APPLICATION OF METHOD OF CHARACTERISTICS IN SEISMIC ANALYSIS

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SUMMARY

In the seismic analysis of nuclear power plants ground response analyses are often essential. For this purpose one-dimensional analyses are generally performed. To account for the nonlinear nature of soils in these analyses, an equivalent linear approach is widely used. In this approach an iterative scheme is employed. In each iteration the soil modulus and damping values are selected corresponding to a fraction of the peak strain and then remain constant throughout the earthquake duration in that iteration. The final soil properties are obtained after a satisfactory convergence is achieved. This iterative procedure has been found to give adequate results in many cases; however, it causes a few problems sometimes, especially for a strong input motion and for high frequencies. It is possible that part of this problem may be corrected if a truly nonlinear analysis is used. For this purpose several different methods can be used. In this paper the application of the method of characteristics is evaluated and the results are compared with those obtained from the equivalent linear approach. The soil profile is modeled by horizontally layered semi-infinite medium and the seismic excitation is introduced at the rock-roll interface. It is assumed that the soil profile is subjected only to the vertically traveling shear waves. In the nonlinear analysis the soil stress-strain characteristics are represented by Ramberg-Osgood relationship.

The comparison of results shows that the equivalent linear method consistently gives higher maximum shear stresses and accelerations at all depths within the soil profile. The comparison of shear stress time history at various layers, however, shows that the shear stresses from nonlinear and equivalent linear methods yield comparable equivalent uniform stresses. It is also found that the method of characteristics gives higher surface response spectrum at high frequencies and lower spectrum at lower frequencies than the equivalent linear method.

In addition, the comparison of results from the two methods shows that the difference in numerical results depends to a great extent on the intensity of excitation. For example, it is found that the results obtained for moderate shaking using the equivalent linear method are only slightly different from those obtained by the nonlinear approach. However, the discrepancy in the results obtained from the two methods increases with the intensity of excitation.

A study is also conducted to determine the effects of adding viscous damping to the hysteretic damping in the two analyses. It is found that the viscous damping has no effect on the maximum values of shear stresses in the nonlinear analysis. However, the surface acceleration response spectrum at high frequencies is substantially reduced. On the other hand for the equivalent linear method the addition of viscous damping has negligible effect on either the maximum shear stresses or the surface acceleration response spectrum.
1. **Introduction**

In the seismic analysis of nuclear power plants, the ground response analysis is often essential. Usually the type of analysis performed is a one-dimensional free field analysis. In many cases, this kind of analysis can be useful in providing an estimate of the design motions at the site in terms of design response spectrum. In addition, this analysis can also be used to determine the stresses and displacements which have developed in the soil strata due to seismic excitation for assessing the site liquefaction potential.

For a site where the subsurface consists of nearly all horizontal, homogeneous soil layers of uniform thickness, the soil response due to a horizontal excitation at the soil-rock interface can be modeled by a shear beam subjected to vertically propagating shear waves. In many cases the nonlinear behavior of the soils can be an important consideration in the ground response analyses and equivalent linear approach is very often used to account for the nonlinearity of the soils. In the equivalent linear approach an iterative scheme is employed. In each iteration the values of the shear modulus and damping ratio of the soils are selected based on a fraction of the peak strain in the previous iteration and remain constant throughout the duration of the motion. The final soil properties and other pertinent results are obtained after a satisfactory convergence is achieved. This equivalent linear method has been found to yield adequate results in many studies. However, this has not always been true, especially for a strong earthquake with high frequency content. It is possible that in some cases use of a truly nonlinear analysis may give better results. Several different numerical schemes can be used to perform nonlinear analysis. This paper investigates one of these schemes, the method of characteristics. In addition, an example problem is analyzed using this method and its results are compared with those obtained from the equivalent linear approach.

