BULK COOLANT CAVITATION IN LMFBR CONTAINMENT
LOADING FOLLOWING A WHOLE-CORE EXPLOSION

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SUMMARY

An LMFBR core undergoing an explosion transmits energy to the containment in a series of pressure waves and the containment loading is determined by their cumulative effect. These pressure waves are modified by their interaction with the coolant through which they propagate. In liquids in their normal state the main effect of the interaction is to steepen the fronts of waves leading to shock formation but in liquids which have undergone bulk cavitation the interaction is much stronger and is expected to be dissipative and dispersive. Since reflections of initial pressure waves from the core at free surfaces lead to the establishment of a state of tension and consequently of bulk cavitation over large volumes of the coolant, it is necessary to model both the induction of bulk cavitation by tension waves and the interaction of pressure waves with cavitated liquid in realistic containment loading calculations. This paper sets out the progress which has been achieved in such modelling and first indications for the effect of bulk coolant cavitation in LMFBR containment loading.

A model describing the interaction between cavitation bubbles and a host liquid subject to time-varying pressure fields, including the physical process of momentum conservation, phase change, heat conduction and mass diffusion is presented and used to a) determine which of the various physical processes involved dominate the results. It is shown that if we are only interested in bulk effects momentum transfer is the chief factor for the pressures and timescales of interest. b) determine the effect of the state of purity of the liquid. The main characterising parameters are the initial radii of nucleation sites and their number density, the latter being very influential. c) identify the important differences between model water and reactor sodium as far as cavitation is concerned. These are chiefly the lower surface tension and higher concentration of initially larger nucleation sites in the sodium.

Based on the foregoing a simple model including only momentum conservation and surface tension is created for wave-propagation investigations to find the degree of cavitation induced by short-duration waves and the smoothing and dissipating effect of cavitated liquid on initially sharp pressure pulses. Results are applied heuristically to the containment loading problem in models and reactors with and without internals. The effect of bulk cavitation is significant in each case. Finally the simple model in conjunction with a Lagrangian hydrodynamics code is applied to a model test. Several values of the number density of nucleation sites are used to demonstrate the importance of this parameter (not measured in the experiment).

Conclusions may be briefly summarised:
- Bulk cavitation must be included in realistic containment loading calculations.
- Phenomenological models of cavitated liquid without memory are inappropriate. The best approach is to model bubble dynamics directly, including at least momentum conservation and surface tension.
- The containment loading resulting from a given explosion is sensitive to the state of preparation of the coolant. The number density of nucleation sites should therefore accompany the results of model tests.
1. Introduction

The conversion of the energy of an LMFBR core undergoing an explosion to the deformation energy of the primary containment and sensitive internals such as heat removal devices is a very inefficient process. Most of the energy ends up as heat in the coolant, with the remainder being apportioned among various sinks including the cover gas, non-sensitive portions of the reactor structure and (the main concern of those who calculate containment loadings) those parts of the reactor structure whose survival after the accident is essential to effective containment of the post-accident situation. Any physical mechanisms which affect the energy transmission and conversion processes significantly, are thus of interest from the point of view of containment safety and it is the task of this paper to draw attention to the mechanism of coolant cavitation.

Consider that stage of the usual postulated accident sequence which commences with the explosive disruption of the core. The core emits pressure waves with peak magnitudes of several tens of MPa and these propagate out radially until they encounter some element of the reactor structure. As in all wave-structure interactions processes of transmission, reflection and absorption take place simultaneously, but for those components which are expected to survive the accident more or less intact, their inertia and strength and the very different acoustic impedances of steel and coolant ensure that transmission is of relatively minor importance. Energy not absorbed is reflected, and in fact in most cases a considerable portion of the incident energy is reflected. The resulting reflected pressure waves can become focussed and concentrated, and may result in high stresses and deformations in some portion of the containment perhaps not designed to withstand them. The general containment problem is thus one of fluid-structure interaction in the presence of multiply-reflected waves, and the number of reflections necessary before progressive absorptions render the energy content of a particular wave negligible rises as the number and complexity of reactor internals increases, for example in progressing from simple models to the real reactor in all its three-dimensional structural complexity.

