

## MATERIAL PROBLEMS IN ACCIDENT ANALYSIS OF PRESTRESSED CONCRETE REACTOR VESSELS

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### SUMMARY

Due to their very high energy absorption capability, as well as their inherent safety advantages, prestressed concrete reactor vessels are presently being keenly studied as the basic barrier to contain hypothetical core disruptive accidents in a fast breeder reactor. An extensive program, concerned with the basic material problems in predicting the accident response of the vessel, has been under way at Northwestern University (under the sponsorship of NSF and ERDA).

One problem investigated is the nonlinear constitutive behavior and failure criteria for concrete. Previously, a comprehensive theory, called endochronic theory, has been shown to satisfy all basic currently known features of test data. Nevertheless uncertainty still exists with regard to non-proportional loading paths, for which good test data are lacking at present. Yet, these paths are most important for predicting material instabilities. A disconcerting feature is that there is no unique theory describing data for proportional loading tests alone; e.g., non-associated plasticity with dilatancy, internal friction, combined with the theory of fracturing material, can describe most proportional loading data equally well as the endochronic theory, while for non-proportional loading there is considerable difference. An extension of the endochronic theory which correlates best with general experimental evidence and includes fracturing terms is given, and a comparison with vertex-type hardening in plasticity is made. Finally, application in a finite element analysis of compression failure of a wall due to localized strain-softening bands is discussed.

A second problem which must be analysed in accident situations is the high temperature shock on the concrete walls (due to liquid sodium, up to 850 °C). Refining a previous crude formulation a rational model for calculating moisture and heat transfer and pore pressures in concrete subjected to thermal shock is presented. It employs full thermodynamic properties of water, considers dehydration of concrete, changes in porosity and in sorption isotherms, as well as heat convection and thermal moisture flux. The formulation results in a system of complex nonlinear diffusion equations for temperature and pore pressure, which are solved by finite elements. The model is calibrated by means of test data. For the purpose of failure analysis of the vessel, creep at high temperature, including the transitional thermal creep must be considered. Finally, a model in which the formulations for heat and moisture transport, for creep and for nonlinear stress-strain behavior are combined to calculate compression failure due to a thermal shock, is outlined. The ultimate purpose is the prediction of spalling, possibly the explosive spalling, of the wall.

In conclusion, a new design concept, in which the concrete vessel is completely dehydrated and kept hot throughout its service life in order to substantially improve its response to thermal shock as well as liquid sodium contact, is described.

## 1. Objectives

Prestressed concrete reactor vessels are not presently designed for nuclear accidents. However, in view of the recent public concern about safety of nuclear plants, it may become necessary to design them to sustain certain hypothetical core disruptive accidents of extremely low probability. Analysis of such accidents is particularly needed in case of liquid metal cooled fast breeder reactors. So far, only steel vessels have been used as a containment of these reactors, but it appears that the capability to contain an accident may be improved if the primary steel vessel is surrounded by a prestressed concrete vessel providing a secondary containment and energy absorption barrier.

The basic uncertainties in predicting the response of a concrete vessel to a core-disruptive accident consist in the knowledge of concrete behavior — the nonlinear triaxial stress-strain relations, the laws of cracking and damage of reinforced concrete, and the response to high temperature exposure. An extensive program, concerned with these problems, has been in progress at Northwestern University, and the purpose of this paper is to give a summary of some of the recent achievements.

## 2. Safety Advantages and the Concept of Hot-Dried Concrete Vessel

As far as failure due to internal pressure is concerned, PCRV offers certain distinct advantages over steel vessels [1]. They consist chiefly in the fact that the failure mode of a prestressed concrete vessel is certain to be ductile rather than brittle, and that the energy absorption is higher than for a steel vessel designed for the same pressure. A finite element study of the process of dynamic failure of a prestressed concrete vessel subjected to a pressure pulse in a core disruptive accident is reported in another paper at this conference.

