement may cause a flaw, which posed no threat to the fresh hexcan's integrity, to be a critical crack under abnormal loading conditions that might occur toward the end of the hexcan's life in the reactor core.

Under such conditions it is expected that linear elastic fracture mechanics will apply. This theory involves a mathematically singular stress field at a crack tip, where the strength of singularities is measured by the stress intensity factors K_I, K_{II}, and K_{III}, representing the tensile, in-plane shear, and anti-plane crack-opening modes, respectively. Here we consider only flaws which are long relative to their depth of penetration of the hexcan wall, and we restrict our attention to hexcans of the dimensions given in Fig. 1 and to cracks along inside corners or outside midflats of the hexcan, as suggested in Fig. 2. These are locations of relative maxima in the circumferential tensile stress in hexcan sections loaded by uniform internal pressure, Fig. 3(a), or by non-uniform local loads, such as those represented in Fig. 3(b), extending for some length along the hexcan section. In each of these cases a plane strain elasticity problem results. The corner crack is definitely the worst case problem under the loading conditions of Fig. 3(a), and it can be argued that, under a non-uniform loading, as in Fig. 3(b), a crack in a corner remote from the loading might pose as great a threat to the hexcan's integrity as a midflat crack under the same loading. This latter contention follows from the observation that, e.g., although outside midflat cracks penetrating 10% of the hexcan wall have the same K_I as inside corner cracks that penetrate 15% of the wall, the relative inaccessibility of the corner to inspection and the greater cold-working of the corner material could make it necessary to assume deeper cracks in less tough material at the corner location. Hence we concentrate here on corner cracks. Midflat cracks may be readily considered from standard handbook K_I calibrations (e.g., [1]) for straight beams.

The unique geometry of hexcans is not among those treated in handbooks, however, and it is necessary to calibrate cracked hexcan corners for loadings of interest in reactor safety. As a first approximation to determining K_I for corner-cracked hexcans loaded as in Fig. 3(a), the corner stress distribution was determined by curved-beam theory, and the cracked corner was variously modeled as a ring, a strip, and a C-shaped fracture toughness specimen [2].

Such analysis yielded the bounds on K_I shown in Fig. 4 for a hexcan with a 3 mm wall and a 116 mm flat-to-flat outside dimension. A subsequent finite element analysis has demonstrated that, of the various elementary means of modeling the cracked corner, the best K_I results are given by the cracked strip loaded with the hexcan stress distribution calculated from curved beam theory. Both the finite element results and those based on the cracked strip model are also given in Fig. 4. The curve of stress intensity factors determined by a simple integration based on a handbook calibration for a cracked strip [2] is seen to be a very good approximation for cracks which penetrate deeper than 20% of the haxcan wall and gives a conservative estimate of the stress intensity factors for more shallow cracks.

The finite element results for the uniformly pressurized hexcan have been taken as the reference solution for cracked hexcans loaded by a class of non-uniform loads, such as that shown in Fig. 3(b). From the stress intensity factor calibration provided by the reference problem, a weight function has been constructed which enables one to compute by simple quadratures stress intensity factors for corner cracks in hexcans loaded by any load system distributed symmetrically with respect to the crack plane. Although not all hexcan loadings of interest are symmetric in this way, it is expected that the simplifying assumption of symmetry will give good first approximations, with a minimum amount of computation, to stress intensity factors for other than uniform loading.

The weight function technique is illustrated below by the example of a hexcan loaded by concentrated forces on opposite midflats remote from a cracked corner. Stress intensity factors computed by this means agree well with finite element results for the same problem.

Stress intensity factor calibrations based on cracked strips whose crack faces are loaded by the hexcan corner stress distribution give suprisingly good results, considering the simplicity of the calculation. However, such calibrations are expected to be limited to relatively thin structures, such as the LMFBR hexcan, and to give poor results for thicker sections where bending is not a predominant mode of deformation.

