A CONTRIBUTION TO FRACTURE CRITERIA FOR GRAPHITE

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SUMMARY

The calculation of stresses and strains in graphite core components of the high temperature reactor is generally performed using the rules of linear elasticity. As a consequence, large stress concentrations are found. This effect would not be so pronounced if the analysis were done non-linearly and non-elastically. In order to make a statement about the fracture strength of the component, the largest calculated stress is compared with the strength obtained from a uniaxial tensile test specimen. This criterion is conservative in nature but inadequate to predict failure in the general case.

In this paper two alternative methods to solve the problem of failure of graphite components are proposed:

— The most reliable test of fracture strength of a component is clearly the mechanical loading of a true replica of the component under conditions simulating the service conditions. However, such experiments are prohibitively expensive and, therefore, the strengths of the components are generally compared with those of classical specimen types viz. tensile, bend, torsion or compact tension (CT) specimen. We propose to use test specimens whose shapes and stress distributions are nearly the same as those of the components in regions where peak stresses are found. The two similar stress distributions are fitted to each other by means of a linear least squares fit. This yields two quantities, the one being the fracture strength of the tested component and the other expressing the degree of similarity between the tested section of the component and the test specimen. The procedure is explained for the case of stresses in block type nuclear fuel elements.

— The second procedure to evaluate fracture strengths is a new fracture criterion based on a linear elastic calculation of the energy density proportional constant $k^2$, averaged over an element of volume $\Delta V$. The quantities $k^2$ related to different parts of the component are then weighted according to the Weibull statistics and summed over the total volume of the component $V$ to yield a risk value

$$R = f(V) \times \frac{1}{n} \sum_{i=1}^{n} \left( k_i^2 \right)^{n/2}$$

The material constants $m$ and $\Delta V$ can be determined experimentally. For critical loads a critical risk $R_c$ is deduced from the formula. It is shown in the paper that the formula relates fracture strengths from so far four types of test specimen (including tensile, bend and CT-specimen).

Results so far obtained show that the practical applications of these procedures should give a reasonable improvement in the fracture criteria for graphite components. Further experimental investigations are necessary to support the theoretical approach and possibly improve its dependence on volume.
1. Introduction

The presently accepted failure criterion is to compare the tensile strength with the maximum calculated elastic stress. This criterion is safe so far as the tensile strength is the lowest strength the material shows and, depending on geometry, the calculated maximum elastic stress can assume numbers as large as infinity. Any test geometry other than that of the tensile test specimen like for instance the bend test yields a higher strength of the material. On the other hand elastic calculations for geometries which include notches yield very high maximum stresses. Although the criterion is conservative, the limitations imposed on the designer are unnecessarily high. Therefore it is desirable to give a more precise prediction of failure to which safety margins may be applied.

With respect to graphite in the core of a nuclear reactor the problem is aggravated by the fact that the secondary stresses, induced by differential shrinkage and differential thermal expansion, have to be compared with the primary stresses present in an externally loaded test specimen like the uniaxial tensile test specimen. In the former case the calculation of stresses always implies the use of stress-strain relationships while in the latter case this is not necessary. One could perform the best possible stress calculation on the component including a non-linear stress-strain relationship and an inelastic behaviour of the material. However the comparison of the stresses so calculated with the strength of the material, would require an equal amount of labour on the side of the test specimen. The term strength is commonly understood as the largest stress a test specimen can withstand in an experiment. In order to deduce the strength from the critical load value a calculation becomes necessary which by definition is linear elastic. If this calculation was performed under the same conditions as the one for the component, i.e. non-linearly and inelastically, the problem of finding a fracture criterion would be different, and possibly more promising.

This paper does not adopt the procedure described. Instead the stress analysis of the component is of the linear elastic type, and on the basis of this assumption two fracture criteria are developed in steps. In the following two sections a basis for the first fracture criterion is considered. This criterion involves specific, experimental actions, and it is outlined in section 4. The preparatory work for a formula permitting the calculation of strengths for arbitrary geometries from two material constants is contained in sections 5 and 6. This formula is presented in section 7.

