DEVELOPMENT OF ADVANCED METHOD FOR FUEL SEISMIC ANALYSIS

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SUMMARY

The method of seismic vibration analysis of PWR fuel (FUVIAN2) has already been reported: SMIRT-2 Paper K 6/10. The method has further been developed later (FUVIAN3) with special concern with grid impact model, because it still had a problem of predicting too conservative grid impact force although displacement prediction was very appropriate. In addition, another computor code FLOAN (Fuel Local Stress Analysis) was developed so that we could analyse internal stresses in the fuel assembly under seismic vibration.

This paper explains briefly about the procedures of fuel seismic evaluation in which the combined FUVIAN and FLOAN methods are applied and then explains about the details of analytical methods of both codes together with various verification tests for them.

On the seismic evaluation of fuel assembly, first the colliding vibration of fuel assemblies is analysed by using FUVIAN and the maximum grid impact force and the worst fuel assembly deformation are obtained. The impact force can be evaluated by comparing it to grid crush strength. Next, for the worst deformation, FLOAN analyses detailed stresses and strains in the fuel components such as fuel rod and control rod guide thimble.

The method of FUVIAN3 is basically the same in governing equations of vibration as of FUVIAN2, but it is essentially improved in impact force calculational model. In FUVIAN2, each fuel assembly was treated as single beam fashion with a pair of impact elements at each grid position. But in FUVIAN3, the fuel assembly is treated as it consists of several sub-beams connected each other with spring network which corresponds to a fuel grid, so that we can deal with the complex grid-rods interaction mechanism in distributed mass manner. The validity of above method was confirmed for both impact force and displacement predictions by full scale fuel assembly impact test.

In FLOAN model, a fuel assembly is idealized such that it consists of elastic beams (fuel rods and guide thimbles) and rigid bodies (nozzles and grids) in which any arbitrary connecting conditions of free, fixed and elastic can be available. The non-linear effect arising from spring and dimple type rod support is treated as an equivalent elastic connection. Firstly, natural frequencies and the corresponding modal shapes are calculated using two-dimensional (axial and lateral) finite element formulations. Then the stresses at each local point of fuel rod and guide thimble are calculated from the modal deformation taking vibrational inertia force into account. From these results and the decomposed modal amplitudes for the worst fuel assembly deformation obtained by FUVIAN, the acutal seismic stresses are calculated. To verify the FLOAN method full scale fuel assembly vibrational test was performed and good agreement was found for both vibrational characteristics and stresses.

1. Introduction

In order to clearify the colliding vibration of fuel assemblies, which might take place in PWR under seismic condition, FUVIAN1 (Fuel Vibration Analysis) was developed. Successibly it was improved as FUVIAN2 [1] so that it could make more realistic and economical analysis. Recently the requirements for fuel seismic analysis has been getting severe and the more advanced method of analysis became necessary. Since FUVIAN2 had been recognized to give too high impact force prediction comparing with the experiment, it was again improved mostly concerning impact calculational model and was named FUVIAN3. Although the code is capable of analizing fuel grid impact forces and the worst vibrational deformations of fuel assembly, it doesn't deal with the stress analysis inside the fuel assembly for the worst deformations. Then FLOAN code was developed to deal with the problem. Now, the seismic design evaluation of PWR fuel can be done utilizing the combined FUVIAN and FLOAN methods and they will be explained in this paper.

2. Improved Method of Colliding Vibration Analysis, FUVIAN3

The basic formulations other than impact model in FUVIAN3 is almost the same as in FUVIAN2, therefore, overall feature or details of the method should be refered to reference [1]. The major points to be mentioned for the improvement are as follows:

(1) Basic equations of motion for fuel assemblies are the same as those of FUVIAN2.

(2) In FUVIAN2 each fuel assembly was treated as a single beam fashion with a pair of impact elements attached at each grid position. But in FUVIAN3, a fuel assembly is treated as it consists of several subbeams connected together with spring net work, so that we can deal with the complex grid-rods interaction mechanism as realistically as possible.

(3) The envelope surface of an actual fuel grid usually have irregularity and this affects to lower the impact spring rate and then impact forces. FUVIAN3 can deal with the above feature.

2.1 Impact model of FUVIAN3

In the impact model of FUVIAN2, all the fuel rods and control rod guide thimbles in a fuel assembly were represented by a equivalent single beam, in which no relative displacement between rod and rod was assumed, therefore the impact stiffness was measured by fixing the inserted rods by rigid frame as seen in Fig.10 of reference [1]. However, actually the fuel grid deforms under impact, so the above assumption is not correct and moreover the grid-rods interaction mechanism is much more complicated in actual condition than was treated in FUVIAN2. Taking the above points into consideration, FUVIAN3 impact model was settled as below.

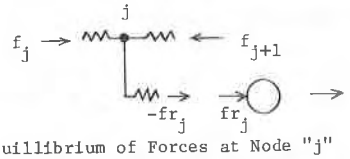
(1) Fuel Rods-Grid Interaction Model in Single Fuel Assembly.

Fig. 1-(a) shows an actual PWR fuel grid and (b) shows the model that describes the x-directional interaction for (a). And (c) shows the model which is modified from (b) for the easier treatment in mathematical calculation. Further, (d) explains that the working spring between a rod and grid differs depending on the direction of force acting to rod. Finally (e) explains that the model (e) is just equivalent to the model (d) if kr is adequately chosen from ks or kd depending on the direction of the force. Model (e) is actually used in FUVIAN3 as grid-rods interacting mechanism in single fuel assembly. But on the engineering stand point an array of rod in a grid could be grouped up to a rather small number, n_d (number of groups of rod), to reduce calculational time, so in practical calculation in FUVIAN3 n_d is taken as less than 6. And the one group of rods corresponds to a subbeam mentioned previously in 2-(2)

(2) Multi-Fuel Assemblies Interaction Model at a Given Grid Location.

