A MODEL FOR CAPACITY CONTROL OF A REINFORCED CONCRETE SHELL SECTION

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SUMMARY

A statical analysis of a concrete shell yields results in the form of

bending moments	M_{xx}, M_{yy}, M_{xy},
in-plane forces	N_{xx}, N_{yy}, N_{xy},
transverse shear forces	M_{xz}, N_{yz}.

The design of the section to resist these stress resultants safely is not straight-forward. This paper suggests a method for computing a state of stress in concrete and reinforcement being in equilibrium with the given stress resultants. This is done by a nonlinear analysis considering realistic constitutive relations both for concrete and for reinforcement. The shell thickness is divided into an optional number of concrete layers. Within each layer, biaxial constitutive relations based on test results are established in the principal strain directions. A formulation which also incorporates the stress-strain curves specified in several design codes (ACI 318-71, CP 110, NS 3473 etc.) is used.

The stress-strain relations for each reinforcement layer are also based on test curves. A bilinear approximation is used.

The equilibrium equations are established by a summation of the contributions from each concrete and reinforcement layer. An iterative solution procedure is used until convergence is achieved.

The model ignores the influence from transverse shear forces. The method satisfies equilibrium, but violates to some extent compatibility, as the strains assumed in the capacity control are not compatible with the displacements in a traditional shell analysis based on a linearly elastic shell theory.

The reinforcement is adjusted so to get sufficient capacity or eliminate superfluous steel. This is performed stepwise. First, modifications for not exceeding the concrete capacity are performed if necessary. Next, too highly strained reinforcement layers are modified. Finally, the reinforcement ratio of layers with too large capacities is reduced. After each modification, a state of equilibrium is computed if possible.

The method has been used in a computer program which has been tested on offshore shell structures. The method has been compared with other more conventional methods currently used in design. The experience with the model is very satisfactory.

1. Introduction

When designing the reinforcement of a concrete shell, the section forces M_{xx}, M_{yy}, M_{xy}, N_{xx}, N_{yy}, N_{xy}, N_{xz} and N_{yz} are given from the statical model. At this stage, the problem is to choose suitable reinforcement for these ULS forces. A state of deformation with stresses in equilibrium with the given forces must be found without exceeding the largest allowable strains. This is in fact a very complicated problem. In order to achieve methods which can be used in practice, simplifications are necessary. A common simplification which will also be adopted here, is to neglect the transverse shear forces N_{xz} and N_{yz} in the solution. A separate check for ample capacities for these forces must therefore be performed afterwards.

Most of the solutions of the problem obtained by the simplification above, are based on some plasticity theory. In the model presented here the equilibrium state is found by a nonlinear analysis taking arbitrary stress-strain relations both for concrete and for reinforcement into account. For convenience this model is based on the finite element method. The computations then encompass only one shell element with constant strains and stresses over the area (by definition).

2. Analytical Model

2.1 Stiffness Matrix of the Shell Element

In Fig.1 a shell element representing the sections under consideration in shown in the xy-plane. The displacement components u,v and w of a point can be expressed by

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} N_u & 0 & -zN_{w,x} \\ 0 & N_v & -zN_{w,y} \\ 0 & 0 & N_w \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{bmatrix} \qquad (1)$$

where N_u, N_v and N_w are the interpolation functions, and $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are the displacements of the nodes. For further description of the shell element, see ref.[1].

The strains are expressed by

$$\mathbf{e}_{xy} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} = \begin{bmatrix} N_{u,x} & 0 & -zN_{w,xx} \\ 0 & N_{v,y} & -2zN_{w,yy} \\ N_{u,y} & N_{v,x} & -2zN_{w,xy} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{bmatrix} = \mathbf{BU} \qquad (2)$$

The stresses are expressed by a straindependent material matrix D_s and the strains (see section 2.2).

$$\mathbf{s}_{xy} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = D_s \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = D_s \mathbf{e}_{xy} \qquad (3)$$

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From the basic equilibrium equations (principle of virtual work) the stiffness relations can be developed

$$\int_V B^T D_s B dV U = R \tag{4}$$

Here R is the load vector containing the consistent nodal forces. This non-linear equation (D_s dependent on U) is solved by iteration. During the first iteration cycle, linearly elastic material is assumed. Later on D_s is computed according to the current state of strains. The integration over the height is then performed numerically, see sections 2.2 and 2.3, and the area integration analytically. The solution is considered to have converged when the changes in all strain components on both sides of the shell element are smaller than a prescribed value (10^{-6} is considered to be acceptable when the reinforcement is not modified).