2. Description of the Finite Element Analysis

The finite element method has frequently been used to determine stress intensity factors in linear elastic fracture mechanics. Application of the method has ranged from the use of a very dense mesh of ordinary elements near the crack tip to the use of relatively coarse "singular elements" which are based on the asymptotic near tip displacement field. The first approach is very costly and inefficient, since many extremely small elements must be employed. The second method is much more economical and has been used with success.

Recently, Benzley [3] presented an analysis of crack problems in elastic bodies using isoparametric quadrilateral elements with a singular displacement field near the crack tip. A finite element computer code based on this work, CHILES [4], was obtained to solve the cracked hexcan problem. Stress intensity factors computed with this code appear to be in error by well under 5% when compared with analytic solutions of several test problems.

Several numerical experiments were performed to select a suitable finite element mesh. The grid chosen for one-twelfth of the hexcan is shown in Fig. 5. One-half of the hexcan must be modelled in this fashion in order that the problem of a crack in a single corner may be solved. The total grid for the half hexcan consists of 1700 elements with 3762 degrees of freedom. Stress intensity factors for the uniformly pressurized hexcan as well as the point loaded hexcan were calculated as a function of crack length. Typical program execution time for the cracked hexcan problem was less than two minutes on an IBM 370/195.

3. A Weight Function for the Corner-cracked Hexcan

The finite element results in Fig. 4 have been fit with a fifth order polynomial to serve as the reference K data from which a weight function has been constructed as described below. Stress intensity factors for the cracked hexcan loaded in a non-uniform way, such as in Fig. 3(b), may now be computed by simple quadratures.

If the stress intensity factor K for a corner crack in the uniformly pressurized hexcan of Fig. 3(a) is known as a function of crack length a, then the stress intensity factor K_I for the same hexcan loaded by the non-uniform loading of Fig. 3(b) is given by (e.g., [5,6])

$$K_I = \int_o^a \sigma(x)\ h(a,x)\ dx \tag{1}$$

where σ(x) is the stress distribution across the crack plane in an unflawed hexcan due to the nonuniform loading and where the weight function

$$h(a,x) = \frac{H}{K}\ \frac{\partial u(a,x)}{\partial a} \tag{2}$$

is computed from the reference problem's crack face displacement u, which is a function of both the crack length a and the distance x from the crack mouth. The material constant H is equal to $E/1-\nu^2$ for plane strain.

Excellent results for cracked strips, holes, and rings have been obtained using the simple representation [7]

$$u(a,x) = \frac{\sigma_o}{H\sqrt{2}} \left\{ 4F(\tfrac{a}{L})\ a^{\frac{1}{2}}\ (a-x)^{\frac{1}{2}} + G(\tfrac{a}{L})\ a^{-\frac{1}{2}}\ (a-x)^{3/2} \right\} \tag{3}$$

where σ_o and L are a characteristic stress and length of the problem, and where F is the dimensionless stress intensity factor $K/\sigma_o(\pi a)^{\frac{1}{2}}$. Since Eq. (1) yields an identity for the reference case, when $K = K_I$, the function G was computed in Ref. [7] to be given by

$$G(\tfrac{a}{L}) = [I_1(a) - 4F(\tfrac{a}{L})\ a^{\frac{1}{2}}\ I_2(a)]a^{\frac{1}{2}}/I_3(a) \tag{4}$$

with

$$I_1(a) = \pi\sqrt{2}\sigma_o \int_o^a [F(\tfrac{a}{L})]^2\ a\ da$$

$$I_2(a) = \int_o^a \sigma(x)\ (a-x)^{\frac{1}{2}}\ dx \tag{5}$$

$$I_3(a) = \int_o^a \sigma(x)\ (a-x)^{3/2}\ dx$$

The important advantage of this technique is that K_I may be computed from simple quadratures involving only the stress σ(x) and the dimensionless stress intensity factor F of the reference problem, given in Fig. 4.

Straight- and curved-beam theory may be used to determine σ(x). A comparison of computed stress distributions through the hexcan corner, Fig. 6, shows curved beam theory to be quite adequate for this purpose. These finite element results, from a grid with twenty elements through the hexcan wall, give essentially the same stress distribution as the ten-element grid shown in Fig. 5. It is for this reason, with similar results for the cracked hexcan, that the ten-element grid was considered fine enough in subsequent computations.

