ANALYTIC MODELING OF THE IMPACT OF SOFT MISSILES ON PROTECTIVE WALLS

K. HORNYIK

Department of Nuclear Engineering, Oregon State University,
Corvallis, Oregon 97331, U.S.A.

SUMMARY

Analytic models are derived from principles of conservation of energy and momentum as well as common engineering assumptions to describe the perpendicular impact of deformable missiles on yielding walls with an ideal plastic behavior. Such modeling is of interest in making risk analyses for nuclear plants where the impact of crashing aircraft or wreckage from nearby explosions must be considered.

The description of the soft missile postulates the existence of a stationary deformation zone in the immediate vicinity of the wall. Deformation is confined to those portions of the missile which enter into this zone. While earlier authors have used this assumption in conjunction with repeated application of the momentum equation, this study also considers energy conservation as applied to missile and wall. The concept of a crushing-energy per unit missile length is introduced, which may be seen as an equivalent to the crushing-load used in previous work. In addition, the model allows for residual kinetic energy of the deformed portion of the missile as they might leave the impact zone. This residual energy is subject to certain limitations due to the available energy and restriction of reaction forces to compression only.

Based on this model and the assumption of unyielding walls, solutions are obtained for the reaction-force history from the impact of missiles characterized by parameters and conditions covering a wide range of cases of practical interest. The equivalence of parametric variations of the impact velocity and the crushing-energy function is established. Solution of the general case requires numerical techniques for which a simple algorithm is devised. This is applied to the case of the impact of a FB-111 jet aircraft, which commonly has been employed as a reference case in reactor safety studies. While for high impact velocity the reaction-force-history approaches the shape of the crushing-energy function, significant deviations are found at lower velocities.

To include the effect of yielding walls, this study considered the motion of the coupled system as well as the uncoupled system. The latter case—which has been considered in past studies—results from obtaining the reaction-force history from a rigid wall analysis and then applying it to a yielding wall. The degree of conservatism introduced by this obvious approximation is exposed by evaluation of sample cases using the more rigorous coupled system model for which numerical algorithms also are provided. The assumptions regarding the wall include ideal plastic behavior and permit application of yield line theory and a corresponding failure criterion.

In view of the need for simple yet reliable models for the analysis of missile impact on protective barriers and the complexity of the relevant phenomena, the models presented here are seen as a significant improvement over previous work, as they are based on a wider range of physical laws and are formulated in terms of observable quantities. In this manner, they afford greater accuracy, a higher degree of confidence in the results, and better insight into the impact mechanisms.
1. **Introduction**

Various safety aspects of nuclear installations require consideration of flying objects impinging on structural barriers. Postulated events include: aircraft crashes, tornado-generated missiles, flying wreckage from nearby explosions and from catastrophic failure of turbines or similarly hazardous components inside the plant. Although this problem has been studied extensively in connection with weapons effects, there appears a new element in the above applications: the postulated events are recognized as being rare and corresponding safety analyses are based in part on probabilistic considerations. This necessitates that the deterministic elements of the analysis be as simple as possible while retaining all essential physical characteristics.

Consequently, one brackets impact problems between the "hard" missile case and the "soft" missile case, predicated on the assumption a rigid/soft missile and a soft/rigid barrier, respectively, as described by J.D. Stevenson [1]. The "soft" missile approach appears applicable to the impact of crashing aircraft, as well as certain postulated tornado missiles (automobile, steel pipe) and wreckage from explosions; objects which are characterized by their potential for a high degree of deformation. This is the case which will be considered here.

The ultimate objective is the assessment of the integrity of the barrier. This is accomplished in two steps. First, one calculates the reaction force on the barrier as a function of time, assuming a perfectly rigid barrier. Then one analyzes the barrier response subject to this reaction force history. The validity of the effective uncoupling of the response of the colliding bodies hinges on certain implied conditions; e.g., period of elastic vibrations of the barrier much longer than impact duration, and effective mass of the barrier much greater than that of the missile. Errors introduced by uncoupling can be investigated by the methods developed in this study.