2.2 Material description - Concrete

The plate thickness is divided into an optional number of layers. For plane stress problems, one layer is sufficient. For ordinary plate and shell problems at least seven layers should be prescribed. D_s is assumed to be constant within each layer and all relations apply to the centreplane of the layer. The integration is thus substituted by a summation.

For each layer, the principal strains are computed from

$$\mathbf{e}_{12} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \end{bmatrix} = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & \sin\alpha\cos\alpha \\ \sin^2\alpha & \cos^2\alpha & -\sin\alpha\cos\alpha \\ -2\sin\alpha\cos\alpha & 2\sin\alpha\cos\alpha & \cos^2\alpha-\sin^2\alpha \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = C\mathbf{e}_{xy} \tag{5}$$

where the angle α between the principal strain direction and the x-axis, see Fig.2, is given by

$$\alpha = 0.5 \text{ Arctg } \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \tag{6}$$

The stress-strain relations are established in the principal strain coordinate system

$$\mathbf{s}_{12} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ 0 \end{bmatrix} = \mathbf{D}_{12}\,\mathbf{e}_{12} \tag{7}$$

σ_1 (and σ_2) are computed according to the stress-strain curve of Fig.3 (identical with the curve given in the codes ACI 318-71, CP110 and NS 3473 if f_t and σ_1^0 is chosen equal to zero)

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$$\sigma_1 = \sigma_1^0 + 1000(f_c - \sigma_1^0)\varepsilon_1 \qquad \text{for } \varepsilon_1 \geq 0 \qquad (8)$$

$$\sigma_1 = \sigma_1^0 + 1000(f_c - \sigma_1^0)\varepsilon_1 (1 + \frac{\varepsilon_1}{0.004}) \qquad \text{for } 0 > \varepsilon_1 > -0.002$$

$$\sigma_1 = f_c \qquad \text{for } \varepsilon_1 \leq -0.002$$

The maximum stress f_c is chosen according to the actual concrete quality. This maximum stress may be kept constant or it may be assumed to be dependent of the strain in the orthogonal direction, according to the formulas

$$\left.\begin{array}{l} f_c = (\sqrt{2.45^2 - (0.6 + \varepsilon_2/0.002)^2} - 1.3)f_{c_0} \\ \quad \geq f_{c_0} \end{array}\right\} \text{for } \varepsilon_2 > -0.0028 \qquad (9)$$

$$f_c = f_{c_0} \qquad \text{for } \varepsilon_2 \leq -0.0028 \qquad (10)$$

where f_{c_0} is the maximum uniaxial stress. This is a close approximation to the isostrain $\varepsilon_1 = -2$ o/oo in Fig.4.

The maximum tensile stress f_t may be chosen equal to zero or may be computed from

$$f_t = \begin{cases} f_{t_0} & \text{for } \varepsilon_2 \geq 0 \\ f_{t_0}(1 + \varepsilon_2/0.0022) & \text{for } 0 \geq \varepsilon_2 > -0.0022 \\ 0 & \text{for } \varepsilon_2 \leq -0.0022 \end{cases} \qquad (11)$$

where f_{t_0} is the maximum uniaxial tensile stress. Eq.11 is a close approximation to the dotted line for maximum tensile stress in Fig.4.

The isostrain $\varepsilon_1 = 0$ in Fig.4 is approximated by

$$\sigma_1^0 = \sigma_1(\varepsilon_1 = 0) = \begin{cases} 0 & \text{for } \varepsilon_2 \geq 0 \\ -180\, f_{c_0}\, \varepsilon_2 & \text{for } 0 > \varepsilon_2 \geq -0.0008 \\ (0.144 - 63(\varepsilon_2 + 0.008))f_{c_0} & \text{for } -0.0008 > \varepsilon_2 > -0.003 \\ -0.2826\, f_{c_0} & \text{for } \varepsilon_2 \leq -0.003 \end{cases} \qquad (12)$$

