HYDRODYNAMIC AND ELASTOPLASTIC STRUCTURAL ANALYSIS OF FAST BREEDER REACTOR CORE ACCIDENT

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SUMMARY

This paper describes the principles and examples of applications of an explicit Lagrangian coupled finite difference-finite element code HEMP-ESI developed in order to calculate the structural consequences of hypothetical core disruptive accidents (HCDA) in nuclear reactors.

The explicit solution algorithm of the finite difference scheme used to discretize the hydrodynamic fluid domains is shown to be very similar to that used for the solution of the finite element discretized shell structures, hence permitting an easy and efficient coupling. The fluid domains are simulated explicitly according to M. L. WILKINS finite difference formulation, valid for hydrodynamic calculations. The branched shell structures are simulated by a finite element approach after T. BELYTCHKO which takes into account the elasto plastic behavior and the large displacements, rotations of the structures. A slide line technique enables the coupling between the fluid and the shell structures:

— when in contact with the fluid shells are accelerated normally to their direction by the fluid

— the fluid in contact with the shell is allowed to accelerate independently tangentially to the shells and are projected normally on them.

Two examples of simulation show the applicability of the method to nuclear reactor core safety analysis (test problem):

1) core explosion in a loop-type reactor including a shell containment. The calculation shows the energy absorbing function of the shell and enables the evaluation of the forces acting on the reactor containment

2) Hypothetical Core Disruptive Accident in a fast breeder reactor. The calculation shows the main features of this accident:

— lifting of the liquid sodium above the explosion and impact on the cover head inducing upward deformations

— radial outflow of the sodium which induces large deformations of the inner and outer shell

— zones of compressive circumferential stresses in the main shell at the junction of the spherical head and the cylindrical part.

1. Introduction

A Hypothetical Core Disruptive Accident (HCDA) in a fast-breeder reactor (pool type) involves several physical phenomena :

- Expansion of the core-gas bubble in the liquid sodium
- Propagation in the coolant of pressure waves which induce two types of fluid motions : one is the upward lifting of the sodium which impacts against the cover head of the reactor and the other is a radial outflow which induces possible large loadings applied to the inner shell structures and the main thin shell vessel of the reactor primary containment.
- Large displacements of the different shell structures, eventually plastification and/or buckling of these structures.

To evaluate numerically the effects of such an accident a Lagrange computer code HEMP/ESI, adapted from previous works by M. WILKINS |1| and T. BELYTSCHKO |2|, has been developed in order to take into account both hydrodynamic and structural features of the physical phenomena involved. This code uses a Lagrangian finite difference techniques to simulate the hydrodynamic fluid domains and a Lagrangian finite element technique to simulate the large displacements behaviour of the complex elastoplastic branched shell structures inside the reactor primary containment.

The paper describes two applications of HEMP/ESI involving the evaluation of structural damages in nuclear reactors :

a) the simulation of a core explosion in a loop-type reactor with a metallic cylindrical shell containment for energy absorption.

b) the calculation of a HCDA in a fast-breeder reactor which shows the capabilities of an explicit Lagrangian finite difference - finite element approach to assess quantitatively the consequences of such an accident.

2. The coupled Lagrangian Finite Element - Finite Difference approaches

2.1. The finite difference approach for the hydrodynamic domain

The usual Lagrangian equations governing the hydrodynamic equilibrium in an axisymmetric domain are (1) :

The equation of conservation of momentum :

$$\rho g_x - \frac{\partial(P+q)}{\partial x} = \rho\ddot{x}$$
$$\rho g_y - \frac{\partial(P+q)}{\partial y} = \rho\ddot{y} \qquad \ldots\ldots\ldots \quad (1)$$

The equation of continuity :

$$\frac{\dot{V}}{V} = \frac{\partial\dot{x}}{\partial x} + \frac{\partial\dot{y}}{\partial y} + \frac{\dot{y}}{y} \qquad \ldots\ldots\ldots \quad (2)$$

The energy equation :

$$\dot{E} = -(P+q)\,\dot{V} \qquad \ldots\ldots\ldots \quad (3)$$

The equation of state :

- for pressure $P = f(V,E)$
- for the artificial viscosity $q = g(\dot{V},\dot{x},\dot{y})$ .... (4)

Where x is the coordinate along the axis of symmetry, y the radial coordinate, p the pressure, V the relative volume, ρ the density, g_x and g_y are the components of a gravity acceleration.