2. Stress Distributions

The most reliable test of the fracture strength of a component is clearly the mechanical loading of a true replica of the component under conditions simulating the service conditions. However, if the loading is induced by irradiation of neutrons in nuclear reactors such experiments are prohibitively expensive.
Thus a compromise is required, and in general this is achieved by replacing the true replica by the classical specimen types e.g. tensile, bend, torsion or compact tension specimens. The maximum tensile stress occurring in the test specimen is compared with that occurring in the component. The problem, however, is the ambiguity resulting from the fact that there are as many strengths as there are test specimens. Therefore it appears reasonable to suggest a compromise which is more in favour of the true replica of the component. If the special shape of that part of the component in which maximum stresses occur is retained for the test specimen, and if the mechanical loading of that test specimen is such that a similar stress distribution to that in the component is generated, then the test is more representative. Furthermore the fore-mentioned ambiguity is removed.

The two requirements for a better compromise may be expressed differently. Since the mechanical loading and the boundary of the test specimen are sufficient to determine its complete stress distribution, it is sufficient to say that for more representative testing a similar stress distribution should be retained. The new requirements also imply that the comparison is no longer carried out between the maximum tensile stresses but between the two similar stress distributions surrounding the location at which maximum stresses occur. Furthermore, since the ambiguity was removed by introducing test specimens with comparable stress distributions, it is logical to say that different stress distributions are responsible for different strengths of the material. With respect to failure the material responds differently if it is loaded with different stress distributions.

The essential proposition for a new fracture criterion is already made in the preceding paragraph. More details for actual application are given in section 4. In the following section 3 the remaining concessions made in the compromise described above are discussed.

3. Primary and Secondary Stresses

There is one important difference between the process of failure in the component and the corresponding process in the test specimen. This difference is not taken into account by comparison of stress distributions. The quantities which are compared with each other are both stresses i.e. quantities defined by the ratio force per unit area. The information about the origin of the force - if it is caused internally (secondary stresses) or externally (primary stresses) with respect to the body - is not included in the stress distribution.

Observation shows, however, that in the case of secondary stresses, the failure seldom occurs catastrophically. A crack is initiated but does not propagate further. The stress distribution which caused the crack collapsed in the process of crack initiation. This statement is particularly true for graphite which does not display a totally brittle fracture. Therefore, the comparison between the component and the test specimen as proposed gives an answer to the question: at which load is the crack initiated? The test specimen being under external but otherwise equivalent load would disintegrate if the strain applied was not controlled.
Furthermore the build-up of stresses in the component occurs during continuous neutron irradiation. The irradiation changes material properties such as the strength significantly, and for this reason it would be desirable to design a test specimen which is made from irradiated material. However it is possible that there is a correlation between the irradiation induced changes in the tensile strength and those of other strengths. A knowledge of such a correlation would help to decrease the costs involved in the experiments.

4. An Experimental Fracture Criterion

The comparison between the stress distributions of the component and the corresponding test specimen can be performed by means of a linear least squares fit. If for instance the results of two plane-strain calculations were obtained via the finite element method, two sets of nodal point stresses are available:

\[
\sigma_i^c = \left\{ c_{(i)}^{(l)}, c_{(i)}^{(l)}, c_{(i)}^{(l)} \right\}
\]

\[
\sigma_i^s = F \left\{ 0_{(i)}, 0_{(i)}, 0_{(i)} \right\} = F \sigma_i^0
\]

with \( i = 1, 2, \ldots, n \), \( c \) - component, \( s \) - specimen

The set of stresses from the test specimen \( \sigma_i^s \) has been split into two factors: a normalization factor \( F \) and a normalized stress distribution \( \sigma_i^0 \). Such a factorization is always possible if for instance \( F \) is chosen to be the externally applied force. \( \sigma_i^0 \) is the set of stresses calculated for the component.