Further loading which follows the pressure pulse and must be considered in the accident analysis is the heat shock due to contact of liquid sodium with the liner of a prestressed concrete vessel (and possibly also the effect of contact of concrete with liquid sodium). To improve the performance of a PCRV due to this type of loading, a novel design concept — a hot dried concrete vessel — is proposed. This concept, which would not be allowed by the present ASME-ACI standard [2], involves forced drying of the concrete walls (i.e., removal of evaporable water), so as to minimize the detrimental effects of water in exposure of concrete to heat and to liquid sodium, and keeping the concrete walls continuously at about 120°C, in order that the low water content be maintained throughout the service life, and that the temperature dilatation in case of an accident be made smaller. The drying of the walls of the vessel (about 6 to 12 ft. in thickness) is achieved by drying ducts spaced about 1 ft. apart. Without heating the concrete, drying would take years to complete, but circulation of air which is 120°C hot would allow completion of drying within about a week. The drying could be preceded by circulation of steam through the same ducts, so that the concrete undergoes low pressure steam curing and attains high strength before the drying. During the service life, continuous circulation of gas through the same ducts would be used to keep concrete temperature constant in time and uniform across the concrete wall. The exterior surface of concrete would be insulated, and the interior surface would be protected in the usual manner by a steel liner overlying a layer of sacrificial insulating concrete, equipped with cooling tubes (for service cooling as well as post-accident heat removal). This latter arrangement would be similar to the recent Austrian hot PCRV project (not dried) [3]. Studies of steel relaxation for that project indicated that the prestress loss of

tendons in hot concrete can be kept within acceptable limits.

### 3. Endochronic-Fracturing Constitutive Model

For prediction of the response of concrete vessels to the pressure pulse, a realistic nonlinear triaxial stress-strain relation, including strain-softening, is needed. Previously [4], a new constitutive relation, called endochronic, was shown to agree with test data much better than the previously used stress-strain relations of plastic or hypoelastic type. In this theory, the inelastic strain increments  $de_{ij}''$  are characterized in terms of increments  $d\zeta$  of intrinsic time — a non-decreasing scalar variable whose increments depend on strain increments  $d\varepsilon_{ij}$ ; see Ref. [4]. In further work the previously published endochronic model [4] has been improved, introducing the following stress-strain relation

$$de_{ij} = \frac{ds_{ij}}{2G} + de_{ij}^{pl} + de_{ij}^{fr}, \quad d\varepsilon = \frac{d\sigma}{3K} + d\varepsilon^{pl} + d\varepsilon^{fr} \quad (1)$$

in which  $e_{ij}$ ,  $s_{ij}$  = deviators of strain tensor  $\varepsilon_{ij}$  and stress tensor  $\sigma_{ij}$ ,  $\varepsilon = \varepsilon_{kk}/3$ ,  $\sigma = \sigma_{kk}/3$ ,  $G$  = shear modulus,  $K$  = bulk modulus. The inelastic strains are split in two components:  $e_{ij}^{pl}$ , the plastic strain, and  $e_{ij}^{fr}$ , the fracturing strain. The former is due to plastic slip, while the latter is due to microfracturing (microcracking) and is analogous to the fracturing strain used by Dougill in a non-endochronic model [5]. The plastic strains are formulated as

$$de_{ij}^{pl} = \left( \frac{s_{ij}}{2G} - ke_{ij}^{pl} \right) \frac{d\zeta}{Z_1}, \quad de^{pl} = d\lambda + d\lambda' \quad (2)$$

$$d\zeta = F_1(\zeta, \varepsilon, \sigma) d\varepsilon, \quad d\xi = \sqrt{\frac{1}{2} de_{ij} de_{ij}}, \quad d\lambda = F_2(\lambda, \varepsilon, \sigma) d\varepsilon \quad (3)$$

$$d\lambda' = \frac{\sigma}{3K} \frac{d\zeta'}{Z_0}, \quad d\zeta' = \frac{d\xi'}{f_1(\xi')}, \quad d\xi' = |d\varepsilon_{kk}| \quad (4)$$