The secant moduli D_{11} and D_{12} are computed from

$$\left.\begin{array}{l} D_{11} = (\sigma_1 - \sigma_1^0)/\varepsilon_1 \\ D_{12} = \sigma_1^0/\varepsilon_2 \end{array}\right\} \text{for } \varepsilon_1 > -0.002 \qquad (13)$$

$$\left.\begin{array}{l} D_{11} = f_c(\varepsilon_2 = 0)/\varepsilon_1 \\ D_{12} = (f_c - f_c(\varepsilon_2 = 0))/\varepsilon_2 \end{array}\right\} \text{for } \varepsilon_1 \leq -0.002 \qquad (14)$$

The stress and the secant moduli in the second direction are computed in the same manner. In order to get a symmetric matrix, D_{12} and D_{21} are inserted by the average of the two computed values. D_{11} and D_{22} are then modified so as to maintain the same stress level.

D_{33} is inserted in order to ensure some shear stiffness of cracked regions. This has a very beneficial effect on the computation procedure. D_{33} is kept proportional to $(D_{11} + D_{22})$

$$D_{33} = 0.5(D_{11} + D_{22})\frac{G}{E} \tag{15}$$

where E and G are the moduli of elasticity and shear of uncracked concrete.

The stress-strain relations Eq.(7) are transformed to the xy-coordinate system

$$\mathbf{s}_{xy} = \mathbf{C}^T \mathbf{s}_{12} = \mathbf{C}^T \mathbf{D}_{12}\, \mathbf{C} \mathbf{e}_{xy} = \mathbf{D}_s^c \mathbf{e}_{xy} \tag{16}$$

2.3 Material Description - Reinforcement

Each reinforcement layer is treated in a similar manner as described above for the concrete layers. Here the direction of the reinforcement bars is used instead of the principal strain direction. The idealized reinforcement layer is given the thickness μT where μ is the reinforcement ratio and T the plate thickness, see Fig.5.

The strain in a reinforcement bar is given by

$$\varepsilon_u = [\cos^2\alpha \;\; \sin^2\alpha \;\; \sin\alpha\cos\alpha] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \mathbf{C}_1 \mathbf{e}_{xy} \tag{17}$$

Further, the stress is expressed by the secant modulus E_{su} and the strain ε_u

$$\sigma_u = E_{su}\varepsilon_u \tag{18}$$

where E_{su} is computed from the stress-strain curve of Fig.6 (identical with the curve in the Norwegian Code NS 3473). The maximum stress f_s is chosen according to the quality of the reinforcement steel. Transformed into the xy-coordinate system, the following equation is derived

$$\sigma_{xy} = \mathbf{C}_1^T \sigma_u = \mathbf{C}_1^T E_{su} \mathbf{C}_1 \mathbf{e}_{xy} = \mathbf{D}_s^r \mathbf{e}_{xy} \tag{19}$$

3. Design of Reinforcement

Before the computations are started, a reasonable amount of reinforcement should be assumed. The reinforcement ratio of each layer may be changed and new layers inserted during the computations in order to provide sufficient capacity or eliminate superfluous reinforcement. No reduction or no increase of the reinforcement ratio may be specified for each layer.

When the largest allowable concrete compressive strain has been exceeded, the reinforcement layer in the compression zone with the bars oriented closest to the principal compressive strain direction is sought. If the angle between those two directions is smaller than 45^o, the reinforcement ratio of the layer is increased with a prescribed factor. If the angle is larger, a new layer oriented in the principal direction of compression and with a prescribed reinforcement ratio and distance from the outer face is inserted. The computations are then repeated until there is sufficient capacity in the concrete.

When the maximum allowable reinforcement strain has been exceeded, the reinforcement ratio of the highest strained layer is increased with a prescribed factor. The computations are again repeated until the strain restrictions are not violated.

The reinforcement ratio of the less strained reinforcement layer is reduced with a prescribed factor if the stress is smaller than the design stress f_s. Again the computations are repeated until a satisfactory equilibrium state is found.

If the assumed reinforcement is far from the "optimum" (as defined by the program), the optimization process may be rather time-consuming. This can be avoided by running the program twice. In the first run, none or only a few changes in the reinforcement should be specified. Based on the results from the first run, a more correct reinforcement can be given as input to the second run.