The minimization of the quantity

\[
S^2 = \sum_{i=1}^{n} \left( \sigma_i^c - \sigma_i^0 \right)^2 P_i \sum_{i=1}^{n} P_i \]

by variation of the normalization factor \( F \) determines both the value of \( F \) and the value of \( S^2 \) uniquely. The weights \( P_i \) are chosen in order to select that part of the stress distribution which contains the largest stresses. For actual application, an agreement has to be found about how to use the weights. A similar agreement should be reached in fixing a limit in \( S^2 \) viz. \( S^2 \). For values \( S^2 = S^2_{\text{F}} \), the test specimen is called representative otherwise the test specimen cannot be used for the component.

The parameter \( F \) determines the actual load on the stressed component. If in an experiment the test specimen is loaded critically the value of \( F \) becomes \( F_c \). The ratio \( F/F_c \geq 1 \) indicates whether fracture is likely to occur or not. The parameter \( F \) need not necessarily be chosen to be the externally applied force. Instead it can be chosen to be the maximum stress value of the stress distribution. In this case \( F_c \) can be interpreted as for instance the uniaxial tensile strength or the bending strength. If the maximum stress is not a finite number, as in the case of the CT-specimen the quantity \( F_c \) can be interpreted as the critical stress intensity factor. Although \( \sigma_i^0 \) was called normalized stress distribution, depending on the choice for \( F \), the units for \( \sigma_i^0 \) are not necessarily stress units.
Thus, the presently proposed experimental fracture criterion incorporates the classical approaches as special cases including the tests used in fracture mechanics. The additional requirement is that only those specimens can be applied which yield a sufficiently low value of $S^2$.

In an example the stress distribution in a ligament between a fuel channel and a cooling channel in a block-type nuclear fuel element (fig. 1) and the stress distribution of the test specimen shown in fig. 2 were compared. The stress distribution was weighted by $P = 0.7$ for tension and $P = 0$ for compression following the usual treatment of fracture statistics /1/. The value of $S$ was 2.3 N/mm² and $F/F_c$ was 75%. In a second step the value of $S = 2.3$ N/mm² could have been reduced by designing a more representative test specimen.

In most cases the above described fracture criterion requires the design of a new test specimen the strength of which has to be determined experimentally. This somewhat lengthy procedure could be eliminated if it was possible to correlate the strengths of various test specimens. An attempt to achieve this is described in the subsequent three sections 5-7.

5. A Bounded Region for the Stress Distributions

In order to collect more information about the stress distributions generally, it is useful to determine and consider their characteristic features. For this purpose three particular test specimens were selected viz. the uniaxial tensile, the bend, and the compact tension test specimens. They will occasionally be referred to as specimens 1, 2 and 3, respectively.

One of the characteristic features of the three specimens is the value of strength that is derived from them elastically. Specimen 1 shows the lowest strength of all specimens that are in common use. The strength of specimen 2 which is the bending strength is higher. The highest value of strength, i.e. infinity, is observed in connection with the CT-specimen. In general the value infinity is not utilizable for any actual application. Therefore in practice fracture mechanics was introduced, which is a mathematical framework to handle the infinitely large strengths. For the present consideration however it is useful to stick to the definitions made viz. the strength is the largest value of stress observed in a test specimen that is analysed by a linear elastic calculation. With respect to the strength as a characteristic feature we conclude that the three specimens may be placed in an order starting from specimen 1 through to specimen 3. Additionally we conclude that there are no further specimens with a lower strength than that of specimen 1, and also there are no further specimens with a higher strength than that of specimen 3.

There are more characteristic features which by closer investigation lead to the same statements. The stress distribution of specimen 1 is a homogeneous distribution. By definition there is no distribution which is more homogeneous than that from specimen 1, and to our knowledge, there is no specimen with a stronger singularity than that in specimen 3.
A similar approach would be to consider the number of terms which are necessary for a power series to describe the stress distributions. In the case of specimen 1 the stress distribution is a constant which requires one term. The stress distribution of specimen 2 contains a linear variation with one coordinate which in general requires two terms. Depending on the required accuracy the expansion series for specimen 3 must be extended to a very large number of terms because of the singularity involved.

The conclusions are that there exists a bounded region for all specimen types. The beginning and end of this region are marked by specimen 1 and specimen 3, respectively. All other test specimens may be placed into this bounded region in an orderly fashion. Strictly speaking these statements only apply to test specimens with plane stress distributions since only such specimen were considered.