where  $F_1$  is a deviatoric plastic hardening-softening function, which makes  $de_{ij}^{pl}$  prevail over  $de_{ij}^{fr}$  at large hydrostatic pressure,  $k$  = kinematic hardening parameter ( $k \approx 0.15$ ),  $d\lambda$  = plastic dilatancy (same as in Ref. [4]),  $d\lambda'$  = hydrostatic plastic compaction, due to large hydrostatic pressure,  $d\zeta$  = deviatoric intrinsic time,  $d\zeta'$  = hydrostatic intrinsic time,  $f_1(\xi')$  = hydrostatic hardening function. The fracturing strains are given as

$$de_{ij}^{fr} = e_{ij} d\kappa, \quad de^{fr} = \beta \bar{\gamma} d\kappa/3 \quad (5)$$

$$\bar{\gamma} = \sqrt{\frac{1}{2} e_{ij} e_{ij}}, \quad d\kappa = F_3(\xi, \sigma, \varepsilon) d\varepsilon \quad (6)$$

where  $\kappa$  = fracturing intrinsic time,  $F_3$  = fracturing deviatoric hardening function, and  $\beta$  = fracturing dilatancy parameter. The elastic moduli are variable, depending on inelastic dilatancy  $\lambda$  (similarly as in Ref. [4]) and, even more importantly, on fracturing strains, similarly as in Dougill's (non-endochronic) model [5];

$$dG = G \frac{-fr}{\bar{\gamma}}, \quad dK = K \frac{fr}{\varepsilon}, \quad \frac{-fr}{\bar{\gamma}} = \sqrt{\frac{1}{2} e_{ij}^{fr} e_{ij}^{fr}} \quad (7)$$

The fracturing dilatancy parameter,  $\beta$ , can be calculated from  $G$  and  $K$ , using Budianski and O'Connell's results on plastic moduli of an elastic solid with microcracks, obtained by the self-consistent model [6]:

$$\beta = \frac{3E}{\gamma} \frac{GK_0}{G_0 K} f_K^1 \left( \frac{G}{G_0} \right) \quad (8)$$

in which  $K_0, G_0$  = initial values of  $G$  and  $K$  and  $f_K^1$  = derivative of Budianski's dependence of  $K/K_0$  upon  $G/G_0$  for a microcracked material.

The foregoing model has been fitted to numerous test data [7] and it has been found to fit the data even better than the previous endochronic model [4]. The improvements consist of the following features:

- 1) Strain-softening is due exclusively to the fracturing strains  $e_{ij}^{fr}, \epsilon^{fr}$  rather than total inelastic strains, and terms  $e_{ij}^{pl}, \epsilon^{pl}$  exhibit no strain-softening, which is physically more reasonable and makes the hardening-softening function  $F_1$  much simpler than before [4]. The fracturing terms vanish (due to  $F_3$ ) at large hydrostatic pressure.
- 2) The change of elastic moduli is related to fracturing using a well-founded microstructural model of Budianski et al. This mainly improves the fit of unloading at very large strain, and also reflects the fact that the effective (tangential) Poisson ratio is decreasing as the microcracks grow, and tends to 0 for a heavily microcracked material.
- 3) The inelastic fracturing strains depend on  $e_{ij}$  rather than  $s_{ij}$ , as is the case for plastic strains. This is more appropriate for strain softening in which stress decreases but strain increases.
- 4) Introduction of endochronic kinematic hardening substantially improves the fit of loops in cyclic straining at very large strain magnitudes. This is important for hysteretic damping.
- 5) Introduction of hydrostatic plastic compaction,  $\lambda'$ , allows modeling the inelastic volumetric strain due to large hydrostatic pressure and caused by the collapse of pores.