4. Example of Use

In Fig.7 section forces, shell thickness and reinforcement ratios are shown for a design case. Some of the computer output from the program described in ref.[3] is shown in Table 1. Maximum 2 changes of the reinforcement are prescribed. The first equilibrium state is found after 9 iterations. All the unbalanced forces are less than 3% of the corresponding given forces. Reinforcement layer no.4 is the less strained layer. Since no strain limits have been exceeded, the reinforcement ratio of this layer is reduced by the given factor 0.8. A new equilibrium state is found after 3 iterations and the same procedure is thereafter repeated once. If an "optimal" amount of reinforcement is sought, a new input with reduced reinforcement should be given to the computer program.

5. Conclusion

The method described here has been tested on offshore shell structures and composed with other more conventional methods currently used in design. The results from the model so far is very satisfactory.

6. References References

[1] BERG, S.: "Nonlinear Finite Element Analysis of Reinforced Concrete Plates". Report No.73-1 Div. of Struct. Mech., The Norwegian Institute of Technology, Trondheim, 1973.

[2] KUPFER, H.: "Das Verhalten des Betons unter zweiachsiger Beanspruchung". Bericht Nr.78, Lehrstuhl für Massivbau, Technische Hochschule, München, 1969.

[3] BERG, S.: CAPCONSHELL - A Program for Capacity Control of a Reinforced Concrete Shell Section. User's Manual. SINTEF, Div. of Structural Engineering, Report No. STF71 A76007, Trondheim, March 1976.