The next step in deriving a formula for conversion of strengths is to establish a mathematical function which is defined on the bounded region and which for instance enables the evaluation of strengths at different locations of the region. Given three locations, namely specimens 1, 2, and 3, the task is then the classical problem of interpolation. An easy solution would be the application of a parabola for interpolation and representation of the strengths within the region, although in this case some kind of scale would be required. A better approach, however, would be to include existing theories about material strengths. A function with these properties is described in the following chapter.

6. An Interpolation Formula

In order to find an interpolation formula for the bounded region of stress distributions the approaches of W. Weibull /1/ and H. Neuber /2/ were studied. The formalism of W. Weibull yields a satisfactory description of the statistical behaviour of the strength. Furthermore the formalism is already capable of converting values of strength between specimen 1 and specimen 2. Recently it has been used to convert the strengths of rings into uniaxial tensile strengths /3, 4/. On the other hand the dependence on volume proposed by Weibull was found to be inadequate /5, 6/.

H. Neuber /2/ suggests the use of average stress values at the point of a notch. The suggestion is based on the fact that for several reasons a linear elastic calculation yields wrong results at this specific location. With respect to fracture strengths it would be more correct to consider the stresses which are averaged over a certain element of volume.

Considering multiaxial stresses a review of data /7/ suggests empirically that the compressive stress will have a negligible effect on the tensile fracture characteristics and the situation should be treated on a criterion for failure at a constant elastic energy:

\[ k^2 = (a_1^2 + a_2^2 + a_3^2) - 2 \nu \left( a_1 a_2 + a_2 a_3 + a_3 a_1 \right) \]

with \( \nu \) Poisson ratio, \( \frac{a_i}{a_j} \) principal stresses, and \( i = 1, 2, 3 \).
For the present approach a selection and alteration of the three ideas mentioned above was used. The basic reference is made to the quantity $k^2$ of equation (2). Therefore the averaging process proposed by Neuber is done with respect to $k^2$ and not with respect to $\Omega$.  

$$k^2 = \frac{1}{\Delta V} \int_{\Delta V} k^2 \, dV$$  

(3)

Such a proposal was recently put forward by J. E. Brocklehurst et al. /9/, but a definition of the extent of the volume $\Delta V$ over which the averaging should be performed presented difficulties.

Using eq. (3) it is necessary to exclude the special ideas about multiaxial stresses presented by Weibull /1/. Weibull's basic description of the failure behaviour is retained in the following equation (4):

$$\frac{r_1}{r_2} = \left( \frac{k_1^2}{k_2^2} \right)^{m/2}$$  

(4)

The risk for failure $r_1$ of a volume $\Delta V_1$ with averaged energy density $k_1^2$ is connected to the risk for failure $r_2$ of an equally large volume $\Delta V_2$ with $k_2^2$ by the relationship (4). The parameters $m$ and $\Delta V$ are material constants. The specific risks $r_1$ and $r_2$ can be obtained experimentally by measuring the rate of failure of $\Delta V_1$ and $\Delta V_2$.

The calculation of the total risk $R$ from a stress distribution $\Omega$ which expands over a volume $V$ can be calculated according to the eq. (5):

$$R = f(V) \times \frac{1}{n} \sum_{i=1}^{n} \left( \frac{k_i^2}{k_0^2} \right)^{m/2}$$  

(5)

The total volume $V$ is divided into $n$ equal parts $\Delta V$ such that $\Delta V = V/n$. For the uniaxial stress distribution, in the limiting case $n \to \infty$ and $f(V) = V$ there would be no averaging ($\Delta V = 0$) and the sum in eq. (5) becomes the integral $\int_{0}^{V} \Omega \, dV$, which apart from another constant is Weibull's definition of the risk. The derivation of eq. (5) and a more detailed description of the whole presentation is given in ref. /9/.

According to eq. (5) the risk of failure $R$ of a specimen depends on two factors. The factor $f(V)$ which contains the dependency on volume is completely unknown so far. The only suggestion made is that of Weibull /1/, $f(V) = V$ which has been proved unsatisfactory /5, 6/. However, since all the remaining terms in eq. (5) can be obtained by experiment and calculation it should be possible to determine the function $f(V)$ empirically.