2. **Equivalent Linear Procedure**

As mentioned above, most of the various procedures that have been developed to compute the response of a soil deposit to earthquake excitation are based on the assumption that the horizontally layered soil underlain by the bedrock can be modeled by a shear beam subjected to a vertically propagating shear wave. For example, Schnabel, Lysmer and Seed [5] developed an analytical method to solve this problem by the equivalent linear approach. In this numerical procedure, the motions within any sublayer are assumed to satisfy the damped wave equation:

\[ \rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} + \mu \frac{\partial^3 u}{\partial z^3 \partial t} \]  

(1)

where \( u \) is the horizontal displacement; \( t \) is time; \( z \) is the depth; and \( \rho \), \( G \), and \( \mu \) are the mass density, the shear modulus, and the viscosity, respectively, of the sublayer. The displacement \( u \) for each sublayer during the
applied base motion is determined by solving eq. (1) using the Fourier transform. Once the displacements have been calculated, other response values such as acceleration and shear stresses are readily obtained. The computer program SHAKE developed by Schnabel, et al. [5] is usually used to perform this analysis. To account for the nonlinear hysteretic behavior of the soil, the shear modulus and damping ratio are adjusted in the SHAKE program to the computed "equivalent" strain level in each iteration.

3. Soil Model for Nonlinear Analysis

Hardin and Drnevich [1] have suggested that the shear stress-strain relation for soils can be represented by a hyperbolic curve. One of the frequently used relationships is the Ramberg-Osgood model [4]. The complete Ramberg-Osgood model is defined by two equations. The backbone curve of the model used for initial loading is expressed by

$$\gamma = \frac{\tau}{G_o} \left[ 1 + a \left( \frac{\tau}{\tau_y} \right)^{R-1} \right]$$

(2)

The skeleton curve used for unloading and reloading from ($\tau_1$, $\gamma_1$) is defined as

$$\gamma - \gamma_1 = \frac{\tau - \tau_1}{G_o} \left[ 1 + a \left( \frac{\tau - \tau_1}{2\tau_y} \right)^{R-1} \right]$$

(3)

In eqs. (2) and (3), $\tau$ is the shear stress and $\gamma$ is the shear strain. $G_o$ is the maximum shear modulus evaluated at very low strain levels and $\tau_y$ is the yield stress of the soil. The other two parameters, $R$ and $a$, are constants which define the size and shape of the hysteresis loop.

The dynamic shear modulus and damping ratio of the soil are determined by field and laboratory tests. For low strain levels ranging from $10^{-6}$ to $10^{-4}$, the resonant column test can be used to obtain the required data. For determining the soil properties in a strain range greater than $10^{-4}$, the strain controlled cyclic triaxial test is often used. In the Ramberg-Osgood soil model the strain dependent secant shear modulus and damping ratio can be derived from eq. (2) as

$$G = \frac{G_o}{1 + a \left( \frac{\tau}{\tau_y} \right)^{R-1}}$$

(4)

$$\lambda = \frac{2(R-1)}{x(R+1)} \left( 1 - \frac{G}{G_o} \right)$$

(5)

Equations (4) and (5) can be correlated with the strain-dependent shear modulus and damping curves used in the equivalent linear approach. This is done by determining appropriate values of the parameters in eqs. (4) and (5). With properly chosen values for $G_o$ and $\tau_y$, conventional curve fitting techniques can be used to obtain the values of $R$ and $a$. An example of this type of curve fitting procedure is given by Jennings [2]. The same correlation
of dynamic soil properties is used in this study.

4. Nonlinear Ground Response Analysis Procedure

Using the Ramberg-Osgood curves to represent the dynamic stress-strain behavior of the soil, the computer program CHARSOIL developed by Streeter, et al. [6] can be used to solve the wave propagation equation. In this approach, the problem of shear waves induced by earthquake motions in linear viscoelastic and strain softening materials is solved by using the method of characteristics. This solution has been described by Papadakis [3] and Streeter, et al. [7]. The equation of motion is written as

\[ \frac{\partial \tau}{\partial t} + \frac{\partial \sigma}{\partial x} = 0 \]  

and

\[ \frac{\partial \sigma}{\partial t} = G \frac{\partial \tau}{\partial x} + \mu \frac{\partial^2 \tau}{\partial x^2} \]  

in which \( \tau \) is the shearing stress, \( \mu \) is the soil viscosity, and \( V \) is the particle velocity.

In the solution of eqs. (6) and (7), the shearing stresses \( \tau \) and particle velocities \( V \) are assumed to be known at the initial time. These initial values of \( \tau \) and \( V \) are used to calculate new values of \( \tau \) and \( V \) throughout the soil profile one time step later using the method of characteristics. The same procedure is repeated for subsequent time steps using newly calculated values.