The weight function has been used to compute K_I for corner cracks in a hexcan loaded by point forces at opposite midflats, and the results are shown in Fig. 7. These are in good agreement with finite element results for the same problem. Figure 7 also shows results based on modelling the hexcan corner as a cracked strip. All three techniques agree closely with each other for this problem. The finite element technique involves the most computational effort, but is the most versatile. The strip calibration is the simplest to employ, but its applicability is limited to relatively thin structures. The weight function technique, while limited to symmetric loading conditions, appears to be more generally applicable and, when coupled with a reference solution from a finite element analysis, appears to be a most efficient means of computing stress intensity factors for a variety of loading conditions of the same body.

4. Conclusions

Plane strain stress intensity factors for corner-cracked hexcans 116 mm (4.575 in.) across flats and with 3 mm (0.120 in) walls have been determined and corroborated by several independent techniques. These results are expected to be applicable to LMFBR hexcans irradiated to high fluences ($\sim 10^{27}$ n/m^2) whose ductility and fracture toughness K_{Ic} have been reduced and whose yield strength σ_y has been increased to such levels that linear elastic fracture mechanics applies. Generally speaking, these material properties must be altered to an extent that the plastic zone size, which at a crack tip is of the order $(K_{Ic}/\sigma_y)^2$, is considerably smaller than the hexcan wall thickness.

5. Acknowledgements

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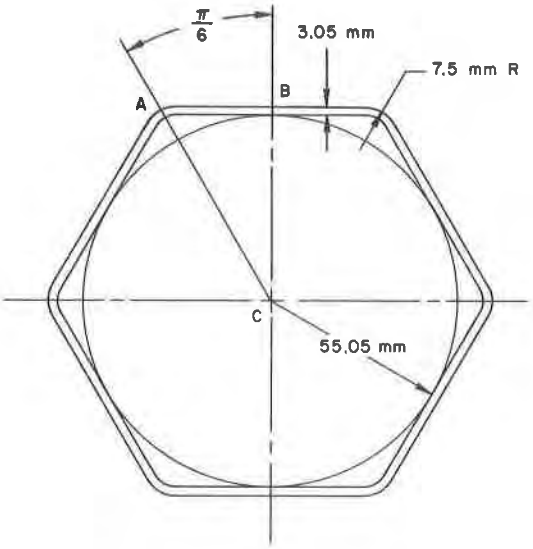

Fig. 1. Cross-section of an LMFBR hexcan, showing dimensions used in computations.

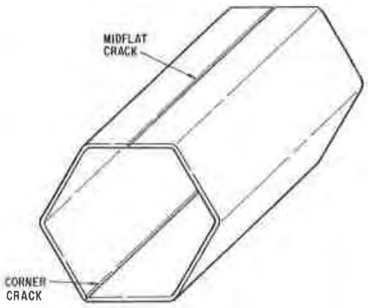

Fig. 2. A length of hexcan with an outside midflat and an inside corner crack.

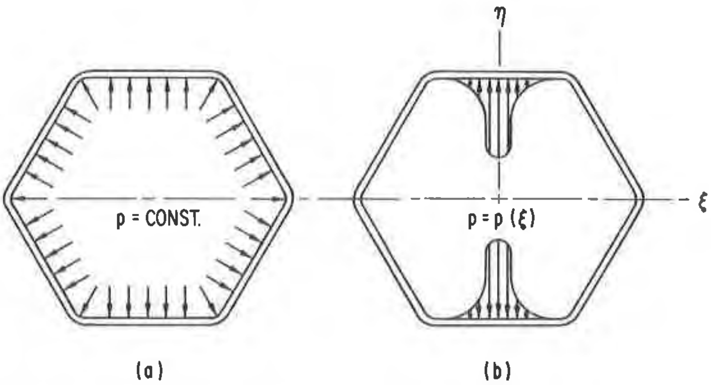

Fig. 3. Two hexcan pressure loadings:
(a) uniformly distributed, and
(b) non-uniformly distributed.

