

## FINITE ELEMENT RANDOM VIBRATION METHOD FOR SOIL-STRUCTURE INTERACTION ANALYSIS

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### SUMMARY

Most current methods of soil-structure interaction are deterministic in nature and consider only a limited sample of possible time histories which approximately fit some given design response spectrum. However, earthquake motions are intrinsically random in nature, and it would be desirable to develop analytical methods which retain this randomness in both the definition of the design motion and the computed response. The authors present a method of this type. In this procedure the seismic environment is defined directly in terms of the given design response spectrum.

Response spectra cannot be used directly for random analysis, thus using extreme value theory a new procedure has been developed for converting the design response spectrum into a design power spectrum. This procedure is reversible and can also be used to compute response spectra from power spectra. This process leads to a random ensemble of design response spectra the distribution of which can be expressed in terms of Confidence limits.

Knowing the design power spectrum the resulting output power spectra and their statistical distribution can be computed by a response analysis of the soil-structure system in the frequency domain. Due to the complexity of soil structure systems, this is most conveniently done by the finite element method. Having obtained the power spectra for all motions in the system, these spectra can be used to determine other statistical information about the response such as maximum accelerations, stresses, bending moments, etc, all with appropriate confidence limits. This type of information is actually more useful for design than corresponding deterministic values.

The authors have developed a computer program, PLUSH, which can perform the above procedures. Results obtained by the new method are in excellent agreement with the results of corresponding deterministic analysis. Furthermore, the probabilistic results can be obtained at a fraction of the cost of deterministic results.

## 1. Introduction

Most current methods of soil-structure interaction are deterministic in nature and consider only a limited sample of possible time histories which approximately fit some given design response spectrum. However, earthquake motions are intrinsically random in nature, and it would be desirable to develop analytical methods which retain this randomness in both the definition of the design motion and the computed response. The authors present a method of this type.

In this procedure the seismic environment is defined in terms of a power spectrum for the motion of a point in the free field. The power spectrum defines an ensemble of motions which have a random distribution of response spectra. This distribution can be determined by extreme value theory in terms of an average response spectrum with appropriate confidence limits. Conversely it is possible to determine an input power spectrum from a given average design response spectrum.

In practice it is convenient to define the seismic environment in terms of a design response spectrum. Thus the authors propose to apply their method by considering this response spectrum as the above mentioned average response spectrum and develop from this the unique input power spectrum. This procedure corresponds to using a band of input response spectra, the distribution of which is determined by the above mentioned theory.

Knowing the input design power spectrum the resulting output power spectra can be computed by a response analysis of the soil-structure system in the frequency domain. Due to the complexity of such systems, this is most conveniently done by the finite element method. Having obtained the power spectra for all required motions in the system, these spectra can be converted to output response spectra or other statistical information about the response such as maximum accelerations, stresses, bending moments, etc., all with appropriate confidence limits. This type of information is actually more useful for design than corresponding deterministic values.

The authors have developed a computer program, PLUSH [10], which can perform the above procedures. Results obtained by the new method are in excellent agreement with the results of corresponding deterministic analyses. Furthermore, the probabilistic results can be obtained at a fraction of the cost of deterministic results.

## 2. Method of Analysis

The method of analysis to evaluate the dynamic interaction between soil and structure is conceptually similar to that proposed by Seed et. al. [13]. The method involves four basic steps and may be summarized as follows (see Fig. 1):

- i) From the average design response spectrum given at some level in the free field, the corresponding input power spectrum is estimated.
- ii) This power spectrum is deconvolved to find the power spectrum at the assumed rigid base of the finite element model.
- iii) Once the rigid base power spectrum is known, response power spectra at different locations in the soil-structure system are obtained, and
- iv) From the response power spectra obtained in step (iii), response spectra and maximum values of accelerations, stresses, strains, moments, etc., are evaluated.

The output may be expressed in terms of mean values with confidence limits at specified uncertainty levels. The accuracy of the output response spectra

will be best if they are computed at the same level of spectral damping as the design response spectrum used in Step (i).