The equations (1) to (4) can be discretized with M.L. WILKINS |1| difference scheme which uses the following approximation formulas (fig. 1) :

$$\frac{\partial P}{\partial x} = \lim_{A\to 0} \frac{\int c\ P(\vec{n}.\vec{x})ds}{A} \quad \text{or} \quad \frac{\partial P}{\partial x} \quad \frac{\int c\ P(\vec{n}.\vec{x})ds}{A}$$

consequently the equation (1) is discretized as follows :

$$\ddot{x} = -\ \frac{\int c(P+q)\ (\vec{n}.\vec{x})ds}{\rho A} + g_n$$

$$\ddot{y} = -\ \frac{\int c(P+q)\ (\vec{n}.\vec{y})ds}{\rho A} + g_y \qquad \ldots\ldots\ldots \quad (5)$$

C is the closed curve I II III IV of area A which surrounds point M where the equilibrium equation is discretized (fig. 1).

Table I shows the explicit time integration algorithm which solves equation (5).

2.2. The finite element approach for the shell structures

The virtual work principle can be used to express the equilibrium of a shell structure :

$$\int_V \rho\ \ddot{u}_i\ \delta u_i\ dV + \int_V \sigma_{ij}\ \delta\varepsilon_{ij}\ dV - \int_V f_i\ \delta u_i\ dV - \int_S \sigma n_i\ \delta u_i\ dS = 0 \quad \ldots\ldots\ldots \quad (6)$$

where $\ddot{u}_i$ are the acceleration coordinates, δu_i the virtual displacements coordinates, σ_{ij} the Cauchy stress, $\delta\varepsilon_{ij}$ the virtual deformation, f_i the body force coordinates per unit of volume and S is the surface boundary where a normal pressure σ is applied (the coordinates of the normal to the surface are n_i).

The exact formulation (6) is discretized by dividing the shell domain into finite elements following a convected elements formulation (2) : in each element (fig. 2) a local system of coordinates is used, one axis being the straight line joining the two nodes of the elements. The axial displacements are assumed to vary linearly between the axial displacements of the nodes and the deflection varies as a cubic between the two nodes. The degrees of freedom used at each node are the translation coordinates and the rotation of the section of the shell (fig. 2). These assumptions lead to the following discretized equations of equilibrium of the shell structures :

$$|M|\ \{\ddot{u}_i\} = \{F_{ext}\} - \{F_{int}\} \qquad \ldots\ldots\ldots \quad (7)$$

where $|M|$ is the mass matrix of the shell structure, $\{\ddot{u}_i\}$ the acceleration vector of all nodes of the structure, $\{F_{ext}\}$ the vector of the externaly applied loads to the nodes of the shell structure, $\{F_{int}\}$ the equivalent nodal load vector of the internal forces of the structure. The fact that the internal forces $\{F_{int}\}$ are evaluated in the rotated deformed position of the shell elements provides a good simulation of large displacements, large rotations of the shell structure.

2.3. The coupling of the hydrodynamic domains with the shell structure slide lines

Table I shows that both explicit algorithm for the hydrodynamic and shell structure are very similar and a coupling between the two is therefore feasible and very efficient (3) :

The coupling is done at the interface according to a slide line logic (fig. 3) : Three possibilities are provided in HEMP/ESI :

a) the hydrodynamic domain is fully tied to the shell structure

b) a void may exist between the hydrodynamic domain and the shell structure. This void can close when there is an impact between them

c) the hydrodynamic domain is tied to the shell along its normal and can slide tangentially to it.

The third case c is considered here and is solved with the following steps (see Table I, Steps 1 and 3) (also fig. 4).

Step 1 (of Table I) : the external force $\{F_{ext}\}$ contains the contributions $\{F_n\}$ of the normal forces applied by the hydrodynamic domain on each shell elements. The mass matrix $|M|$ of the shell structure contains the normal masses contribution of the hydrodynamic domain sliding along the shell elements. The nodal points of the shell elements are accelerated according to the modified masses and external forces (i.e. A becomes A'B', in fig. 4).

Step 3 (of Table I) : the sliding nodes of the hydrodynamic domain are accelerate tangentially to their shell elements (i.e. P becomes P') and projected normally to them (i.e. P' is projected in P").

3. Examples of applications

3.1. Response of a loop-type reactor to a core explosion

The first simulation consist in the dynamic response of a cylindrical loop type reactor which has the geometry shown on figure 5 : in the simulation the concrete containment shell is considered as rigid and the cylindrical metal shell is considered as flexible with an elastoplastic behaviour including strain hardening. The explosion is simulated by an initially sperical bubble of perfect gas with an initial pressure of 40 bars.