The problem is further complicated by the existence in the system of at least two free surfaces, at the interface between coolant and core bubble and at that between coolant and cover gas. Pressure waves reflected at these free surfaces (of course again with very little transmission) propagate back into the coolant as tension waves. The direct interaction of such tension waves with structures is here unimportant. The degree of cohesion between structure and coolant is generally too weak for the structure to be "pulled back" and the bond breaks with the formation of a thin layer of vapour and previously dissolved or adsorbed gas, which is reclosed when the next pressure wave arrives. Much more important is the interaction of tension waves with pressure waves and with the coolant itself. There is generally some destructive interference with pressure waves, and if they meet head-on the pressure waves can be neutralised (a phenomenon familiar in shock-tube calculations).
but more influential from the reactor containment point of view is the ability of tension waves to cause cavitation in the coolant if their magnitudes are more than very modest (over 0.5 MPa or so). The cavitated coolant then interacts strongly with subsequent pressure waves, spreading and lowering pressure peaks with overall dissipation and consequent reduction in the stresses in those structures at which the pressure wave subsequently arrives. Calculations for model tests (Jones [11]) have shown that in simple rigid geometries the mechanism described here is the principal dissipative process and without it (i.e. if the coolant is assumed non-cavitating) the coolant mass oscillates indefinitely in a jelly-like manner. The effect on safety of this dissipative mechanism must in general be beneficial but in any case in adequate calculation of the containment loading following an HGDA bulk coolant cavitation and its consequences must be modelled. This paper and the accompanying oral presentation set out the progress which has been achieved in such modelling and first indications for the significance of bulk coolant cavitation in LMFBR containment loading.

2. The Modelling Problems

The usual procedure for producing numerical solutions to the containment loading problem involves a discretisation of the coolant volume into cells or elements and stepwise solution of the associated discretised equations of mass, momentum and energy conservation. The set of equations is completed by an equation of state linking the pressure, specific volume and specific energy of the coolant. If cavitation is to be included in this procedure (so that the appreciable sophistication incorporated in modern finite difference and finite element codes can be taken over without change) it is necessary to use a much more versatile equation of state which takes into account the dynamics of the cavitation bubbles existing in each calculational cell. Such an "equation of state" obviously has time-dependence, memory and internal variables and is more correctly termed a constitutive relation. The modelling problem is thus reduced to that of finding a constitutive relation which adequately represents the behaviour of liquid under arbitrary variations in the applied pressure, positive or negative. Unfortunately, experimental data of the type required are very scanty in the tensile half of the p-V plane even for water, and the experimental situation is further confounded by the dependence of such parameters as the tensile strength of liquids on the state of purity and the nature and form of the impurities, i.e., on the nucleation site spectrum. What is being examined experimentally is in each case some aspect of the metastable behaviour of a liquid containing dissolved gas, dust particles etc. and such behaviour is of very great variety (Skripov [2]). In an effort to fill the void the author has created a model describing a uniform mass of liquid containing nucleation sites of specified radius and number density and which treats the processes associated with cavitation bubble growth in some detail. The model is embodied in the computer program CAVBUBL, the contents of which are described below. The idea behind its creation is to apply to the mass of liquid
being modelled arbitrary external pressure histories and to calculate the resulting variation in specific volume. From the effects of parameter variation on the system paths in p-V-time space the dominant physical processes in the pressure ranges and time-scales of interest can be determined.

3. The Physical Processes in Cavitation Bubble Growth and their Modelling in CAVBUBL

The physical processes taking place and the form of modelling chosen in each case are as follows:

a) Compression of the liquid surrounding the bubbles.
   A constant sound-speed pressure-volume relationship is used.

b) Conservation of momentum for the bubbles.
   This results in a Rayleigh equation with the inclusion of surface tension and viscosity, and an internal pressure for the bubble. The local compressibility of the liquid is not modelled.

c) Phase-change between liquid surrounding the bubbles and vapour within the bubbles.
   This is kinetic theory limited, with the temperature and pressure within the bubble being determined by an energy balance assuming perfect mixing, and the interface temperature as in d).

d) Heat transfer within the liquid.
   This supplies the latent heat for phase-change and is calculated by an integral approximation to the heat-diffusion equation as in Theofanous et al. [3].

e) Diffusion of dissolved gas into the bubbles.
   This is kinetic theory limited, the gas concentration at the interface being arrived at via f).

f) Mass diffusion within the liquid.
   An integral approximation is used for the diffusion equation as in d).