The analysis may be performed iteratively to allow for the strain-dependent nature of nonlinear soils [12]; i.e. in each iteration the analysis is linear but the soil properties are adjusted from iteration to iteration until the calculated strains are compatible with the soil properties used in the analysis.

Steps (ii) and (iii) may be attained by means of the following well-known input-output relationship derived from random vibration theory:

$$P^r(\omega) = |H(\omega)|^2 P^i(\omega) \quad (1)$$

where  $P^i(\omega)$  is the input power spectrum;  $H(\omega)$  is the complex transfer function which is a characteristic of the system;  $P^r(\omega)$  is the response power spectrum; and  $\omega$  is the frequency.

The complex transfer functions may be obtained directly from the equation of motion written in the frequency domain. For instance, the equation of motion for a single degree of freedom system is:

$$\ddot{x}(t) + 2\beta\omega_0 \dot{x}(t) + \omega_0^2 x(t) = -\ddot{y}(t) \quad (2)$$

where  $x(t)$  is the displacement of the mass relative to the support and  $\ddot{y}(t)$  is the given acceleration time history at the support;  $\omega_0$  and  $\beta$  are the natural frequency and damping ratio of the system, respectively. Making the substitution  $\ddot{y}(t) = \exp(i\omega t)$  and  $x(t) = H(\omega)\exp(i\omega t)$  in eq. (2) and cancelling the  $\exp(i\omega t)$  terms, the complex transfer function for relative displacement is obtained as:

$$H(\omega) = \frac{-1}{\omega_0^2 - \omega^2 + 2i\beta\omega_0\omega} \quad (3)$$

where  $i = \sqrt{-1}$ .

Complex transfer functions for multidegree of freedom systems cannot be defined in such a simplistic way. However, they may be obtained from the equations of motion of the system written in the frequency domain.

The degree of complexity of the complex transfer function is directly influenced by the complexity of the analytic model used to represent the soil-structure system. Lysmer et al. [6, 7] have developed a numerical procedure for a soil-structure model such as that presented in Fig. 2. This model includes viscous boundaries on the planar sides of the slice used for analysis to simulate the propagation of wave energy in the direction perpendicular to the axis of the slice; energy-transmitting boundaries are placed at the lateral boundaries to simulate the dynamic effects of the semi-infinite visco-elastic horizontally layered soil system beyond the finite element region.

The equation of motion for the above finite element representation of the soil-structure system can be written

$$[M] \{\ddot{u}\} + [K] \{u\} = -\{m\}\ddot{y} - \{v\} + \{F\} - \{T\} \quad (4)$$

where  $[M]$  and  $[K]$  are the usual plane strain mass and stiffness matrices, respectively, of a slice of unit thickness;  $\{u\}$  are the displacements of the nodal points relative to the rigid base and  $\{m\}$  is a vector related to  $[M]$  and the direction of the rigid base acceleration,  $y(t)$ . Material damping may be included by forming  $[K]$  from complex moduli:  $G^* \approx G \exp(2i\beta)$ , where  $\beta$  is the fraction of critical damping.

The forces  $\{V\}$  due to the viscous boundaries on the planar sides of the slice are according to Hwang et. al., [4]:

$$\{V\} = \frac{1}{L}[C](\{\dot{u}\} - \{\dot{u}_f\}) \quad (5)$$

where  $L$  is the thickness of the slice,  $[C]$  is a diagonal matrix containing the damping characteristics of the dashpots attached to the model, and  $\{\dot{u}_f\}$  are the free field velocities.

The forces  $\{F\}$  which are given by

$$\{F\} = [G]\{u_f\} \quad (6)$$

simply represent the forces acting on a vertical plane in the free field and they involve no horizontal transmission of wave energy;  $[G]$  is a simple frequency-independent stiffness matrix formed from the complex moduli in the free field.

The forces  $\{T\}$  which are related to horizontal energy transmission through the lateral boundaries are given by

$$\{T\} = ([R] + [S])(\{u\} - \{u_f\}) \quad (7)$$

where  $[R]$  and  $[S]$  are symmetric frequency-dependent boundary stiffness matrices initially introduced by Lysmer and Drake [5] and Waas [15]. These matrices represent the exact dynamic effect of the semi-infinite viscoelastic soil system shown in Fig. 2 at both ends of the model and account for radiation losses in the horizontal directions within the slice of analysis.