Multi-fuel assemblies collision model can easily be derived by extending the previous model and by introducing non-linear spring connections at gap points (gaps between adjacent fuel grids) which work only for contraction, as shown in Fig.2. Fig.2(a) shows the basic model for multi-fuel assemblies interaction and (b) shows the model rearranged from the above for the convenience of mathematical treatment of related forces and displacements. The detailed relationships between the forces and displacements should be refered to Table 1.

(3) Equations for the multi-Fuel Assemblies Interaction Model.



Equillibrium of Forces at Node "j"
Contruction is (+) for f_j
x direction is (+) for fr_j

If we consider the equillibrium of forces at node j referring to the left figure, we will get next relations.

$$f_j - f_{j+1} - fr_j = 0$$

Referring to Table 1 for f_j & fr_j we get

$$(z_{j-1} - z_j - \delta_j)kg_j - (z_j - z_{j+1} - \delta_{j+1})kg_{j+1} - (z_j - x_j)kr_j = 0$$

$$\underbrace{-kg_j\, z_{j-1} + (kr_j + kg_j + kg_{j+1})\, Z_j + kg_{j+1}\, z_{j+1}}_{①} \underbrace{+ kg_j\delta_j - k_{j+1}\,\delta_{j+1}}_{②} \underbrace{- kr_j x_j}_{③} = 0$$

where $z_o = 0$

$z_{n+1} = 0$

In above equations kg and kr are known constants and the value of x_j can be taken from the solution of fuel rod displacement at the end of the previous time step. Then δ can also be known, therefore the terms ② & ③ are known. Taking the above into account we can finally get next type equation for multi-fuel assemblies collision model at a given grid position.

$$\begin{bmatrix} kg_1 + kr_1 & & & & & \\ + kg_2 & - kg_2 & & & & \\ - kg_2 & kg_2 + kr_2 & & & & \\ & + kg_3 & - kg_3 & & & \\ & - kg_3 & & & & \\ & & & - kg_{n-1} & & \\ & & -kg_{n-1} & kg_{n-1} + kr_{n-1} & & \\ & & & +kg_n & -kgn & \\ & & & -kg_n & kg_n + kr_n & \\ & & & & +kr_{n+1} & \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \bullet \\ \bullet \\ \bullet \\ z_n \end{bmatrix} = \begin{bmatrix} - kg_1\delta_1 + kg_2\delta_2 \\ - kg_2\delta_2 + kg_3\delta_3 \\ \bullet \\ \bullet \\ \bullet \\ -kg_n\delta_n + kg_{n+1}\delta_{n+1} \end{bmatrix} + \begin{bmatrix} kr_1x_1 \\ kr_2x_2 \\ \bullet \\ \bullet \\ \bullet \\ kr_nx_n \end{bmatrix}$$

or simply

$$[K_G]\{Z\} = \{D\} + [\,K_R]\,V \qquad \text{.....} \quad (1)$$

$$\{Z\} = (z_1, z_2, \ldots\ldots, z_n)^T$$

$$\{V\} = (x_1, x_2, \ldots\ldots, x_n)^T$$

$$\{D\} = (d_1, d_2, \ldots\ldots, d_n)^T$$

$$d_j \equiv - kg_j\delta_j + kg_{j+1}\delta_{j+1}$$

Since the right hand side of above equation is known, {Z} can theoretically be solved. However kr_j should be decided in consistent with the boundary condition of z_j in Table 1. Therefore eq. (1) should be solved by iteration method. Now that {Z} has been solved, interacting forces f_j and fr_j can easily be determined by using Table 1. By applying the above calculation for every grid location of fuel assemblies, we can complete the instantaneous colliding interaction calculation of whole system for a given time step.

As to the treatment for restitutional characteristic of fuel grid, the frictional damping method is used as in FUVIAN2. Namely eq. (6) of reference [1] is applied to all impact related springs kg_j and kr_j.

2.3 Multi-Fuel Assemblies Colliding Vibration

The equations governing the colliding vibration in FUVIAN3 is almost the same as in FUVIAN2 which is described in eq. (7) of reference [1] as

$$[M]\{\ddot{X}\} + [C]\{X\} + [K]\{X\} = - M\{\ddot{X}_o\} + \{F\} \qquad \text{.....} \quad (2)$$

$\{X\} = (x_1, \theta_1, x_2, \theta_2, \ldots\ldots, x_m, \theta_m)^T$: Nodal displacement of a beam relative to barrel

$\{\ddot{X}_0\} = (\ddot{x}_{01}, \ddot{\theta}_0, \ddot{x}_{02}, \ddot{\theta}_0, \ldots\ldots, \ddot{x}_{0m}, \ddot{\theta}_0)^T$: Seismic acceleration of barrel by absolute coordinate, common to all beams

$\{F\} = (f_1, M_1, f_2, o, f_3, o, \ldots, f_m, M_m)^T$: Supporting force for f_1, M_1, f_m, M_m and grid-rods interacting force for the other f_j's

m : Number of node in a fuel assembly

In FUVIAN2, above eq. was applied to each fuel assembly. However, in FUVIAN3 one fuel assembly is treated as composed of n_d subbeams, therefore, eq.(2) is applied to each subbeam and grid-rod interaction force in {F} in eq.(2) should be derived by rearranging the solution f_j (determined previously in lateral array) in beam lengthwise. Further, the total number of beams in whole system is taken as n_dxN instead of N (number of fuel assemblies) in FUVIAN2. As to the vibrational characteristics of a sub-beam to be inputted, the same values as a single fuel assembly is used. k_g is determined from compression test of a fuel grid, k_r and k_d are also measured by spring and dimple stiffness tests.