2. **"Soft" Missile Model**

The conceptual model for "soft" missiles has been applied in this context by various authors, e.g., J.L. Haley et al. [2], and in reference [3]. It is assumed that the major portion of the missile behaves as a rigid body while deformations remain confined to a small zone near the barrier. The earlier studies mentioned above establish the relation for the reaction force on the basis of applying the equation of motion twice: once to the entire missile and then to the rigid portion only; also involved is the crushing load P, defined as the force required to "crush" a cross-sectional slab located at distance x from the nose of the missile in the direction of motion (assumed perpendicular to flat barrier). The present model involves both conservation principles as pertaining to momentum and to energy. The latter has been neglected in past studies, although one can expect this principle to be of great relevance.

Application of the momentum equation in the direction of motion (x-direction) yields:

\[ F = \frac{d}{dt} (m \cdot v) \]

Consistent with the assumption of the small deformation zone, into which mass enters and in which mass remains confined without any further change of momentum in the x-direction, one can develop this equation as
\[ F = \frac{\text{d}m}{\text{d}t} + m \frac{\text{d}v}{\text{d}t} \]  

(1)

where \( v \) is the velocity (in the \( x \)-direction) of the rigid portion of the missile, \( m \) is its mass, and \( \frac{\text{d}m}{\text{d}t} \) is the rate at which mass enters the deformation zone. This term can be written as

\[ \frac{\text{d}m}{\text{d}t} = \frac{\text{d}m}{\text{d}x} \frac{\text{d}x}{\text{d}t} = -\mu v \]

where we have defined the linear mass density \( \mu = \frac{\text{d}m}{\text{d}x} \). Thus, we obtain for the reaction force:

\[ F = -\mu v^2 + m \frac{\text{d}v}{\text{d}t} \]

(2)

The conservation of energy principle can be formulated as:

\[ \text{d}K = 0 \]

where \( K \) encompasses all forms of energy significant to the collision process. In particular, we consider the following breakdown:

a) kinetic energy of the rigid portion of the missile in the \( x \)-direction:

\[ \text{d}E_1 = \frac{1}{2} m v^2 \]

b) mechanical work expended in deforming (crushing) the mass as it enters the deformation zone; using an ideal elasto-plastic deformation model, this term can be written as:

\[ \text{d}E_2 = \sigma_y \varepsilon_y \text{d}x = P_y \varepsilon_y \text{d}x = e \text{d}x \]

where \( \sigma_y \) and \( \varepsilon_y \) are yield stress and yield strain respectively and \( A \) is the cross-sectional area of the mass entering into the deformation zone (= area in contact with barrier). The force \( P_y = \sigma_y A \) thus corresponds to the crushing load used by other authors. The quantity \( e \) is the total deformation work absorbed per unit length at coordinate \( x \).

c) residual (kinetic) energy of the mass after deformation; assuming that motion of this mass is confined to directions perpendicular to the \( x \)-direction (i.e., mass is confined to the deformation zone), that the deformed mass moves as incoherent small pieces, and that the velocity of these pieces is in proportion to the momentary incoming velocity of the missile, we can write:

\[ \text{d}E_3 = \frac{1}{2} m \text{v}^2 \text{f} \]

where \( \sqrt{\text{f}} \) is the constant of proportionality between \( v \) and the residual velocity.

Expressing the infinitesimal change of energy \( \text{d}E \) over the increment \( \text{d}x \) and introducing the above relations with appropriate signs, we get:

\[ -\text{e} = \frac{1}{2} \mu v^2 \quad (1-\text{f}) + m \frac{\text{d}v}{\text{d}t} \]

(3)

This equation determines the motion of the missile when initial velocity \( v_0 \) and the missile properties \( e, \mu, \) and \( f \) are known functions of the \( x \)-coordinate, while the reaction force \( F \) is obtained from equation (2). It is convenient to substitute for the derivative term in this equation by using equation (3):

\[ F = - \left[ \frac{1}{2} \mu v^2 \cdot (1+\text{f}) + \text{e} \right] \]

(4)

For the special case that \( f = 1 \), equations (3) and (4) reduce to:

\[ -e = m \frac{\text{d}v}{\text{d}t} \]

(5)

and:

\[ F = - \left[ \mu v^2 + e \right] \]

(6)
Assuming further that ε_{v} ≡ 1, then e → 0 and these equations become identical to those derived in earlier references [1] [2], as mentioned previously. The meaning of f ≠ 0 is, of course, that mass entering the deformation zone gets merely deflected by 90° but does not lose any kinetic energy.