5. Example Problem

To compare the equivalent linear method with the method of characteristics, an example problem is analyzed by both procedures. The problem consists of 200 ft. layer of clay underlain by bedrock. The soil profile is subjected to an earthquake excitation at the soil-rock interface. The time history used is the first ten seconds of the El Centro earthquake record of 1940, N-S component. In order to investigate the effect of the earthquake intensity on the soil response, the recorded earthquake is scaled to two different levels of peak acceleration: 0.15\( g \) and 0.4\( g \).

Figures 1 and 2 show the strain dependent shear modulus and damping values used in the equivalent linear approach of SHAKE. The shear modulus at very low strain levels (10^{-6} in./in.) is 11,000 ksi.

For comparison purposes, the nonlinear soil properties used in CHARSOIL are selected such that they are compatible with those used in SHAKE. The curve fitting technique described in Section 3 is used to determine the constants \( a \) and \( R \) in the Ramberg-Osgood curves. For a value of 2.2 ksi for \( \tau_y \), the \( a \) and \( R \) are 1.0 and 3.0, respectively.

The effect of viscosity on the CHARSOIL results is investigated by varying its value from zero to 62,278 lb-sec/ft^2. The latter value is chosen to be equivalent to 4% hysteretic damping. The correlation between the viscosity and hysteretic damping is given in Section 7.
Discussion of Results

6.1 0.15g Earthquake

The comparison of the response spectra of the ground surface motions obtained from SHAKE and CHARSOIL for the 0.15g earthquake is shown in Figure 3. No viscosity is considered in CHARSOIL. It is found that the maximum ground surface acceleration from CHARSOIL is smaller than that from SHAKE. However, the two response spectra match well at all frequencies except in the high frequency range. It is generally recognized that SHAKE filters out the high frequencies of the earthquake motion and yields lower response values in the high frequency range. As seen in Figure 3, the response spectrum from SHAKE shows a flat response for frequencies higher than 10 cps. This indicates that there is practically no frequency content beyond 10 cps in the ground surface motion generated by SHAKE. This phenomenon can be partly attributed to the fact that SHAKE overdamps the high frequency response. In SHAKE a constant damping is selected based on the equivalent strain which is a fraction of the peak strain in each iteration. Therefore the frequencies associated with strains lower than this equivalent strain are overdamped. Since high frequencies usually develop very low strains they, therefore, are overdamped in the equivalent linear approach. This problem does not occur in the nonlinear CHARSOIL approach, where the damping is consistent with the stress level at each time step.

The maximum strains along the depth are presented in Figure 6 and the maximum stresses are presented in Figure 8. It can be seen that the maximum strains obtained from both methods are in reasonably good agreement. Although the maximum strains are close, the maximum stresses obtained from CHARSOIL are about 20% to 25% lower than those obtained from SHAKE. This may be because a constant shear modulus based on a fraction of the peak strain is used in SHAKE for each iteration. Therefore the modulus used in SHAKE for the peak stress is much higher than the actual modulus at that stress level. Again, this problem does not exist in the nonlinear approach used in CHARSOIL. Therefore, the maximum shear stresses will be higher in SHAKE than in CHARSOIL as shown in Figure 8.

6.2 0.4g Earthquake

In order to verify the validity of observations made for the case of a strong earthquake motion, the same example problem is used with a 0.4g earthquake excitation. It is found that the comparison of results from SHAKE and CHARSOIL for a strong earthquake is similar to that obtained for the moderate earthquake. The maximum ground surface acceleration obtained from SHAKE is higher than that from CHARSOIL and the response spectra as shown in Figure 4 indicate similar differences in the high frequency range. However, the magnitude of difference in this case is slightly larger. This should be expected because stronger shaking causes the soil to become softer and thus high frequencies are further filtered out in SHAKE.
This trend is also observed in the comparisons of maximum shear strain and stress results obtained from the two methods as shown in Figures 7 and 9, respectively. The stress time histories at a depth of 150 ft. from the two methods are compared in Figure 5. The pattern of these stress time histories is similar except for the peaks. It is noted that the peaks of the larger stress cycles from CHARSOIL are somewhat lower than those from SHAKE. However, the peaks of the smaller stress cycles from CHARSOIL are found to be higher than those from SHAKE. Since the maximum shear stress occurs instantaneously in the time history, it alone is not of much significance. Instead, the cumulative damage caused by the entire time history is the important factor in the ground response analysis. It is seen from Figure 5 that the number of stress cycles of the two time histories are of the same order and the differences in stresses are of such magnitudes that they will probably produce a comparable degree of damage. Therefore, the two methods will yield similar results as far as the damage due to shear stress in the soil profile is concerned.