2.4 Grid Impact Surface Irregularity Effect.

Surface irregularity effect is taken into impact model by modifying outermost kg_j of each fuel assembly in following manner. In Fig. 3(a), let's assume that grid surfaces have irregularity of $\pm\ \delta g$ and that the irregularity effect (impact spring rate would be reduced) works only while opposit side contact surface is located between $\pm\ \delta g$ as seen in the figure. If the contact surface goes to further contraction, the effect will vanish and the spring behave as regular one. Taking a simple case of n_d=1, relationship between contractive force and deformation is presented by non-lenear spring as shown by solid line in (b), Where Kg = 2K and K is compressive stiffness of single grid. Further, for n_d=3 the problem may be treated as shown in (c). Since irregularity is located only on grid surfaces, the modification should be applied just to outermost spring elements, kg_1 and kg_4 for the case (c).

2.5 Experimental Verification of Impact Model of FUVIAN

14x14 type full size fuel assembly impact test was conducted to confirm the validity of FUVIAN3 impact model. Fig.4 shows the comparison of impact forces between calculated and experimental, and Fig.5 shows the similar comparison concerning the mid-grid displacements at the impact. And Fig.6 shows the comparison of impact force predictions of FUVIAN3 with that of FUVIAN2 in which considerably big improvement is observed.

3. Vibrational Stress Analysis of PWR Fuel Assembly, FLOAN

FLOAN is a computer code to calculate vibrational stresses on the components in a PWR fuel assembly. It is also available to predict vibrational characteristics of the fuel assembly that is necessary as FUVIAN input. The code is applicable up to 20x20 type PWR fuel assembly.

3.1 Calculational Model of FLOAN

In FLOAN model, a fuel assembly is idealized such that it consists of elastic beams (fuel rods and control rod guide thimbles) and rigid bodies (nozzles and grids) in which the connecting conditions between elastic and rigid members or supporting condition are adequately chosen from free, fixed and elastic, in each direction at each nodal point.

Fig.7 shows an example of FLOAN calculational model for a 14x14 type PWR fuel assembly. Actually, it has (14x14-16) fuel rods and 16 guide thimbles in it, but they are projected in X-Y plane so that we can deal with the problem by 2 dimmensional finite element method. So, each member, other than nozzles and grids in the model, represents all depthwise members. The locations of guide thimbles are intentionary shifted a little to aboid double nodes. The bending rigidity of fuel rod containning pellets was almost the same as that of fuel clad by the experiment, and nonlinear effect of spring and dimple type supporting (see Fig.1 -(a)) is treated as an equivalent elastic connection. Because the model is axisymmetric, only upper half is treated putting appropriate supporting conditions on the axis of symmetry. Since the whole structure is entirely elastic and linear, we have to explain only briefly about its mathematical treatment. As for the treatment of rigid bodies in elastic system reference [2] was refered.

Governing equation of natural vibration for whole system is a form as next.

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\} \qquad \ldots\ldots \quad (3)$$

[M],[K] : Mass and stiffness matrices for whole system

{X} : Displacement vector for whole system

Assuming that we have the next type solution for eq.(3),

$$\{X\} = \{A\} \sin \omega t$$

we can easily determine the natural frequency ω and modal shape {A} (normalized) for each vibrational mode as

$$\omega_1, \omega_2, \ldots, \omega_j \text{ and } \{A\}_1, \{A\}, \ldots, \{A\}_j \qquad \ldots\ldots \quad (4)$$

Next, the stresses in fuel component can be derived from ω_j and $\{A\}_j$ by following manner.

Equation of motion for a given member is

$$[m]\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\end{Bmatrix} + [k]\begin{Bmatrix}x_1\\ x_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix} \qquad \cdots\cdots \quad (5)$$

$[m]$, $[k]$: Memberwise mass and stiffness materices (6x6)

$\{X_1\} = (x_1, y_1, \theta_1)^T$, $\{X_2\} = (x_2, y_2, \theta_2)^T$: Displacements at both eands of a member

$\{F_1\} = (fx_1, fy_1, M_1)^T$, $\{F_2\} + (fx_2, fy_2, M_2)^T$: Force acting at both ends of a member

If the system is vibrating in j-th mode, the vibration of a given member is as next form.

$$\begin{Bmatrix}X_1\\ X_2\end{Bmatrix} = \begin{Bmatrix}a_1\\ \vdots\\ a_6\end{Bmatrix}_j \sin \omega_j t$$

Where, $a_1 \ldots a_6$ should be picked up from corresponding elements in $\{A\}_j$. Substituting above into eq.(5) and solving for F_1, F_2 we get

$$\begin{Bmatrix}F_1\\ F_2\end{Bmatrix} = \left[-\omega_j^2 [m] + [k]\right]\begin{Bmatrix}a_1\\ \vdots\\ a_6\end{Bmatrix} \sin \omega_j t$$

Now that we have got the magnitude of end forces (pre-sine term) stress at any location on the member can be derived by conventional method.

3.2 Experimental Verification

In order to verity the method of FLOAN, experiments on 14x14 type fuel assembly were conducted. Fig.8 shows the comparison of natural frequencies between predicted and experimental, and simular comparison about a skelton assembly (without fuel rods) is also presented in Fig.9 for the further confirmation. Good agreements are found in the both cases. Dotted lines show that the fuel assembly can not be dealt by any of the conventional equivalent beam concept. Fig.10 shows one of the predicted modal shapes, 3rd mode, and Fig.11 shows the comparison of several modal shapes between predicted and exprimental, in which we can also observe good agreements. Finally, Fig.12 shows comparison of stresses on fuel rods between predicted and exprimental for the first mode, and a rather good agreement can be seen.