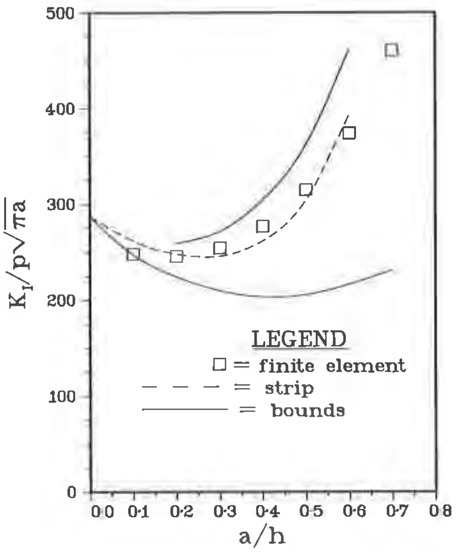

Fig. 4. Dimensionless stress intensity factors for corner cracks of depth a through a hexcan wall of thickness h for the case of uniform internal pressure p.

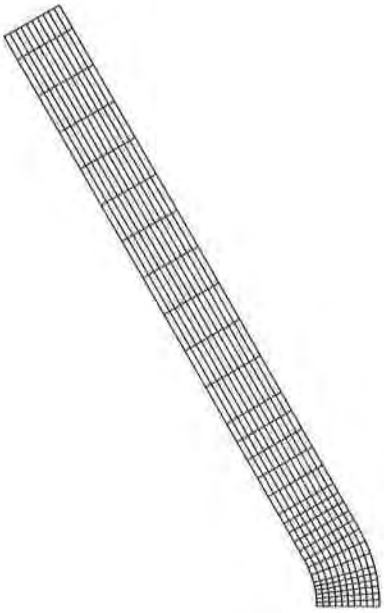

Fig. 5. Finite element grid employed to model one-twelfth of a hexcan.

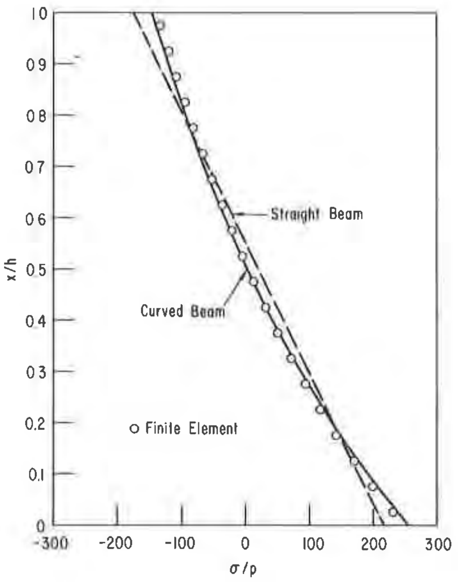

Fig. 6. Comparison of calculated circumferential stress distributions σ(x) through a hexcan wall of thickness h for the case of uniform internal pressure p.

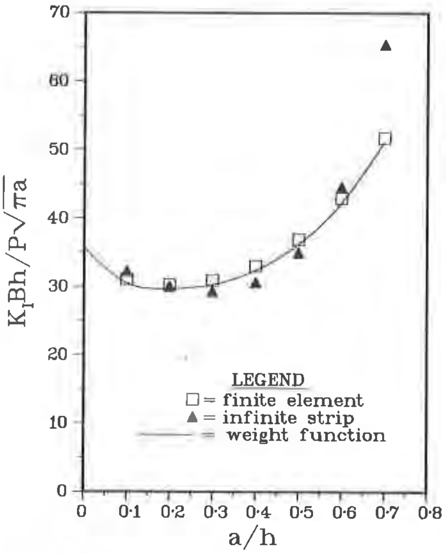

Fig. 7. Dimensionless stress intensity factors for corner cracks of depth a in a hexcan loaded by concentrated forces P/B per unit length along opposite midflats remote from the crack.