The nucleation sites from which the cavitation bubbles grow are modelled as hard spheres in which a small amount of gas is assumed to be trapped so that bubble growth will be initiated when the external pressure has fallen to the "tensile strength" set by the surface tension and the nucleation site radius. The above modelling relations result in 8 ordinary differential equations coupled in a non-linear manner, which are solved by a 4th order Runge-Kutta technique. Special approximations are employed for the very early stages of bubble growth and for the situation in which the bubble collapses rapidly onto the nucleation site. In the first case an analytical relationship valid at small times is used, while in the second a form of "numerical damping" is employed to smoothe out the collapse, which from a macroscopic point of view is unimportant.

Tests using CAVBUBL in the manner described in paragraph 2, show that as far as bulk cavitation is concerned, under the pressures and on the time-scales associated with containment loading heat transfer and mass diffusion have little effect and the dominant pro-
cess is simply expansion and contraction of bubbles under the constraint of liquid inertia. The slides of the associated presentation provide numerical evidence of this, but with hindsight it is possible to produce a semi-analytical justification also. Consider the situation in which unit mass of a liquid of density $\rho$ and sound speed $c$ containing $N$ nucleation sites of initial radius $R_m$ is subjected to a tension $p$. Cavitation bubbles grow at the sites to a radius $R$ so as to bring the pressure close to zero once more, and it is easily shown that $R$ satisfies:

$$p + \frac{4}{3} \pi \rho c^2 N R^3 = 0.$$  \hfill (1)

The timescale of bubble growth is fixed by the fact that under expansion from small radii the Rayleigh equation is dominated by the term containing the radial velocity squared. There results for the characteristic time $t^*$

$$\left(\frac{\frac{4}{3} \pi c^2 N}{\rho c} \right)^2 \rho p t^* + \left(\frac{c}{c}\right)^3 = 0$$  \hfill (2)

showing that $t^*$ depends only on the $1/6$ power of the pressure. Numerically for water with $N = 1000 \text{ kg}^{-1}$ and $p = 10 \text{ MPa}$ we find $R = 1 \text{ mm}$ and $t = 0.01 \text{ ms}$.

Since nucleation site radii $R_m$ are of order 0.001 ms the bubble volume increases by a factor of $10^7$ in 0.01 ms. The internal pressure of gas or vapour thus drops by several orders of magnitude, far too fast for vapour or gas to flow in to compensate, whether limited by thermal diffusion or not, since the timescale associated with the kinetic theory limited transfer of mass from wall to bubble is of the order of milliseconds for bubbles of radius $R$. The neglect of compressibility except in the bulk sense, not tested for validity numerically, may also be justified by order of magnitude arguments except at very small radii, at which cavitation bubbles have little effect on the bulk behaviour of the coolant.

Omission of compressibility also results in neglect of the waves emitted by individual collapsing bubbles. These waves are in general incoherent due to the range of possible nucleation site radii in real liquids, and the spherical expansion of the waves results in only weak signals at containment boundaries.

We now turn to the influence of the state of purity of the liquid. The chief characteristics are the gas content, the surface tension (including the effect of impurities) and the nucleation sites. The remarks above indicate that except when the bubble is small the gas content is not important, and the same applies to the surface tension. However, the tensile strength of a liquid, i.e. the maximum sustainable tension before cavitation begins, is obviously of importance in describing e.g. spalling in containment loading problems and it depends on the small-radius behaviour. If $\sigma$ is the surface tension and $R_m$ the mite radius then the tensile strength is

$$p_m = -2\sigma/R_m + p_i$$  \hfill (3)

where $p_i$ is the internal pressure composed of liquid vapour pressure plus the pressure of
dissolved gas in the liquid. The precise meaning of $\sigma$ depends on the type of nucleation site postulated and $\sigma$ could refer to liquid/vapour or liquid/site. In any case experimentally (Travena [41]) $p_m$ is of the order of $-0.5$ MPa for tap water and given the lower surface tension of sodium at temperature and the expected higher gas content under reactor operating conditions (although not in most test rigs) a still smaller tensile strength can be expected for reactor coolant.