4. Combined FUVIAN & FLOAN Method for PWR Fuel Seismic Analysis

Overall seismic evaluation of fuel assemblies is done by using FUVIAN and FLOAN in a manner as shown in Fig.13 Firstly the vibrational characteristics (natural frequencies and modal shapes) and modal stress intensities per unit amplitude are calculted by using FLOAN. Then they are transfered to FUVIAN and used to calculate stresses on fuel components for the worst seismic deformation. Sample output of decomposed modal shapes of the seismic deformation is shown in Fig.14. Essentially, the combined FUVIAN and FLOAN method calculates grid impact forces, the worst deformation of fuel assembly and local stresses for the deformation, and they are all adequately evaluated with the design considerations.

5. Conclusion

In order to deal with increasing requirements for PWR fuel seismic analysis the method of analysis has further been progressed by improving FUVIAN and by newly developing FLOAN. They are combined together and adequately utilized for the seismic design of PWR fuel in Japan.

6. Reference

[1] Nuno, H., "A Method of Colliding Vibration Analysis of PWR Fuel Assemblies Under Seismic Condition", The 2nd SMiRT, K6/10, Berlin, Sept. (1973)

[2] Livesley, R.K., "Matrix Methods of Structural Analysis", Pergamon Press Ltd. (1964)

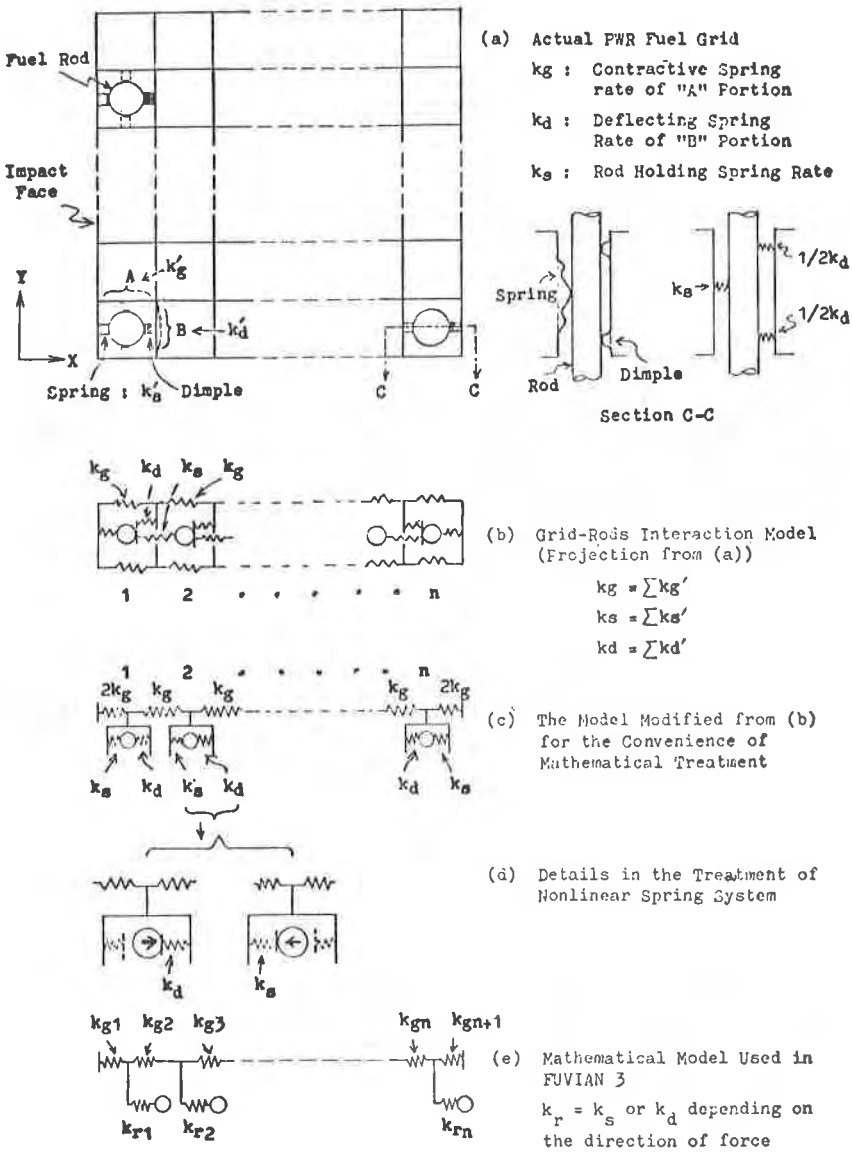
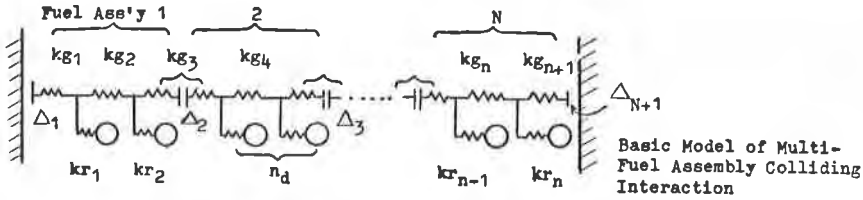


Fig. 1 Grid-Rods Interaction Model in a Fuel Assembly

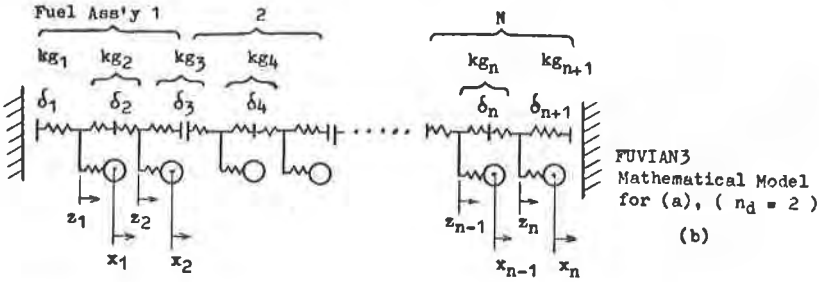


N : Number of Fuel Assemblies

n_d: Number of Fuel Rod Groups in a Fuel Assembly

n : $n = N \times n_d$, Total Number of Rod Groups

Δ : Initial Gap Between Adjacent Fuel Assemblies



$\delta_1 = \Delta_1$, $\delta_2 = 0$, $\delta_3 = \Delta_2$, ..., $\delta_n = 0$, $\delta_{n+1} = \Delta_{N+1}$