From these discussions, it can be concluded that for both moderate and strong earthquakes, the two methods provide comparable results. However, the equivalent linear approach consistently yields higher maximum ground surface accelerations and peak shear stresses although the response spectrum in the high frequency range is smaller.

7. Effect of Viscous Damping

In general, most of the damping in the soil is hysteretic damping and there is little viscous damping. In SHAKE, the maximum shear strain governs the selection of damping for the entire time history and it usually falls in such a range that the hysteretic damping is considerable in magnitude. Therefore, addition of the viscous damping, which is usually small, will not significantly affect the SHAKE results. On the other hand, the damping used in CHARSOIL depends on the instantaneous stress or strain level as dictated by the Ramberg-Osgood curves. Since the hysteretic damping is very small in the low strain range, it is expected that the addition of viscous damping will have significant effect on the CHARSOIL results, especially in high frequencies where the strains are smaller.

To determine the effect of adding viscous damping to the hysteretic damping, the two methods are used again to analyze the example problem with the 0.4g earthquake. A four percent damping is added to the damping curve shown in Figure 2 for SHAKE. In CHARSOIL the viscosity is directly used. The relation between the viscosity, \( \mu \), used in CHARSOIL and the hysteretic damping, \( \lambda \), used in SHAKE is defined as

\[
\mu = \frac{2G\lambda}{\omega}
\]

(8)

where \( G \) is the shear modulus and \( \omega \) is the circular frequency.

It is generally recognized that the viscosity \( \mu \) varies with frequencies of the excitation. However, CHARSOIL allows one to use only constant viscos-
ity values. In this study, the fundamental frequency of the soil column was selected to obtain the constant viscosity value from eq. (8) corresponding to 4% hysteretic damping. The value obtained is 62,278 lb-sec/ft².

It is found that the viscous damping has little effect on the maximum values of shear stresses and strains in the nonlinear analysis as shown in Figures 7 and 9. This is because at the maximum shear stress and strain level, the hysteretic damping is in dominant proportion and has more influence than the viscosity.

The results for the response spectrum at the ground surface obtained from CHARSOIL with and without viscosity are shown in Figure 4. It is noted that the addition of viscosity substantially reduces the response in the high frequency range, whereas the response at low frequencies is not affected much. This is due to two reasons. The small strains associated with high frequencies correspond to small hysteretic damping and, therefore, addition of viscous damping affects the response in the high frequency range significantly. Furthermore, the equivalent hysteretic damping at high frequencies for the selected value of viscous damping, which is based on the fundamental frequency of the soil profile, is much higher than 4%, as evident from eq. (8).

8. Conclusions

The results from the equivalent linear method used in the SHAKE program and the method of characteristics used in the CHARSOIL program are compared. An example problem is analyzed to demonstrate the difference in these two methods and the effect of various parameters on the analysis. The results of this problem lead to the following conclusions:

1. Both SHAKE and CHARSOIL give comparable maximum strains along the soil depth.
2. The maximum stresses obtained from CHARSOIL are lower than those obtained from SHAKE with the maximum difference of about 20% to 25% at the soil-rock interface.
3. SHAKE yields higher maximum ground surface acceleration than CHARSOIL.
4. CHARSOIL with no viscous damping gives higher response spectrum at the ground surface in the high frequency range than SHAKE.
5. The ground surface acceleration response spectrum generated by CHARSOIL is lower than that generated by SHAKE in the entire frequency range for the case in which viscous damping equivalent to 4% hysteretic damping is added.
6. Viscous damping has little effect on SHAKE results.
7. The stress time histories from the two methods are very similar in shape and in the number of cycles.
References


Figure 1. Shear Moduli Used in Example Problem

Figure 2. Damping Ratios Used in Example Problem
Figure 3. Ground Response Spectra for 0.15g Earthquake
Figure 4. Ground Response Spectra for 0.4g Earthquake

Figure 5. Shear Stress Time Histories at Depth of 150 ft.
Figure 6. Maximum Shear Strains for 0.15g Earthquake

Figure 7. Maximum Shear Strains for 0.4g Earthquake

Figure 8. Maximum Shear Stresses for 0.15g Earthquake

Figure 9. Maximum Shear Stresses for 0.4g Earthquake