Figure 5 shows the deformed shapes of the bubble explosion and shell at 15 ms. The pressure wave induced by the explosion leads to large elasto-plastic deformations of the shell which thus absorbs part of the energy generated by the explosion. As the shell deforms it increases its radial velocity and that of the surrounding water mass : this leads to a large impact pressure with peaks of about 200 bars, on the adjacent concrete containment (fig. 6).

3.2. Response of a fast breeder reactor (Pool-type) to a core explosion

The second calculation consists in a typical test problem of the response of a fast breeder reactor (Pool-type) to a HCDA. The structures of the reactor are simplified as shown on figure 7 : The problem is considered axisymmetric and all the structures are simulated by finite shell elements. The explosion bubble is a low density charge. The maximum pressure is of the order of 150 bars.

Figure 7 shows the main features of the accident :

While the bubble expands it induces several flows :

- an upward sodium flow impacts the cover head which is up lifted. Right after the impact a large radial pressure is applied to the upper cylinder of the main vessel which deforms radially with large circumferential deformations.
- a radial sodium flow induces large elastoplastic deformation of the inner shells
- a downward flow applies large loads on the core support shell which displaces downwards.

The main vessel shows a global tendency to dilatate in order to accomodate the pressure loads. However, in the knuckle between the spherical head and the cylinder, high compressive circumferencial stresses develop (fig. 8) which indicate a buckling tendency which would occur with formation of circumferencial waves for the real three dimensional situation.

4. Conclusion

To benefit from existing proven Lagrangian techniques for hydrodynamic calculations and make use of the efficiency of the finite element method to model complex shell structures, an explicit coupled finite difference - finite element Lagrange code has been developed to simulate the main phenomena occuring during hypothetical core disruptive accidents in nuclear reactors. The main physical and geometric features of the problem have be realistically modeled and the practicality of the numerical tool established.

The development of the HEMP/ESI code is still in its early stage concerning applications to fast breeder safety analyses and improvements are in preparation to better simulate the dynamics of the explosion bubble growth, and of the large rotational flow in perturbed areas (corners of the structure), where improved rezoning and time integration techniques can be devised.

Aknowledgments

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References

1 Wilkins, M.L. "Calculation of elastic-plastic flow" Methods in Computational Physics Vol. 3, Fundamental Methods in Hydrodynamics (1964).

2 Belytschko, T. and Hsieh, B.J. "Nonlinear transient finite element analysis with convected coordinates" International Journal for Numerical Methods in Engineering, Vol. 7, 255-271 (1973).

3 Dubois, J.J., Chedmail, J.F., de Rouvray, A.L. "A coupled model for the Finite Difference Finite Element analysis of hydrodynamic and elasto-plastic problems". International Conference on Innovative Methods in Engineering. Versailles Mai 1977.

4 M. Fortunato, M. Falgayrettes, A. Meyer-Heine, "Codes de calcul et programmes expérimentaux pour l'étude des déformations dynamiques de confinement d'un réacteur rapide". To be presented at the SMIRT 4th Conference San Francisco.

TABLE I

Explicit solution algorithm of the finite difference hydrodynamic and the finite elements shells calculations