x_j : Displacement of j-th Fuel Rod Group

z_j : Displacement of Node j in Grid

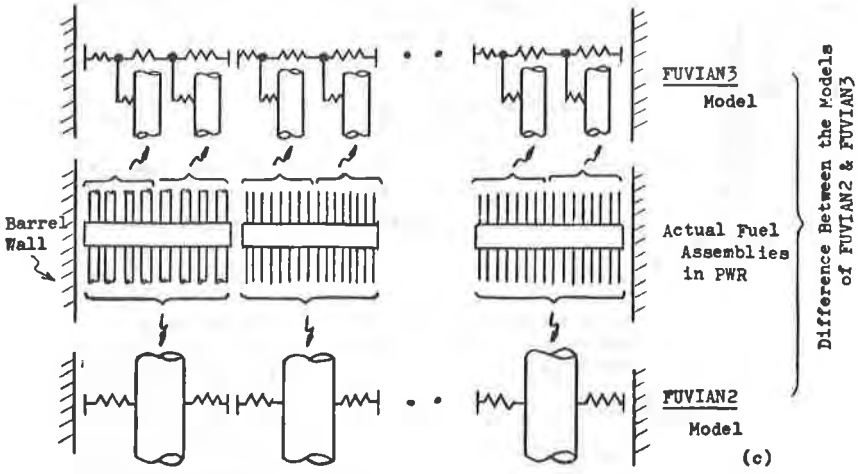
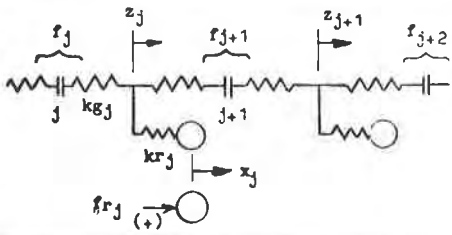


Fig. 2 Multi-Fuel Assemblies Colliding Interaction Model of FUVIAN3 & Its Comparison with the Model of FUVIAN2

Table 1 Grid-Rods Interaction Forces, f_j & fr_j

Location of kg_j	Boundary Condition	kg	δ	f_j, fr_j
Between Adjacent Fuel Assemblies (Kg_1, kg_3,... in Fig 2)	Gap is open $z_{j-1}-z_j-\delta_j < 0$	$kg_j = 0$	$\delta_j = \Delta_k$	$f_i = (z_{j-1}-z_j-\delta_j)kg_j$
	Gap is closed $z_{j-1}-z_j-\delta_j \geq 0$	kg_j as given		
Act inter-grid location (kg_2, kg_4...in Fig 2)	For all cases		$\delta_j = 0$	
For all cases				$fr_j = (z_j - x_j)kr_j$



f_j : Compressive force acting on kg_j (Compression : +)

fr_j : Force acting to fuel rods (a fuel rod group) (x-direction : +)