Another special case is obtained when f ≠ 0. Equations (3) and (4) reduce to:
\[ -e = -\frac{1}{2} \mu v^2 + m_{i} \frac{dv}{dt} \]  
and:
\[ F = \left[ \frac{1}{2} \mu v^2 + e \right] \]

This implies that there is no residual kinetic energy of the mass entering the deformation zone. However, this may not be physically possible under the assumptions made here; as can be seen from equation (3), the acceleration \( \frac{dv}{dt} \) must not be positive and therefore:
\[ f \geq 1 - \frac{2e}{\mu v^2} = \epsilon_{min} \]

This says that, if the kinetic energy of the mass arriving at the deformation zone, \( \frac{1}{2} \mu v^2 \), is greater than the energy which can be absorbed by deformation of this mass, e, then there must be residual kinetic energy associated with this mass, as expressed by \( f \cdot \frac{1}{2} \mu v^2 \). Other logical restrictions applying to the value of f are:
\[ 0 \leq f \leq 1 \]

corresponding to the fact that residual kinetic energy cannot be negative nor is it likely to be greater than the kinetic energy at arrival. Additional guidance in determining the quantity f may be obtained from considering the buckling characteristics of the structural elements of the missile.

3. Solutions

Solutions to equations (3) and (4) for general cases can be obtained by numerical integration. Using simple first-order forward extrapolation, one can formulate the following algorithm:

\[ v_{i+1} = v_{i} + (\mu_{i} \cdot v_{i}^2 + F_{i}) \cdot \frac{\Delta t}{m_{i}} \]

\[ m_{i+1} = m_{i} - v_{i} \cdot v_{i} \cdot \Delta t \]

\[ x_{i+1} = x_{i} + v_{i} \cdot \Delta t \]

\[ t_{i+1} = t_{i} + \Delta t \]

\[ F_{i} = \frac{1}{2} (1 + f_{i}) \cdot \mu_{i} \cdot v_{i}^2 + e_{i} \]

where \( \mu_{i}, e_{i}, \) and \( f_{i} \) are the values of \( \mu, e, \) and \( f \) at coordinate \( x_{i} \), respectively. The initial conditions for \( i = 0 \) are: \( x_{0} = t_{0} = 0, m_{0} = \text{entire mass of missile}, v_{0} = \text{initial velocity.} \) The impact terminates when \( v \) reaches zero. This will occur when either all or only part of the missile has entered into the deformation zone, depending on the initial velocity \( v_{0} \) of any given missile.

Solutions to some special cases can be obtained directly and provide useful insights. If the equality in relation 9 is satisfied, then the missile moves with constant velocity \( v_{0} \) and the reaction force is given by \( F = -\mu v_{0}^2 \). The duration of the impact in this case is \( T = 1/v_{0} \) where \( l \) is the dimension of the missile in the z-direction. This also is the asymptotic solution as \( v_{0} \) increases and the deformation energy becomes small relative to the
kinetic energy. Note that \( f \) must approach unity in this case because of relation (9). Explicit solutions also can be obtained for homogeneous prismatic missiles, but are omitted here for the sake of brevity.