Steps of calculation for time t^{n+1}	Hydrodynamic finite differences calculations	Shells finite elements calculations
1) Calculation of the nodal accelerations of every points of the mesh at time t^n	$\ddot{x}^n = \frac{\int c(P+q)\ (\vec{n}\ \vec{x})\, ds}{\rho A}$ $\ddot{y}^n = \frac{\int c(P+q)\ (\vec{n}\ \vec{y})\, ds}{\rho A}$	$\{\ddot{u}^n\}=\|M\|^{-1}\|\{F_{ext}\}^n-\{F_{int}\}^n\|$
2) Explicit calculation of the nodal displacements of all shell nodes at time t^{n+1}		$\{\dot{u}\}^{n+1}=\{\dot{u}_i\}^{n-\frac{1}{2}} + \{\ddot{u}^n\}\Delta t$ $\{u\}^{n+1}=\{u\}^n + \{\dot{u}\}^{n+\frac{1}{2}}\ \Delta t$ $\{x\}^{n+1}=\{x\}^n + \{u\}^{n+1}$ {u} is the displacements vector of all nodes
3) Explicit calculation of the nodal displacements of all hydrodynamic nodes at time t^{n+1}, projection of sliding hydrodynamic points on their neighbouring shells	$\dot{x}^{n+\frac{1}{2}} = \dot{x}^{n-\frac{1}{2}} + \ddot{x}^n\ \Delta t^n$ $\dot{y}^{n+\frac{1}{2}} = \dot{y}^{n-\frac{1}{2}} + \ddot{y}^n\ \Delta t^n$ $x^{n+1} = x^n + \dot{x}^{n+\frac{1}{2}}\ \Delta t^n$ $y^{n+1} = y^n + \dot{y}^{n+\frac{1}{2}}\ \Delta t^n$	
4) Calculation of the increments of strains in all hydrodynamic zones and at each integration points of all shell elements	$V^{n+1}=V^n+\Delta t^{n+\frac{1}{2}}\ 1(\dot{x}^{n+\frac{1}{2}},\dot{y}^{n+\frac{1}{2}})$	$\{\Delta\varepsilon_a^n\}=\{B_a\}\ \|\{u_i^{n+1}\}-\{u_i^n\}\|$
5) Calculation of pressure and viscosity in all hydrodynamic zones and stresses at each point of all shell elements	$P^{n+1}=f\|V^{n+1},\ E^n\|$ $q^{n+1}=g(\dot{V}^{n+1},\ \dot{x}^{n+1},\ \dot{y}^{n+\frac{1}{2}})$	$\{\sigma_a\}^{n+1} = h\ (\sigma_a^n,\ \Delta\varepsilon_a^n)$ (elasto plastic behaviour)
6) Calculation of energy	$E^{n+1}=E^{n-\frac{1}{2}}\ \dot{V}^{n+1}dt(P^{n+1}+q^{n+1}+P^n+q^n)$	
7) Calculation of the applied forces on the shell elements due to the pressure in the neighbouring hydrodynamic zones		$\{F_{ext}\}^{n+1} = \sum_a \int_{l_a} N_a\ P^{n+1}\ dS$
8) Calculation of the internal forces in the shell elements		$\{F_{int}\}^{n+1} = \sum_a \int_{V_a} B_a^T\{\sigma_a^{n+1}\}dV$
9) Go to 1		

Note : N_a is the shape function in element a, B_a is the strain displacement matrix in element a, $n+\frac{1}{2}$ means at time $t^n+\frac{dt}{2}$.