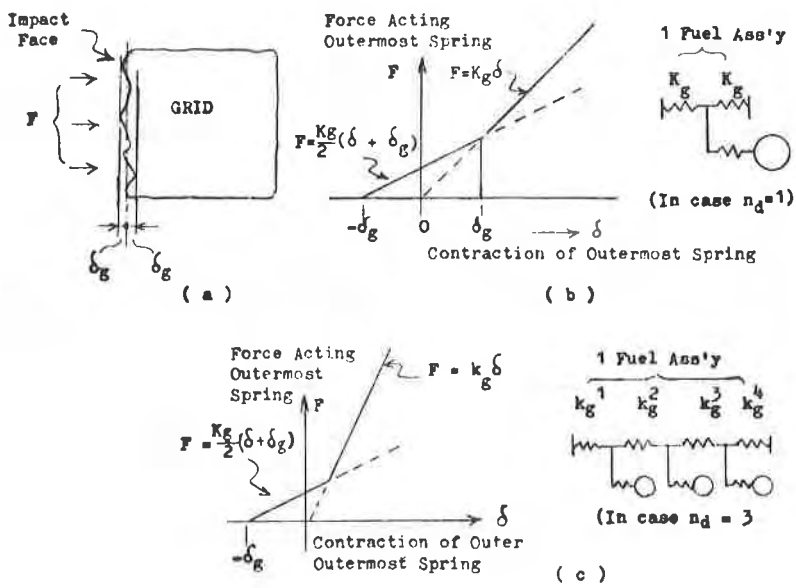


Fig. 3 Treatment for Grid Surface Irregularity Effect

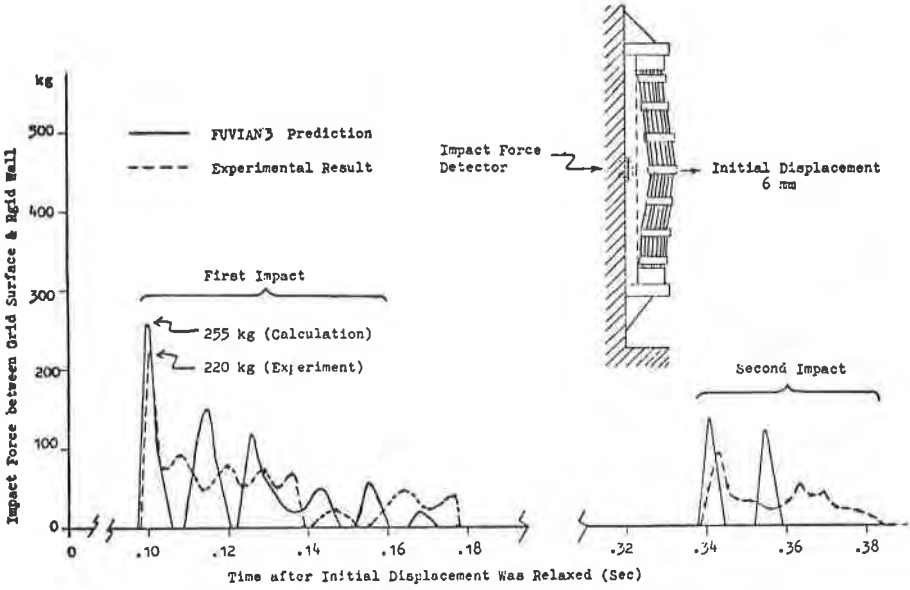


Fig. 4 Comparison of FUVIAN3 Impact Force Prediction with the Experiment

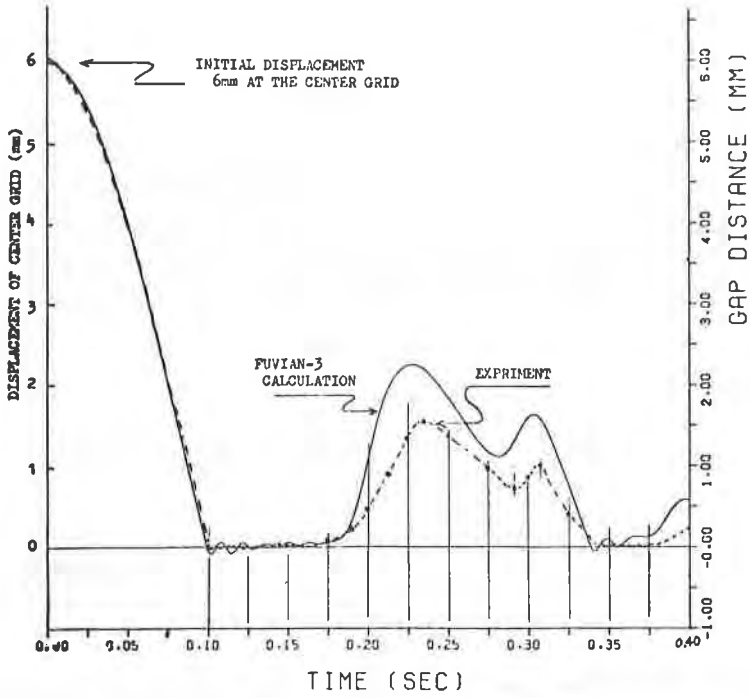


Fig. 5 Comparison of Predicted Displacement of Mid-Grid with Experiment

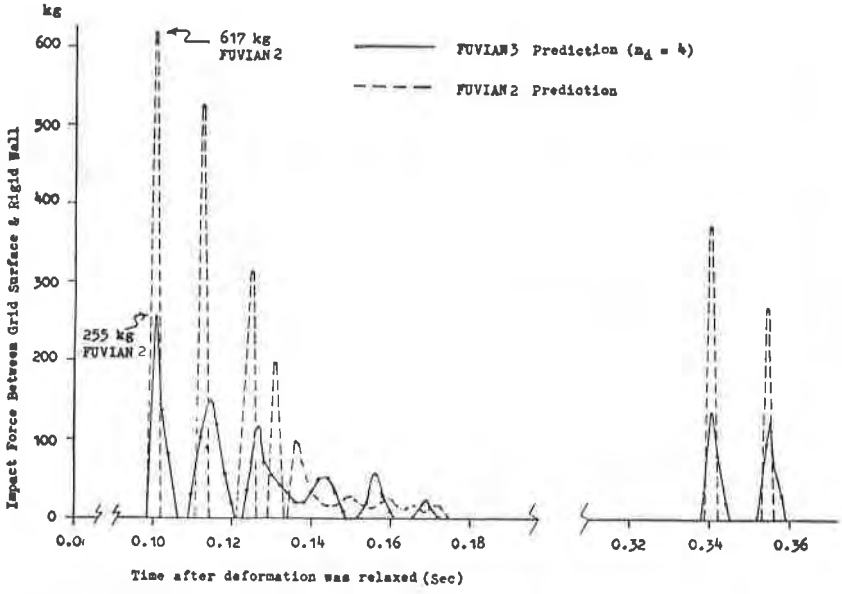


Fig. 6 Comparison of FUVIAN3 Impact Force Prediction with That of FUVIAN2

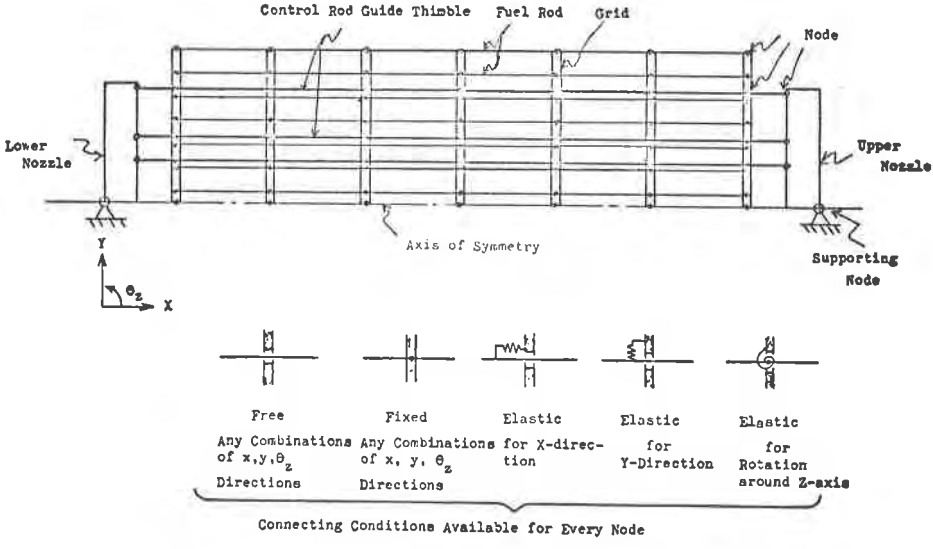


Fig. 7 Calculational Model of FLOAN for the Vibrational Stress Analysis of 14x14 Type Fuel Assembly