The second factor in eq. (5) is an arithmetic mean obtained from the specific risks $r_i$. The specific risks $r_i$ also represent the rates of failure that occur in the different parts $\Delta V_i$ of the specimen. Eq. (5) shows that the probability of failure basically depends on two factors i.e. the volume and the stress distribution. Therefore the bounded region of stress distributions introduced in section 5 can be extended by a further dimension i.e. in volume. However, since for the definition of $f(V)$ further experiments are required, in the present numerical applications the three specimen 1, 2 and 3 were chosen to be of an equivalent.
volume /9/ and \( f(V) \) was put to be \( V \). This does not mean any restriction for the present task to find an interpolating function for the bounded region of stress distributions. In the following section 7 it will be shown that the second factor in eq. (5) is capable of interpolating the region.

7. A Theoretical Fracture Criterion

As described earlier (section 4) it is always possible to split up the stress distribution and therefore also the corresponding distribution of local energy density \( k^2 \), into two factors viz. the normalisation factor \( k \) and the normalized distribution \( \tilde{s}_i \) such that \( k_i = k \tilde{s}_i \). Using the factorized form of the stress distribution, the eq. (5) can be rewritten.

\[
R = f(V) k^m \frac{1}{n} \sum_{i=1}^{n} (\tilde{S}_i^2)^{m/2}
\]  

(6)

For critical loads there exists a critical value of the normalisation factor \( k = k_c \). If \( k_c \) is used on the right hand side of eq. (6) a critical total risk \( R_c \) is obtained.

The theoretical fracture criterion is based on the condition that the critical total risk \( R_c \) should be the same regardless of what type of stress distribution \( s_i \) is used for evaluation of eq.(6). In particular the critical risks of the selected three specimens 1, 2, and 3 should be the same. This condition can be expressed mathematically by

\[
\frac{k^m_{1c} \Sigma_1 (m, \Delta V)}{k^m_{2c} \Sigma_2 (m, \Delta V)} = \frac{k^m_{3c} \Sigma_3 (m, \Delta V)}{k^m_{2c} \Sigma_2 (m, \Delta V)}
\]

(7)

if the influence of the volume is eliminated by having the same volume for all three specimens. For brevity that factor of eq. (6) which contains the arithmetic mean is written as \( \Sigma(m, \Delta V) \). The abbreviated form indicates that the process of forming the arithmetic mean of the rate of failure distribution depends upon the parameters \( m \) and \( \Delta V \). These parameters are said to be material constants but otherwise have not yet been defined.

The definition of the parameters \( \Delta V \) and \( m \) is now given by the eqs. (7). The material constants \( \Delta V \) and \( m \) have to be chosen such that the system of eqs. (7) holds. Since \( m \) and \( \Delta V \) are not involved linearly in \( \Sigma \) there are the possibilities of no solution at all or more than one solution.

Therefore at least one example has to be carried out numerically to show the existence of a solution. The example was chosen to be the double impregnated pitch coke graphite AS2-F-500 with a grain size not larger than 1 mm. The uniaxial tensile strength, the bending strength, and the critical stress intensity factor of this material were readily available.

Before starting the description of the calculations for \( \Delta V \) and \( m \), two properties of the eqs. (5), (6) and (7) should be pointed out.

If the conditions eqs. (7) imposed on the function \( R_c \) given in eq. (6) can be met, the function \( R_c \) produces 3 identical values \( R_c \) located at the beginning, in the middle, and at the end of the bounded region. The function \( R_c \) is by definition a constant function with respect to the three points. Therefore it is probable that the function \( R_c \) in eq. (6) is a constant
with respect to the remaining intermediate points of the region. Also this statement has to be tested by at least one example. Furthermore, if the assumption that \( R_c \) is a constant everywhere is found to be correct the equations of the type (7) solve the problem of converting strengths of specimens or components with arbitrary shape and loading.