A practical case is the impact of an F-111B jet fighter at \( v_o = 200 \) mph, which has been considered variously as conservative case in the design analysis of recent nuclear plants constructed near air traffic centers. The linear mass density \( \mu \) for this aircraft as given by the references [3][5] was used along with various assumptions concerning the deformation energy per unit length, \( e \), and the factor \( f \). As a basic case, \( e \) was taken as equal to the crushing load used in the same references; plots of these quantities are shown in Figure 1 for completeness. The factor \( f \) was assumed to be unity. It is noted that, under these assumptions, the total deformation work which can be absorbed by the aircraft is only 14% of the initial kinetic energy. This results in a reaction force very nearly equal to \( \mu v_o^2 \) as mentioned earlier and as is illustrated by the dash-dotted curve in Figure 2, which is essentially proportional to the plot of \( \mu \) vs \( x \). On the other hand, as \( e \) is increased, one begins to see noticeable deviations from the basic case, generally characterized by higher forces early during the impact and lower forces during the later phases. This compensation, of course, is necessary to satisfy the momentum equation applied to the entire impact which requires that \( \int F \, dt = m v_o \) and which is a constant for all cases considered here. Values of the parameter \( f \) other than unity have not been considered since condition 9 would not admit significantly lower values for the basic case which is of principal interest here.

4. Equivalence Relation

For the impact of a specific missile, one may treat the initial velocity \( v_o \) as a variable parameter. We introduce the scaled quantities:

\[
F^s = F/v_o^2, \quad t^s = t v_o, \quad v^s = \frac{dx}{dt^s} = \frac{dx}{dt} \frac{dt^s}{dt} = v/v_o
\]

and introducing the scaled quantities into the basic equations (3) and (4), we get:

\[
\frac{\dot{v}^s}{v_o^2} = - \left( \frac{\mu}{2} \right) (v^s)^2 (1-f) + m \frac{\ddot{v}^s}{dt^s}
\]

\[
F^s = - \left( \frac{1}{2} \mu (v^s)^2 \cdot (1+f) + \frac{e}{v_o^2} \right)
\]

These equations are identical to the original equations in the unscaled quantities except for the factor \( 1/v_o^2 \) applied to the quantity \( e \). Therefore, one can obtain the (scaled) solution for different values of \( v_o \) directly from the (unscaled) solution for the original case with the quantity \( e \) modified by \( 1/v_o^2 \). In this manner, one can interpret the solutions for various \( v_o \) values also as scaled solutions for various \( v_o \) values when corresponding scale factors for force and time are applied.

5. Coupled System Description

In order to check the validity of the assumption of uncoupled motion of the missile and the barrier, this assumption was dropped and the more general -- albeit more complex -- problem of coupled motion investigated. While keeping the model for the missile unchanged, the following modeling assumptions concerning the barrier were introduced:

a) deformation of the barrier due to the impacting missile can be described by a single coordinate, \( \xi \), which is taken to be the deflection of the impact
point in the direction of missile motion;

b) inertial effects associated with barrier deformation can be
described as linear motion along this coordinate of an effective
barrier mass \( M \);

c) energy is absorbed by the barrier in proportion to the deflection.

These assumptions permit the application of the principles of yield line-theory as
introduced by Hogestad and applied by Kennedy et al. [4]. It is based essentially on
the hypothesis that a loaded thin plate will fail by (plastic) deformation along a line
pattern which is determined by minimizing the virtual work required for deformation. Thus,
with the assumption of an idealized elasto-plastic behavior of the barrier, one can de-
dtermine for a given slab the (static) yield force \( k \) which must be applied at the given
impact location, to cause (plastic) deformation. Elastic deformation of the barrier at
forces below \( k \) is neglected. In the dynamic case which is considered here, a force in
excess of \( k \) is required in order to accelerate the effective mass, \( M \), of the deforming slab.

The failure criterion can be stated simply in terms of the maximum deflection \( \epsilon_y \) of the
impact point, which the barrier can sustain, or, equivalently, the yield energy \( U_y = k \epsilon_y \)
which the barrier can absorb. It is noted that the condition \( k > e \) must be imposed which
is the analytic equivalent to stating that the model pertains to soft missiles impacting on
hard barriers.

As a direct consequence of these assumption, one finds that the impact process may
exhibit three distinct consecutive phases characterized as follows:
Phase 1: reaction force \( F \) at the barrier missile interface less than \( k \);
no deflection of the barrier.

Phase 2: reaction force \( F \) has exceeded the yield-force \( k \); barrier
deflects (accelerates), while missile continues to decelerate.