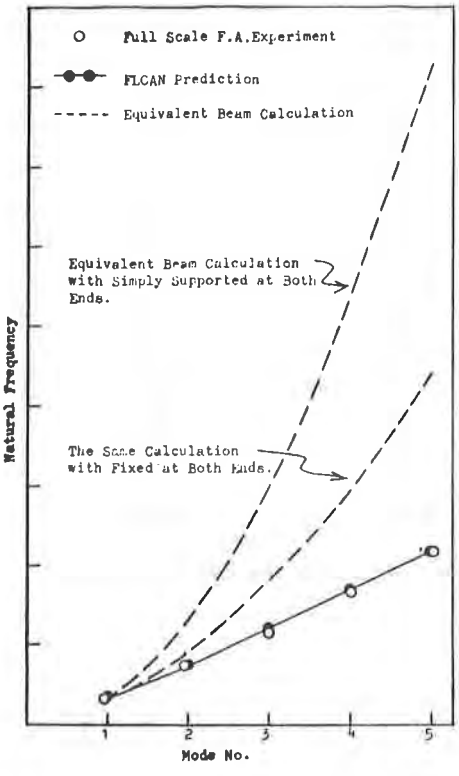


Fig. 8 Comparison of Predicted Natural Frequencies with the Experiment for 14x14 type Fuel Assembly

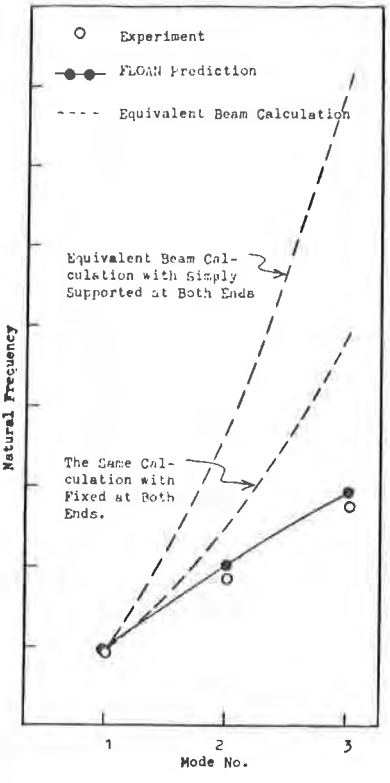


Fig. 9 Comparison of Predicted Natural Frequencies with the Experiment for 14x14 Type Skelton Assembly

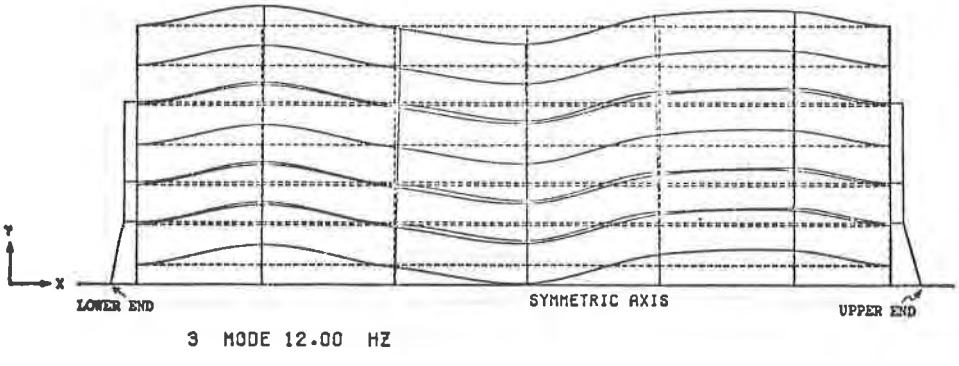


Fig.10 Predicted 3rd Mode Shape of the Fuel Assembly

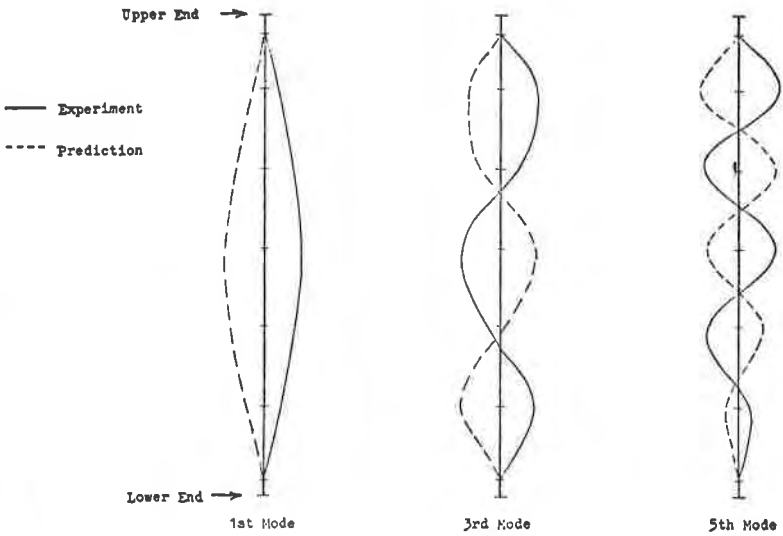


Fig.11 Comparison of Modal Shapes of the Fuel Assembly Between Calculated & Experimental