It is illuminating to compare the endochronic inelasticity with incremental plasticity [8]. For this purpose it is useful to define in the strain space the locus of all vectors  $de_{ij}$  which give the same inelastic strain increment. This locus, called inelastic stiffness locus [8], is a straight line for plasticity theories, while for endochronic theories it is a curve, and particularly for the present formulation, a circle. This means that incremental plasticity gives perfectly elastic response for strain increment vectors which are parallel to the current loading surface, while the endochronic theory gives a much softer, inelastic response. This is physically more reasonable, and it is also similar to the recent vertex hardening models with an infinitesimal vertex. Replacing the circle by a straight line which is tangent to the circle at the end point of vector  $de_{ij}$ , the endochronic formulation can be linearized and brought to the form of incremental plasticity. However, for various directions  $de_{ij}$ , different linearized formulations are obtained. For proportional loading, the endochronic formulation can always be replaced by an equivalent plasticity formulation.

The feasibility of using the present endochronic theory in finite element analysis with step-by-step loading has been verified. It appears that the finite element model is capable of representing the formation of localized strain-softening bands when failure is approached. This is of interest for the analysis of compression failure of a heated concrete wall.

#### 4. Tensile Cracking and Damage

The preceding endochronic formulation exhibits stress-induced anisotropy as far as the inelastic deformations are concerned, but the elastic part of deformation is governed by isotropic relations. This is certainly a simplification because for all stress states but the hydrostatic one, the microcracks are likely to exhibit some prevalent orientation. While

for compressive stress states the inherent elastic anisotropy is probably weak and can be neglected, it is definitely not so for tensile stress states, where most cracks tend to be normal to the maximum principal stress,  $\sigma_1$ .

The modelling of concrete which is fully cracked in one, two or three directions, with cracks being densely and uniformly distributed, is well known. If there are cracks only in one direction, normal to  $x_1$ , then the strain,  $\epsilon_{11}^m$ , of the material between cracks has no relation to strain  $\epsilon_{11}$  of the cracked concrete as a whole, and stress  $\sigma_1$  is zero. Thus, for normal stress and strain components

$$\sigma_{11} = D_1 \epsilon_{11}^m + D_2 \epsilon_{22} + D_2 \epsilon_{33} = 0$$

$$\sigma_{22} = D_2 \epsilon_{11}^m + D_1 \epsilon_{22} + D_2 \epsilon_{33}$$

$$\sigma_{33} = D_2 \epsilon_{11}^m + D_2 \epsilon_{22} + D_1 \epsilon_{33}$$

where  $D_1 = K + 4G/3$ ,  $D_2 = K - 2G/3$ ,  $G =$  shear modulus,  $K =$  bulk modulus. Expressing  $\epsilon_{11}^m$  from the first equation and substituting it in the other two equations, and noting that the stiffness in direction  $x_1$  is zero, one obtains a relation  $\sigma = \underline{D}^c \epsilon$  in which  $\sigma, \epsilon =$  column matrices formed of six stress and strain components,  $\underline{D}^c = [D_{ij}^c] = (6 \times 6)$  stiffness matrix of concrete fully cracked in the direction normal to  $x_1$ ;  $D_{22}^c = D_{33}^c = D_1 - D_2^2/D_1$ ,  $D_{23}^c = D_{32}^c = D_2 - D_2^2/D_1$ ,  $D_{55}^c = 2G$ ,  $D_{44}^c = D_{66}^c = \alpha 2G$ , all other components  $D_{ij}^c$  being zero. Factor  $\alpha$  would be zero if cracks caused complete separation of surfaces; however, due to surface roughness and aggregate interlock this is not so, and  $\alpha$  is generally nonzero ( $\alpha < 1$ ), often taken as  $\alpha \approx 1/2$ . More realistically,  $\alpha$  should be considered to be a function of crack opening width  $w$ , shear strain  $\epsilon_{12}$  (or  $\epsilon_{13}$ ), and crack spacing  $s$ ;  $\alpha = f(w, \epsilon_{12}, s)$ . The stiffness matrix for concrete which is cracked in two directions can be derived similarly.