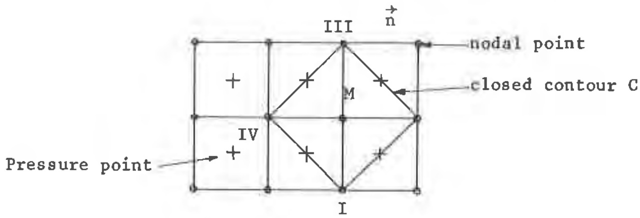

FIGURE 1 : FINITE DIFFERENCE MESH OF HYDRODYNAMIC DOMAIN

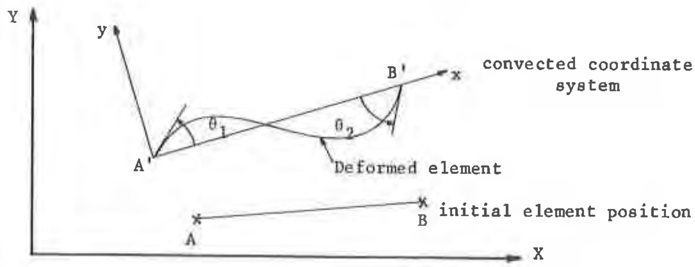

FIGURE 2 : SHELL FINITE ELEMENT

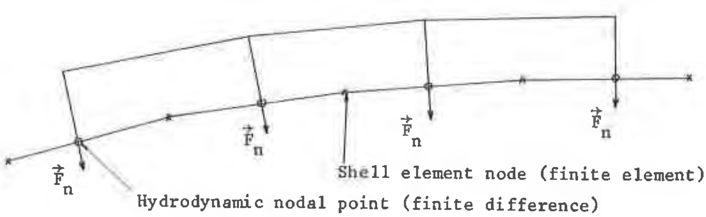

FIGURE 3 : GEOMETRY OF THE COUPLING SLIDE LINE

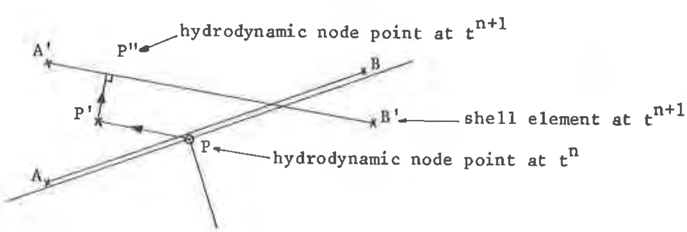

FIGURE 4 : DISPLACEMENT OF A SLIDING POINT A ALONG A SHELL SEGMENT AB

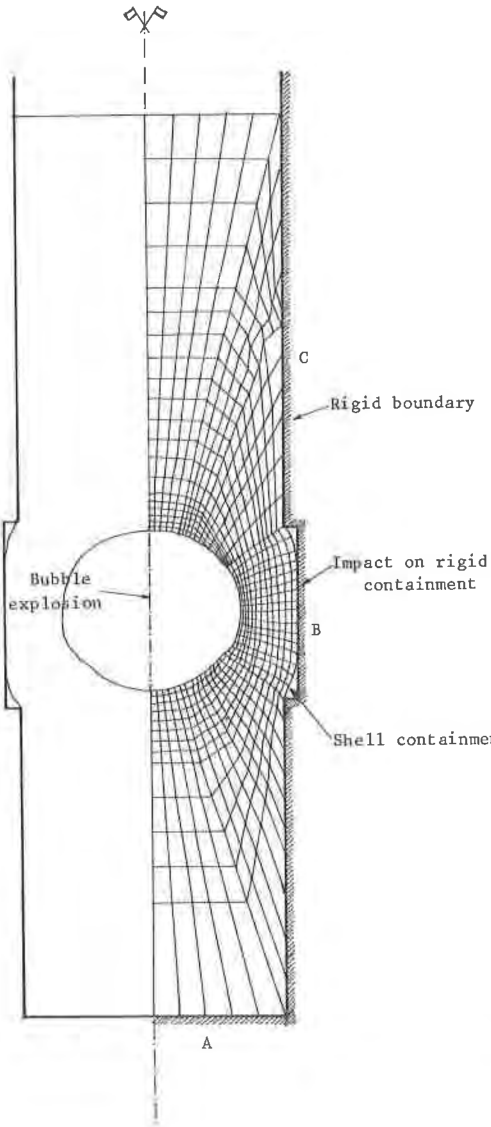

FIGURE 5 : EXPLOSION IN A POOL TYPE REACTOR