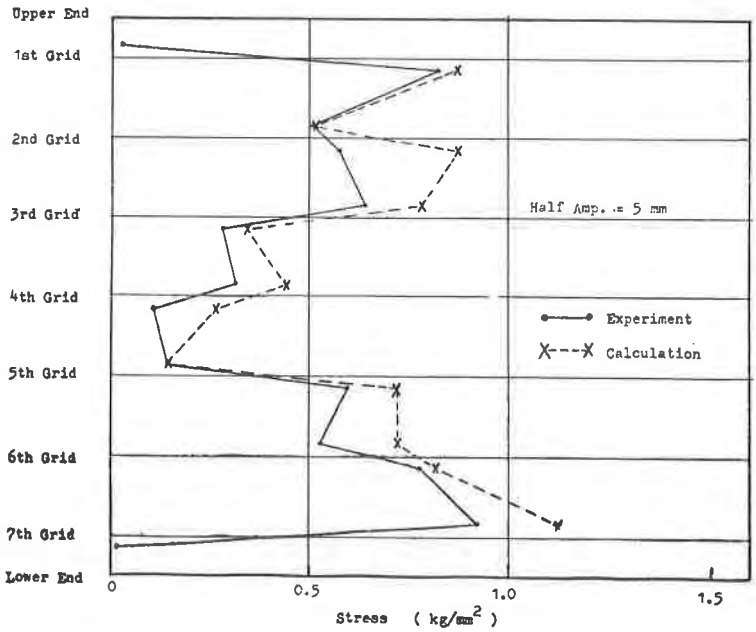


Fig.12 Comparison of Predicted Fuel Rod Stresses with the Experiment for the 1st Mode

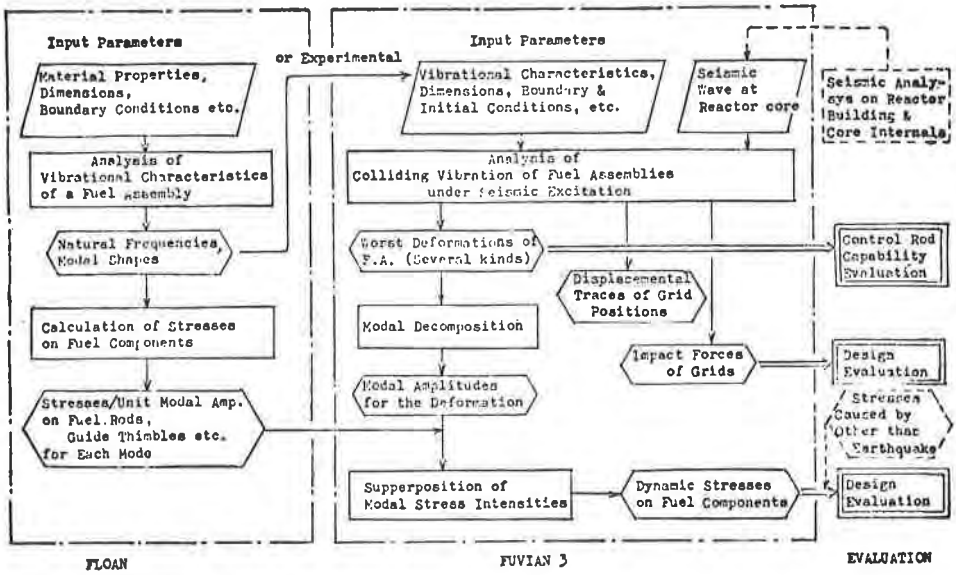


Fig.13 Procedures of Seismic Evaluation of PWR Fues Assemblies by Using Combined FUVIAN & FLOAN

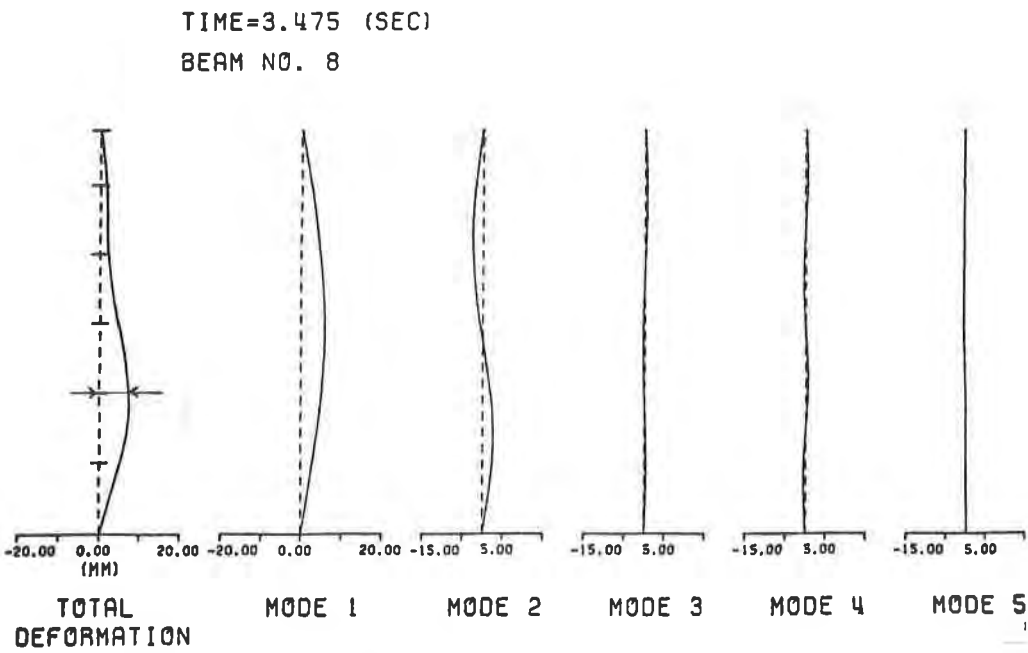


Fig.14 An Example of Decomposed Modal Intensities for One of the Worst Deformations of a Fuel Assembly Caused by Seismec Vibration in PWR