Phase 3: barrier velocity \( (x) \) has reached missile velocity \( v \); from this
instant on, missile and barrier move (decelerate) together
and no more crushing of the missile takes place.

Deceleration of the barrier may start already during Phase 2, if the reaction force \( |F| \)
drops below \( k \) before \( (x) \) reaches \( x \). Similarly, failure of the barrier may take place in
either Phase 2 or 3.

Since there is no barrier deflection during Phase 1, the model equations for this case
are identical to those presented earlier. When the value of \( |F| \) as determined by equation
(4) reaches \( k \), Phase 2 is entered, for which the following model equations are derived:

Momentum equation as applied to missile:

\[
P = -v^2 \cdot \mu \cdot \eta' + m \cdot v^{(1)} \cdot \eta' \cdot v
\]

(11)

where the following notation has been introduced:

\( \eta = x - \xi \) coordinate of missile relative to barrier,
\( \mu = \frac{dm}{d\eta} \) linear mass density of missile at barrier,
\( \cdot \) \( (\cdot) \) derivative with respect to \( x \),
\( (\cdot) \) \( \cdot \) derivative with respect to \( \eta \).

Energy equation applied to missile, analogous to earlier derivation:

\[
\dot{O} = -\mu v^2 \frac{1}{2} \cdot (1-\epsilon) + m \cdot v \cdot v^{(1)} + e \cdot F \cdot \frac{1}{\eta' - 1}
\]

(12)

The last term in this equation arises from the work done by the reaction force on the
yielding barrier.
Finally, we write the equation of motion for the barrier as:

\[ F + k = -N \cdot \frac{d^2\xi}{dt^2} \]  \hspace{1cm} (13)

Equations (11) - (13) constitute the description of the system during Phase 2 in terms of the three unknowns \( F \), \( v \), and \( \nu \) which may be considered as functions of anyone of the variables \( t, x, \eta \), while \( \xi \) may be eliminated from equation (13) by: \( \xi = x - \eta \). The termination of Phase 2 is indicated by the condition \( \eta' = 0 \). Initial conditions are the continuity of all variables at the transition from Phase 1 to Phase 2: \( x = \eta = x_1, v = v_1, \eta' = 1 \), and \( F = -k \), where the subscript 1 denotes values taken at the end of Phase 1. The quantities \( \mu, e, f \) are given functions of \( \eta \) while \( M \) and \( k \) are given constants. The missile mass \( m \) also is a function of \( \eta \) and related to \( \eta \) by: \( m = M - \int_0^\eta u \, d\eta \).

The step-wise integration scheme with a time increment \( \Delta t \), based on first order forward extrapolation can be formulated as:

\[ x_{i+1} = x_i + v_i \cdot \Delta t \]  \hspace{1cm} (14)

\[ \eta_{i+1} = \eta_i + (v_i - v_1) \cdot \Delta t \]  \hspace{1cm} (15)

\[ v_{i+1} = v_i + \frac{k}{M} \cdot \frac{F_i}{k} \cdot \frac{v_i}{v_i} \cdot (1 - \frac{y_1}{v_i}) \cdot \Delta t \]  \hspace{1cm} (16)

\[ y_{i+1} = y_i + \frac{k}{M} \cdot \left[ -\frac{F_i}{k} - 1 \right] \cdot \Delta t \]  \hspace{1cm} (17)

\[ x_i = \frac{1}{k} \cdot \left( x_1 + k \right) \cdot \frac{v_i}{k} \cdot \frac{v_i}{v_i} + \frac{F_i}{k} \]  \hspace{1cm} (18)

and the termination condition: \( \gamma = v \). The quantity \( \gamma \) corresponds to the velocity of the barrier at the impact point, \( i \).