In order to find a solution for the system of eqs. (7), uniaxial values for the tensile strength \( k_{1c} \), the bending strength \( k_{2c} \), and the stress intensity factor \( k_{3c} \) and a large number of test values \( m \) and \( \Delta V \) were used to tabulate the function \( R \) given in eq. (6). It was found that the solution was not very sensitive to \( m \), the optimum value of \( m \) being 7. In fig. 3b the critical risk \( R_c \) is plotted against the material constant \( \sqrt{\Delta V} \). With respect to the tensile test specimen a horizontal line is found which shows that the averaging process with \( \Delta V \) does not make any changes in \( R_c \) because the stress distribution to be averaged is already a constant distribution. The corresponding curve for the CT-specimens starts at infinity for no averaging (\( \Delta V = 0 \)). For increasing values of \( \Delta V \) it is steadily decreasing and crossing the constant line at about \( \Delta V = 1000 \text{ mm}^3 \). The third line represents the critical risk \( R_c \) of the bending test as a function of \( \Delta V \). This line is terminated just before the cross over of the other two lines is reached. This effect is due to the limited total volume of the test specimen. The error bars indicate the range between 50% at the lower and 50% at the upper end of the probability of fracture distribution \( /9/ \). The size of the error bars is such that the solution can be located only between values of about 100 \( \text{mm}^3 \) and 3000 \( \text{mm}^3 \). The corresponding value for \( \Delta V = 1000 \text{ mm}^3 \) is \( R_c = 1.7 \times 10^{11} \text{ N} / \text{mm}^{11} \) with an error bar from \( 4.2 \times 10^{10} \) to \( 6.1 \times 10^{11} \text{ N} / \text{mm}^{11} \). Fig. 3a shows the corresponding variation in \( m \) for a fixed value in \( \Delta V = 1000 \text{ mm}^3 \). The value of \( m \) being a solution of the eqs. (7) is not determined within the tabulated range from \( m = 5 \) to \( m = 10 \).

Fig. 3b also shows a number of symbols corresponding to calculations based on finite element analyses of the CT-specimen and the bending test specimen. The most important results are given as a series of dots which correspond to the test specimen shown in fig. 1. This additional calculation shows that for one more specimen (apart from specimens 1 to 3) the constancy of \( R_c \) given in eq. (6) is confirmed.

8. Conclusions

Two fracture criteria have been formulated. The experimental criterion always requires the special design of a new test specimen of which the strength has to be measured. If the new design has been found to be representative with respect to the component the probability of fracture under a given load may be stated. The procedure can be applied without further preparations.

Concerning the theoretical criterion the procedure is less straightforward. Firstly, as shown by the numerical results of the AS2-F-500 graphite it is not permissible to calculate the critical total risk of specimens which are much smaller than 1000 \( \text{mm}^3 \). Secondly the dependency on volume \( f (V) \) has not yet been measured. In order to find a functional dependence on volume it is suggested to study the strengths of specimen types 1, 2, and 3 including various sizes and individual magnification in the three spatial directions.
A new precise definition of the quantity volume selecting that part of volume which is loaded with paramount tensile stresses may be helpful. This way the function \( f(V) \) can be found empirically and a mathematical representation in form of a polynomial becomes possible. The theoretical fracture criterion was derived by adapting the free parameters of a mathematical function to reproduce experimental values. This way the function given in (6) represents a true phenomenological description of what is observed in experiments. Furthermore the function was chosen to be partly compatible with established theories on failure mechanisms. For these reasons the described approach seems to be a promising way to proceed.

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Fig. 1: Principal Stresses on the Ligament of a Prismatic, Block-type Fuel Element in a High Temperature Reactor

Fig. 2: Specimen Simulating the Critical Region of the Component shown in Fig. 1. Measurements in cm.
Fig. 3: Critical Total Risk as a Function of the Parameters \( m \) and \( \Delta V \)

a) Variation of the Critical Total Risk \( R_c \) with \( m \) for \( \Delta V = 1000 \text{ m}^3 \)

b) Variation of the Critical Total Risk \( R_c \) with \( \sqrt[3]{V} \) for \( m = 7 \)