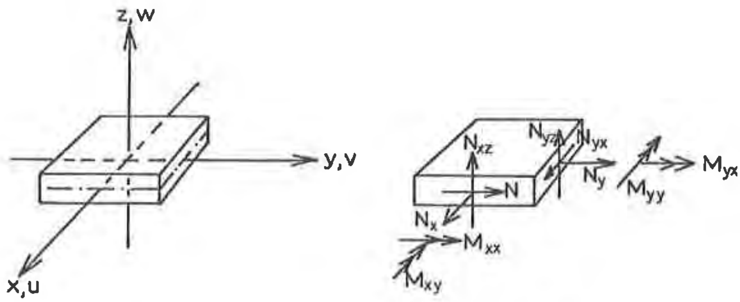

FIG.1 A Shell Element

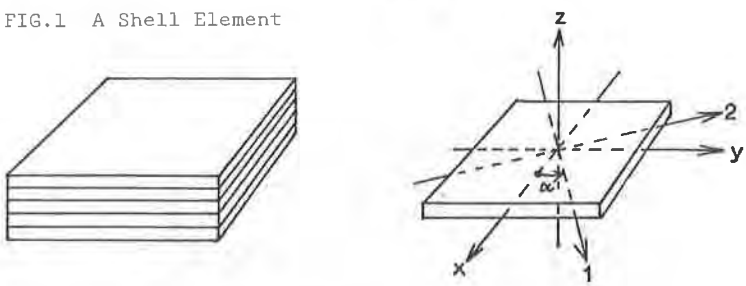

FIG.2 Layer Subdivision of the Element

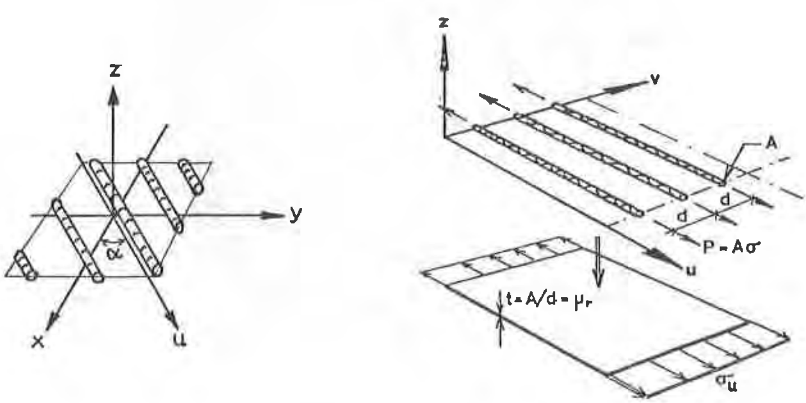

FIG.5 Idealization of a Layer of Reinforcement

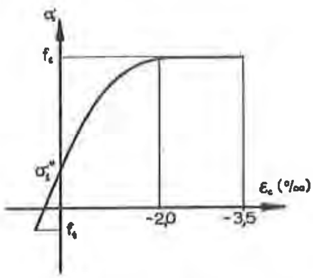

FIG.3 Stress-strain Curve for Concrete

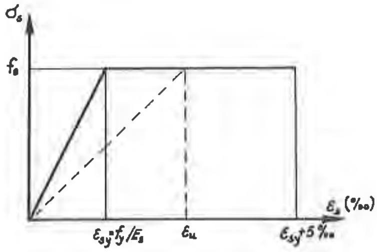

FIG.6 Stress-strain Curve for the Reinforcement (NS 3473)

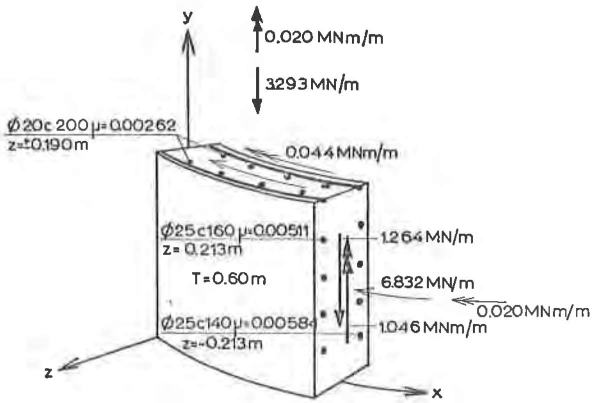

FIG.7 A Shell Section with Combined Forces

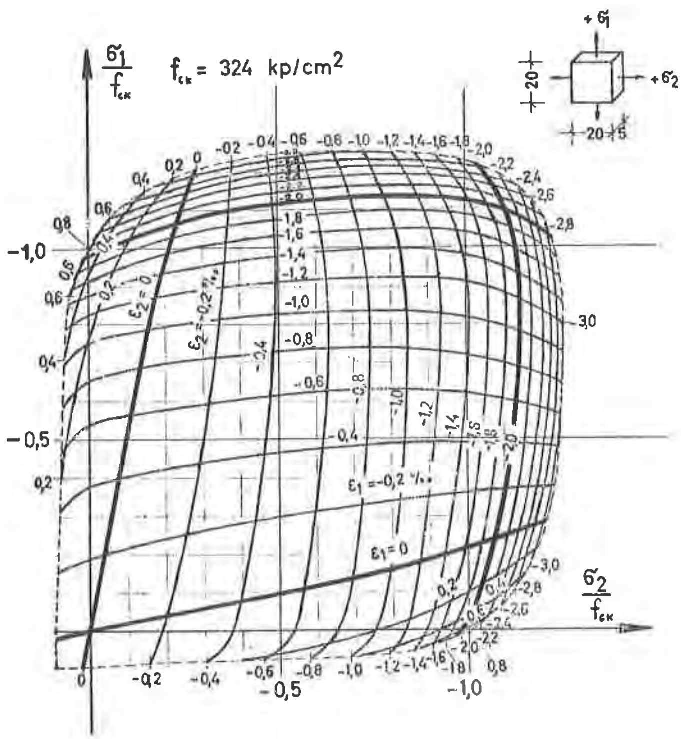

FIG.4 Biaxial Stress-strain Dependence according to Kupfer [2]

TABLE 1   Example of use - Computer Output

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TEST FOR COMBINED SHELL FORCES

LENGTH UNIT:M
FORCE UNIT: MN

CONCRETE QUALITY C  45
FCN= 24.0. ALLOWABLE COMPRESSIVE STRESS FC= 19.2 (GAMMA M=1.25). ALLOWABLE TENSILE STRESS:  .00. SHELL THICKNESS:     .60
BIAX= .00 (ALLOWABLE BIAXIAL COMPRESSIVE STRESS=FC*BIAX*DELTA FCUIAXIAL).
REINFORCEMENT STEEL QUALITY: ST  40
YIELDING LIMIT FY=400.0 MAX. ALLOWABLE STRESS FS=320.0 (GAMMA M=1.25). MODULUS OF ELASTICITY E=  210000.0