Phase 3 is determined simply by the joint decelerating motion of the barrier and the remaining portion of the missile with mass \( m_2 \) as at the end of Phase 2:

\[ k = -(M + m_2) \cdot \frac{d^2x}{dt^2} \]  \hspace{1cm} (19)

The reaction force at the missile barrier interface during Phase 3 is given by:

\[ F = -k \cdot \frac{m_2}{m_2 + M} \]  \hspace{1cm} (20)

It is noted that there is a discontinuity in this force at the transition from Phase 2 to Phase 3. The initial conditions for Phase 3 are: \( x = x_2, v = v_2 \). Equation (19) can be solved directly and yields for the duration of Phase 3:

\[ \Delta t_3 = v_2 \left( \frac{m_2}{m_2 + M} \right) \]  \hspace{1cm} (21)

and the deflection during this final phase:

\[ \Delta x_3 = \frac{1}{2} \cdot v_2 \cdot \Delta t_3 \]  \hspace{1cm} (22)

The total hypothetical deflection of the barrier is the sum of deflections from Phase 2 and 3:

\[ \Delta \xi = \delta \xi_2 + \delta \xi_3 = \xi_2 + \Delta x_3 = x_2 - \eta_2 + \Delta x_3 \]  \hspace{1cm} (23)

If \( \Delta \xi \) is greater than \( \xi' \), failure of the barrier is expected to occur. A restriction during Phase 2 on the parameter \( f \), such as given by relation (9), is no longer necessary since the available energy can also be imparted now to the barrier. Under certain conditions of impact, Phase 3 is never reached. In those cases, wall deflection \( \xi \) reaches a maximum (\( \xi = 0 \)), at which point Phase 2 terminates. Subsequently, the wall will remain stationary while the remainder of the missile continues to crush. This last phase is described by the same model as used for Phase 1, with initial conditions satisfying continuity of relevant variables \( (x, v) \) at the transition point.
6. Sample Case

A missile with simple characteristics was chosen in order to illustrate the basic features of the model. Certain integral parameters were picked to resemble those of the PB-111 aircraft (e.g., total mass, total length). The quantities \( \nu \) and \( e \) were taken to be linear functions of the missile coordinate measured from its tip, starting from zero and rising to maximum values at mid-plane, \( \eta = \frac{1}{2} \), and then dropping back to zero towards the tail of the missile as illustrated in Figure 3. The relevant numerical values also are presented in this figure. The factor \( f \) was assumed to be equal to unity, so as to have correspondence to the cases treated earlier.

The results of this sample case are presented graphically in Figures 4 and 5. For comparison, the no-yield case also is plotted, showing a reaction diagram only slightly different from the triangular shape of \( \nu \); the reason for this is the small amount of energy which is assumed to be absorbed by missile deformation. Three yielding barriers were considered, differing in their effective mass \( M \) (0.1 \( m_o \), \( m_o \), 10 \( m_o \)) but having identical values of \( k = 5 \times 10^5 \) lb. It can be seen from the plotted results in Figures 4 and 5 that only the case: \( v_o = 125 \) ft/sec, \( M = 0.1 \) \( m_o \) satisfies the conditions leading to Phase 3. All other cases result in impacts described by the phase sequence 1-2-1.

7. Comparison with Uncoupled System

The common practice in analyzing the missile-barrier impact is to uncouple the system by evaluating the reaction force on the basis of a perfectly rigid barrier. Subsequently, this force is applied to obtain the response (deflection) of the barrier. It is obvious that energy and momentum are not conserved in the overall system by this analysis. In general, one can expect the results to be conservative (barrier deflection overpredicted) because this treatment leads to a net increase of energy over that which is available originally, i.e., \( \frac{1}{2} m v^2 \).

To demonstrate the degree of conservatism, this approximate analysis is applied here to the sample cases treated in the preceding chapter. The reaction force \( F \) which has been obtained for the no-yield case is now applied to the barrier, for which idealized elasto-plastic behavior is assumed. Therefore, as was the case earlier, the barrier does not deflect as long as \( |F| < k \). Starting from the instant when \( |F| = k \), the barrier motion is described by:

\[
F + k = -M \frac{d^2 \xi}{dt^2}
\]

(24)

The deflection terminates when \( \frac{dF}{dt} \) reaches zero; i.e., when,

\[
\int_{t_0}^{t_1} (F+k) \, dt = 0
\]

(25)

The maximum deflection occurs at this instant and is found by double integration of equation (24). Plots for \( \xi \) as a function of time are given in Figure 6. Total deflections obtained in this manner may be compared with those resulting from the coupled treatment presented earlier (see Table I). As expected the agreement is good for the case \( M/m_o = 10 \) but poor for \( M/m_o = 1.0 \) and very poor for \( M/m_o = 0.1 \).