AT 15 ms

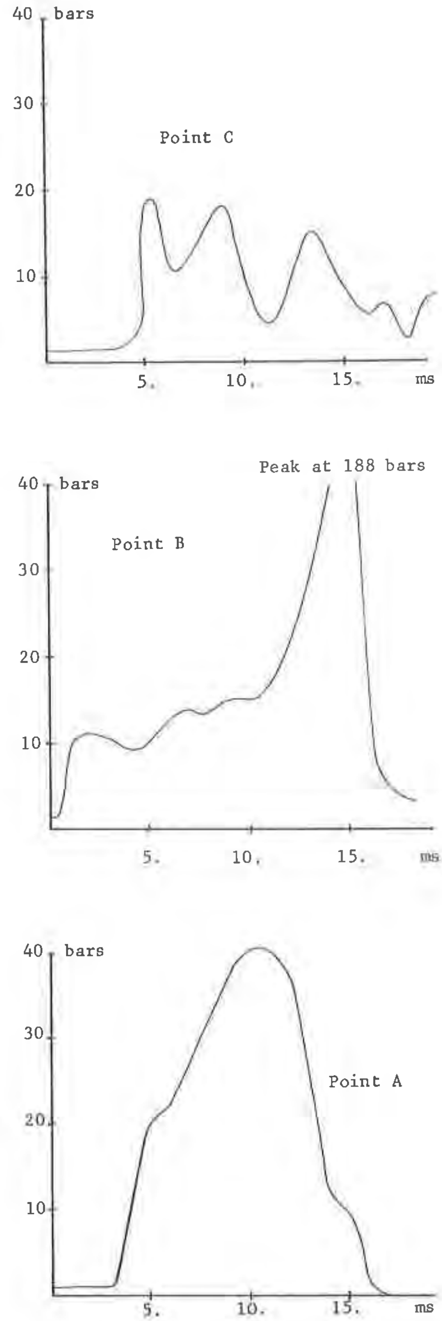

FIGURE 6 : PRESSURE ON REACTOR WALL

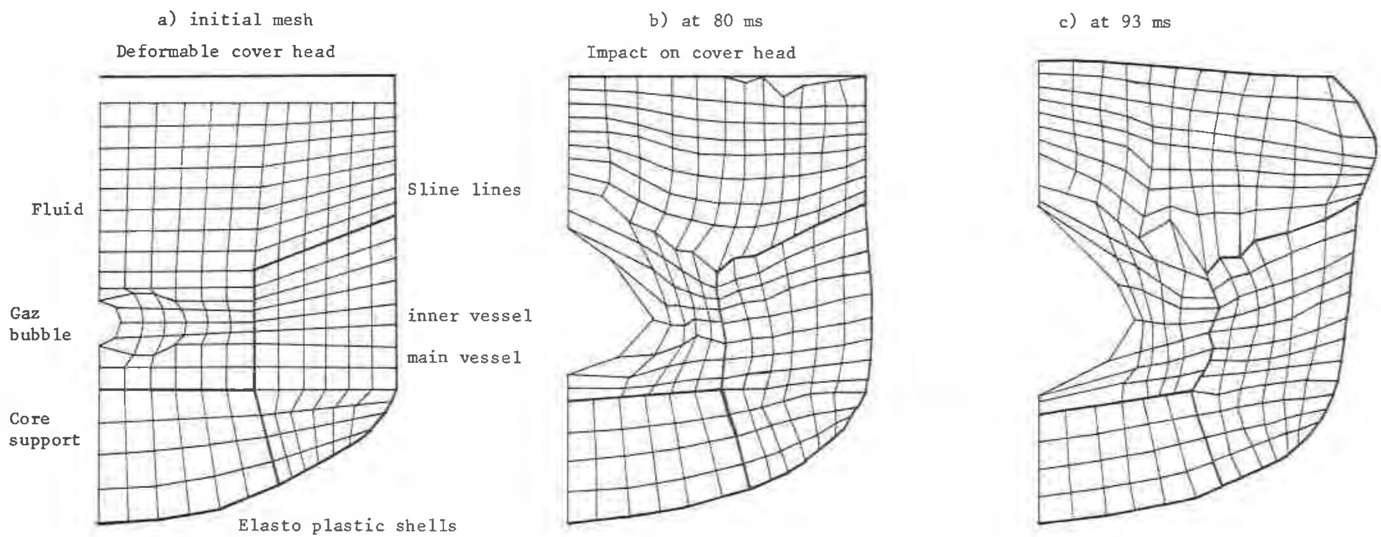

FIGURE 7 : DEFORMED SHAPE OF EXPLOSION BUBBLE AND SHELL STRUCTURES OF A FAST BREEDER REACTOR TEST PROBLEM DURING AN HCDA

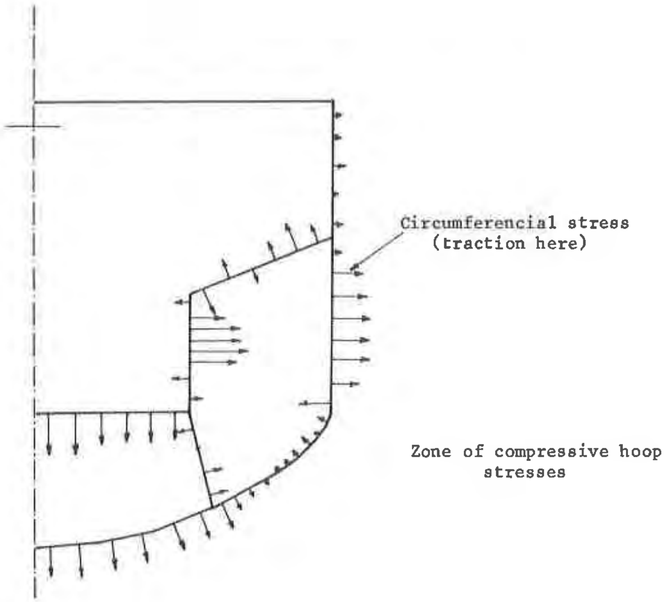

FIGURE 8 : CIRCUMFERENCIAL STRESSES IN THE SHELL STRUCTURES,
EXHIBITING BUCKLING TENDENCY IN KNUCKLE BETWEEN CYLINDER AND SPHERICAL HEAD