NUMBER OF REINFORCEMENT LAYERS: 4
REINF.LAYER NO.  DIRECTION  DIST. FROM CENTREPL.      REINF. RATIO   CHANGE RESTRICTION.(-1=NO ENCREASEMENT, 1=NO REDUCTION)
          1          .00               .21               .005            .000
          2          .00              -.21               .006            .000
          3        90.00               .19               .003            .000
          4        90.00              -.19               .003            .000

THE SECTION IS DIVIDED INTO  7 LAYERS
MAXIMUM NUMBER OF ITERATION CYCLES 14
CONVERGENCE CRITERION    .00500. (LARGEST CHANGE IN A STRAIN COMPONENT ON TOP OR BOTTOM SIDE/.002)

MAXIMUM  2 CHANGES OF THE REINFORCEMENT.
CONCRETE COVER+HALF ROD DIAMETRE OF POSSIBLE NEW REINFORCEMENT LAYER:    .130
REINFORCEMENT RATIO OF ADDITIONAL LAYER WHEN THE CONCRETE CAPACITY IS INSUFFICIENT: .001
REINFORCEMENT RATIO MODIFICATION FACTOR FOR TOO HARD STRAINED REINFORCEMENT:1.250
MODIFICATION FACTOR FOR TOO LITTLE STRESSED REINFORCEMENT: .800

AXIAL FORCE NXX:                             -6.832
AXIAL FORCE NYY:                             -3.293
INPLANE SHEAR FORCE NXY:                     -1.264
BENDING MOMENT MXX:                           1.046
BENDING MOMENT MYY:                            .044
TORSIONAL MOMENT MXY:                          .020
TRANSVERSAL SHEAR FORCE NXZ:                 -1.297
TRANSVERSAL SHEAR FORCE NYZ:                   .029***NXZ AND NYZ ARE NOT CONSIDERED IN THE COMPUTATION.
                                                   ***CAPACITIES FOR THESE FORCES MUST BE CHECKED BY HAND**N9**



*********************************************************************************************************************
ITERATION CYCLE NO.  9


        COMPUTED FORCES      GIVEN FORCES       UNBALANCED
NXX=       -6.820328          -6.832000          -.011672
NYY=       -3.290794          -3.293000          -.002206
NXY=       -1.260799          -1.264000          -.003201
MXX=        1.043743           1.046000           .002257
MYY=         .043585            .044000           .000415
MXY=         .019423            .020000           .000577

CONCRETE
LAYER              SIGMA X                                   SIGMA Y                                      TAU XY

 1      -.1800                 *             -4.9816              I      *          -.9468               I  *
 2      -.6446                 *             -4.8208              I      *         -1.7628               I     *
 3     -4.4981                 I *           -4.4133              I     *          -2.9795               I        *
 4    -11.1852                 I    *        -5.4108              I       *        -2.6607               I       *
 5    -15.7904                 I      *      -5.6664              I       *        -2.4723               I      *
 6    -18.3091                 I        *    -5.8548              I        *       -2.1450               I     *
 7    -18.9691                 I        *    -6.0522              I        *       -1.7923               I    *
                    STRAINS UPPER SIDE                                       STRAINS LOWER SIDE
       EPS X           EPS Y            GAMMA                      EPS X            EPS Y             GAMMA
     .00096430      -.00026300      -.00037444                  -.00235064       -.00012413       -.00054639

REINFORCEMENT
LAYER   DIRECT.         Z          MY          EPS         SIGMA       SIG X        SIG Y       TAU XY
  1       .000        .213      .005110     .000481      80.786      80.726        .000        .000
  2       .000       -.213      .005840    -.001867    -313.691    -313.691        .000        .000
  3     90.000        .190      .002620    -.000238     -39.906       -.000     -39.906        .000
  4     90.000       -.190      .002620    -.000150     -25.131       -.000     -25.131        .000



THE ITERATIONS FINISHED AFTER  9 CYCLES
THE SOLUTION SATISFIES THE CONVERGENCE CRITERION      .0044
REINFORCEMENT LAYER NO. 4 MODIFIED
REINFORCEMENT:

REINF. LAYER   DIRECT.   DISTANCE FROM CENTRE  REINFORCEMENT RATIO
         1        .00              .21               .0051
         2        .00             -.21               .0058
         3      90.00              .19               .0026
         4      90.00             -.19               .0021

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Cont

TABLE 1 Cont.