The foregoing well-known formulation disregards the fact that cracks do not form suddenly but gradually, especially under dynamic loads. So, it is necessary to use a stiffness matrix for a partially cracked concrete. Let  $a_1, a_2, a_3$  be the uncracked area fractions normal to directions  $x_1, x_2, x_3$ . Assume that  $0 \leq a_1 \leq a_2 \leq a_3 \leq 1$ . The cracked area fractions are  $1 - a_1 \geq 1 - a_2 \geq 1 - a_3$ . The area fraction cracked in all three directions is then  $1 - a_3$ ; the area fraction  $a_3 - a_2$  is a fraction cracked in two directions,  $x_1, x_2$ ; the area fraction  $a_2 - a_1$  is a fraction cracked in one direction,  $x_1$ , and the area fraction  $a_1$  is not cracked in any direction. This intuitive reasoning suggests that the stiffness matrix of a partially cracked concrete in arbitrary global coordinates may be taken as

$$\underline{D} = \underline{R}^T \{ a_1 \underline{D}^s + (a_2 - a_1) \underline{D}^c + (a_3 - a_2) \underline{D}^{cc} \} \underline{R} \quad (9)$$

where  $\underline{D}^{cc}$  is the stiffness matrix of concrete cracked in two directions,  $\underline{D}^s$  is the isotropic stiffness matrix of solid (uncracked) concrete, and  $\underline{R} =$  matrix of rotation from crack directions  $x_1, x_2, x_3$  to global coordinates. The effect of cracking is characterized by three quantities  $a_1, a_2, a_3$  associated with three directions  $x_1, x_2, x_3$ . This is a similar situation like for principal stresses. Thus, for eq. (9) to satisfy tensorial invariance restrictions,  $a_1, a_2, a_3$  must be considered to be the extreme values of the area fractions among all possible directions, and one must define damage tensor  $a_{ij}$  such that its principal values are  $a_1, a_2, a_3$ , and their associated principal directions are  $x_1, x_2, x_3$ .

The alternative concept of damage, such that  $\sigma_{ij} = D_{ijmn} \epsilon_{mn}$  with  $D_{ijmn} = a_{ik} D_{kjmn}^s$ , has been also explored. This, however, does not yield matrices  $\underline{D}^c$  and  $\underline{D}^{cc}$  for fully cracked con-

crete as special cases, while eq. (9) is justified mainly by the fact that it yields these special cases. Nevertheless, other approaches may be possible, and eq. (9) should be considered as tentative.

Differentiating the elastic relation  $\sigma = D\varepsilon$ , one obtains the incremental stress-strain relation  $d\sigma = Dd\varepsilon + dD\varepsilon = Dd\varepsilon + d\sigma^{fr}$  where  $d\sigma^{fr} = dD\varepsilon =$  inelastic fracturing stress increments, in which  $dD$  is a change of stiffness due to increment  $da_{ij}$  of the damage tensor and change  $d\beta$  in its principal directions. The rules for calculating  $da_{ij}$  will have to be determined by fitting tensile strain-softening diagrams for concrete.

#### 5. Pore Pressures and Moisture Movement in a Heated Concrete Wall

Another problem of importance for predicting the response of a concrete vessel in accident situations is the rapid heating of concrete walls which would be caused by the liquid sodium. A theoretical model for the pore pressures, heat conduction and moisture diffusion in concrete heated beyond 100°C has been presented at the previous SMIRT conference [9]. In subsequent work, material parameters of this model have been approximately determined by analyzing [10] available test data, which included experimental investigations of rapid high temperature drying at Northwestern University [10], as well as tests of pore pressure and temperature distributions in heated specimens reported by England [11] and by Shevchenko [12]. The test data were fitted using a finite element program for nonlinear coupled heat conduction and moisture diffusion. This analysis furnished, in particular, the approximate form of the sorption equation, relating water content, pore pressure and temperature, and the approximate dependence of diffusivity of moisture in concrete upon temperature. The latter dependence was found to exhibit an increase by two orders of magnitude when the temperature of 100°C is exceeded.

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