8. Concluding Remarks

The model presented here is perceived to comprise a number of significant accomplishments: formulation is based in part on physical conservation principles; those, along with
common engineering concepts employed, permit full correlation between observables and describing quantities. Furthermore, dropping certain simplifying assumptions as used in the past yields results with greater confidence and establishes the amount of conservatism introduced by these assumptions. Finally, the analytic formulation of the model is relatively simple and will cause only marginal increases in calculational cost and effort over those entailed by earlier models. And, although explicit solutions are not obtained, it appears possible to generate parametric solutions, from which specific solutions can be extracted by graphical procedures or interpolation, along with the application of equivalence relations as yet to be established for the coupled system.

Only a small number of pertinent experiments have been reported for which data are available. Their interpretation in view of this model should provide additional guidance in correlating systems characteristics with certain model parameters, as for instance the factor $f$ which was introduced to account for residual kinetic energy of crushed missile parts. Future experiments can be designed to focus specifically on such features.

Acknowledgement

The author would like to acknowledge many helpful discussions of the subject with staff members of the Electric Power Research Institute (G.S. Lellouche, G.E. Sliter) and of Science Applications, Inc. (R.C. Erdmann, S.L. Basin) in Palo Alto, California. This work was performed under the sponsorship of the Electric Power Research Institute.

References


### TABLE 1: Comparison of Impact Models

<table>
<thead>
<tr>
<th>( v_0 ) (ft/sec)</th>
<th>( M )</th>
<th>MAXIMUM BARRIER DEFLECTION ( \max(\text{ft}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>COUPLED</td>
</tr>
<tr>
<td>100</td>
<td>0.1 ( m_0 )</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>( m_0 )</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>10.0 ( m_0 )</td>
<td>0.044</td>
</tr>
<tr>
<td>125</td>
<td>0.1 ( m_0 )</td>
<td>30.9</td>
</tr>
<tr>
<td></td>
<td>( m_0 )</td>
<td>8.7</td>
</tr>
<tr>
<td></td>
<td>10.0 ( m_0 )</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Fig. 1: Distribution of Mass and Crushing Load for FB-111, [5]

Fig. 2: Reaction Force vs. Time, FB-111
missile length: \( l = 100 \text{ ft} \)
missile mass: \( m_o = 10^5 \text{ lb} \)

\[ \mu_{\text{MAX}} = \frac{2m_o}{l} = 2000 \text{ lb/ft} \]
\[ e_{\text{MAX}} = 5 \cdot 10^6 \text{ lb-ft/sec}^2 \]
\[ k = 16 \cdot 10^6 \text{ lb-ft/sec}^2 \]

<table>
<thead>
<tr>
<th>( 0 \leq x \leq \ell/2 )</th>
<th>( \ell/2 \leq x \leq \ell )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \mu ]</td>
<td>[ \mu_{\text{MAX}} \frac{x}{\ell} ] [ \mu_{\text{MAX}} \frac{(1-x)}{\ell} ]</td>
</tr>
<tr>
<td>[ \frac{m}{k} ]</td>
<td>[ m_o \frac{(1-(\frac{x}{\ell})^2)}{2} ] [ m_o \frac{(1-x)^2}{\ell} ]</td>
</tr>
<tr>
<td>[ \frac{e}{k} ]</td>
<td>[ e_{\text{MAX}} \frac{x}{\ell} ] [ e_{\text{MAX}} \frac{(1-x)}{\ell} ]</td>
</tr>
</tbody>
</table>

Fig. 3: Missile Characteristics for Sample Calculation

Fig. 4: Reaction Force vs. Time, Sample Case
Fig. 5: Displacement vs. Time, Sample Case

Fig. 6: Displacement vs. Time, Sample Case, Uncoupled System