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ITERATION CYCLE NO. 3

      COMPUTED FORCES      GIVEN FORCES       UNBALANCED
NXX=       -6.828653          -6.832000         -.003347
NYY=       -3.292130          -3.293000         -.000870
NXY=       -1.261758          -1.264000         -.002242
MXX=        1.045537           1.046000          .000463
MYY=         .043466            .044000          .000534
MXY=         .020111            .020000         -.000111

CONCRETE
LAYER              SIGMA X                                 SIGMA Y                                   TAU XY

1      -.1764             *                  -4.9743            I       *         -.9368            I *
2      -.6379             *                  -4.8219            I       *        -1.7538            I    *
3     -4.5013             I *                -4.4230            I      *         -2.9800            I        *
4    -11.2119             I    *             -5.4307            I        *       -2.6659            I       *
5    -15.8212             I      *           -5.6874            I        *       -2.4813            I      *
6    -18.3062             I         *        -5.8813            I         *      -2.1534            I     *
7    -18.9667             I         *        -6.0842            I         *      -1.7491            I    *

               STRAINS UPPER SIDE                                   STRAINS LOWER SIDE
       EPS X           EPS Y          GAMMA                  EPS X          EPS Y          GAMMA
    .00096928      -.00026271     -.00037154            -.00235992     -.00012630     -.00055223

REINFORCEMENT
LAYER   DIRECT.          Z         MY          EPS        SIGMA       SIG X       SIG Y      TAU XY
  1        .000        .243     .005110     .000484      81.273      81.273       .000        .000
  2        .000       -.243     .005840    -.001874    -314.901    -314.901       .000        .000
  3      90.000        .190     .002620    -.000238     -39.935      -.000      -39.935       .000
  4      90.000       -.190     .002096    -.000151     -25.420      -.000      -25.420       .000


THE ITERATIONS FINISHED AFTER  3 CYCLES
THE SOLUTION SATISFIES THE CONVERGENCE CRITERION      .0016
REINFORCEMENT LAYER NO. 4 MODIFIED
REINFORCEMENT:

REINF. LAYER   DIRECT.   DISTANCE FROM CENTRE  REINFORCEMENT RATIO
      1           .00               .21                .0051
      2           .00              -.21                .0058
      3         90.00               .19                .0026
      4         90.00              -.19                .0017

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ITERATION CYCLE NO. 3

      COMPUTED FORCES      GIVEN FORCES       UNBALANCED
NXX=       -6.830366          -6.832000         -.001634
NYY=       -3.292677          -3.293000         -.000323
NXY=       -1.262180          -1.264000         -.001820
MXX=        1.045917           1.046000          .000083
MYY=         .043502            .044000          .000498
MXY=         .020340            .020000         -.000340

CONCRETE
LAYER              SIGMA X                                 SIGMA Y                                   TAU XY

1      -.1754             *                  -4.9665            I       *         -.9332            I *
2      -.6362             *                  -4.8212            I       *        -1.7513            I    *
3     -4.5023             I *                -4.4294            I      *         -2.9803            I        *
4    -11.2182             I    *             -5.4436            I        *       -2.6679            I       *
5    -15.8279             I      *           -5.7047            I        *       -2.4845            I      *
6    -18.3093             I         *        -5.9038            I         *      -2.1565            I     *
7    -18.9656             I         *        -6.1116            I         *      -1.7516            I    *

               STRAINS UPPER SIDE                                   STRAINS LOWER SIDE
       EPS X           EPS Y          GAMMA                  EPS X          EPS Y          GAMMA
    .00097039      -.00026213     -.00037057            -.00236193     -.00012846     -.00055435

REINFORCEMENT
LAYER   DIRECT.          Z         MY          EPS        SIGMA       SIG X       SIG Y      TAU XY
  1        .000        .213     .005110     .000484      81.383      81.383       .000        .000
  2        .000       -.213     .005840    -.001876    -315.163    -315.163       .000        .000
  3      90.000        .190     .012620    -.000238     -39.921      -.000      -39.921       .000
  4      90.000       -.190     .001677    -.000153     -25.698      -.000      -25.698       .000

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THE ITERATIONS FINISHED AFTER  3 CYCLES
THE SOLUTION SATISFIES THE CONVERGENCE CRITERION      .0019

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