The radius of nucleation sites is thus not of great moment in containment loading problems, but their number density is more influential. Equation (1) shows that increasing $N$ yields finer bubbles with a shorter characteristic time (the variation being as $N^{1/3}$), and this must reflect back on the interaction with travelling waves. Moreover the mean separation of bubbles becomes less and they are more likely to interact, again with consequences for the bulk dynamics. We thus ask the physical significance of $N$, how it can be measured and how water and reactor sodium compare in this respect. When liquid containing a distribution of nucleation site sizes is subjected to a significant tension, equation (3) shows that bubble growth is prevented by surface tension only at the smallest sites. At all the others bubbles begin to grow but they do so faster at the larger sites. The growth relieves the tension locally and the growth of smaller bubbles is suppressed. Thus $N$ is the effective number density of nucleation sites and is generally much smaller than the total number density. It represents the high-radius tail of the size-distribution, the portion of the tail included being determined by some as yet undiscovered relationship involving the applied tension and its mode of application (e.g. whether it is preceded by pressurization or not).

Practically, the way to measure this quantity is to subject the liquid under test to a "typical" tension and then to count the site-density using high-speed photography. So far this has not been done for e.g. the water used in model tests. Lastly, concerning reactor sodium (for which site-counting would need to be done by some non-optical method) the most likely nucleation sites would be gas bubbles, and experience to date seems to indicate that gas entrainment is sufficient for $N$ values to be considerably higher in reactor sodium than in tap-water.

Quantitative remarks must however wait on measurement.

4. A Simplified Constitutive Law

We now return to the problem outlined in paragraph 2, that of finding a constitutive relation for cavitated liquids. The results presented above show that simulation by rate-dependent phenomenological relations such as are used for e.g. viscoplastic materials is not possible because of the inertial aspects of the behaviour. In fact the insignificance of most of the physical processes taking place makes it more convenient simply to simulate numerically the growth and collapse of inertially controlled bubbles, producing a constitutive relation which is physically valid at the cost of carrying one internal variable, the bubble radial velocity. Details of the modelling relation are now presented. The hydrodynamics code
in which the model is to be inserted, in common with most of the others used in containment calculations, requires that when supplied with the previous values of all quantities, the new specific volume and the timestep its "equation of state" module return the new pressure and specific energy. This is the reverse of the mode of operation of CAVBUBL, and requires some manipulation of the Rayleigh equation and a standard equation of state for uncavitating liquid. This latter is taken to be of form

\[ p = (a + b \mu) + c \rho_o (1 + \mu) \]  

(4)

where \( a \), \( b \) and \( c \) are constants characteristic of the liquid, \( \rho_o \) is the initial density, \( e \) the specific energy and \( \mu \) is given by

\[ \rho = \rho_o (1 + \mu). \]  

(5)

Given the old values of the specific volume, pressure and specific energy, the model inverts eq. (4) to obtain the old value of \( \mu \) and hence the old specific volume of the liquid alone. By comparing this with the old overall specific volume the bubble radius on entry to the routine, \( R_o \), is found. It is thus not necessary to store this as an internal variable.

The Rayleigh equation reads

\[ \frac{d}{dt} \frac{3}{2} R^2 = -\frac{p}{\rho} - \frac{2\sigma}{R} \]  

(6)

and this must be solved over the step coupled with eq. (4) and the specific energy equation (7)

\[ de = -p \, dV \]  

(7)

while the specific volume changes linearly from its old value to the new value supplied to the model over the specified timestep. It turns out to be convenient to use in place of \( R \) the auxiliary variable

\[ X = R^3 \]  

(8)

and with this substitution the Rayleigh equation becomes:

\[ \ddot{X} = \frac{X^2}{6X} - \frac{\sigma}{\rho} \frac{3 \dot{p}}{X} X^{1/3}. \]  

(9)

With initial conditions \( X = R^3 \) the above equations are then solved numerically by a substepping process, the old pressure being used to calculate the new \( X \) and this and the new specific volume being used in turn to find the new pressure. \( X \) is passed on from cycle to cycle, and is stored for each calculational cell as an internal variable.

This model has been tested alone and also when functioning as the constitutive relation for a 2-D Lagrangian hydrodynamics code. The resulting combination has been used to examine both the process of cavitation caused by the propagation of a tension wave into virgin liquid and the effect on a pressure wave of interaction with cavitated liquid. The combination has also been used to calculate a well-instrumented model test, and the effect of varying the nucleation site density investigated. The results of all these investigations together with a
heuristic application of the experience gained to more complex model tests and to full-size reactors will be presented orally.

References