Using the complex response method eq. (4) can be written in the frequency domain as

$$\begin{aligned} ([K] + [R]_r + [S]_r + \frac{i\omega_r}{L}[C] - \omega_r^2[M])\{U\}_r = \\ - \{m\}\ddot{Y}_r + ([G] + [R]_r + [S]_r + \frac{i\omega_r}{L}[C])\{U_f\}_r \end{aligned} \quad (8)$$

From this set of equations the displacement amplitudes  $\{U\}_r$  may be obtained for each frequency  $\omega_r$ ,  $r = 0, 1, 2, \dots$ , of the input motion by Gaussian elimination.

The free field motions,  $\{u_f\}$ , appearing in eqs. (5), (6), and (7) are calculated separately on the assumption that the free field consists of horizontal soil layers and that the seismic excitation consists of vertically propagating P- or S- waves. The free field amplitudes,  $\{U_f\}_r$ , are computed from:

$$\begin{aligned} [K_f]_r \{U_f\}_r = -\{m\} \ddot{Y}_r \\ \text{or } \{H_f\}_r = \frac{\{U_f\}_r}{\ddot{Y}_r} = -[K_f]_r^{-1}\{m\} \end{aligned} \quad (9)$$

where

$$[K_f]_r = [K_f] - \omega_r^2[M_f]$$

Here  $[K_f]$  and  $[M_f]$  are the stiffness and mass matrices of the layered system;  $\{H_f\}_r$  is a vector containing the amplification values (complex transfer functions) from the rigid base acceleration to the layer displacements. Substitution of eq. (9) into eq. (8) gives the equation of motion in its final form:

$$[K]_r \{U\}_r = \{P\}_r \ddot{Y}_r \quad (10)$$

where  $[K]_r$  is the frequency-dependent stiffness matrix:

$$[K]_r = [K] + [R]_r + [S]_r + \frac{i\omega_r}{L} [C] - \omega_r^2 [M] \quad (11)$$

and

$$\{P\}_r = ([G] + [R]_r + [S]_r + \frac{i\omega_r}{L} [C]) \{H_f\}_r - \{m\} \quad (12)$$

is the load vector corresponding to unit amplitude of the rigid base motion. The complex transfer functions for the soil-structure system may be obtained by setting  $\ddot{Y}_r = 1$  and  $\{U_r\} = \{H(\omega_r)\}$  in eq. (10). Thus:

$$[K]_r \{H(\omega_r)\} = \{P\}_r \quad (13)$$

which can be solved for  $\{H(\omega_r)\}$  by Gaussian elimination for all frequencies  $r = 0, 1, 2, \dots$  of the input motion.

Eqs. (1), (9) and (11-13) will furnish the means to attain steps (ii) and (iii) of the method of analysis.

### 3. Relationship Between Response and Power Spectra

Relations between average response spectra and power spectra (needed to accomplish steps (i) and (iv)) have been proposed by several researchers Rosenblueth and Bustamante, [8, 11]; Housner and Jennings, [3]; Gasparini and Vanmarcke, [2]. Even though these relations were obtained under the assumption that the Gaussian random process is white noise, they may be extended, with little loss in accuracy, to cases where the random process has a power spectrum which varies slowly with frequency, Clough and Penzien [1].

In cases where the random process is not white noise, the power spectra estimated by means of these relations are different from the actual power spectra corresponding to the given response spectra. Since the area under the power spectral curve is a measure of the energy contained in the process, the input energy (i.e. level of dynamic excitation) will thus be misrepresented by the estimated power spectrum. Therefore, the use of a power spectrum estimated in this fashion would be equivalent to studying the seismic behavior of the soil-structure system under consideration, to an excitation level different from that actually existing. Accordingly, the values of maximum accelerations, strains, stresses, moments, etc., obtained would be different from those that would actually develop under the given excitation.

In order to avoid this problem, Romo-Organista [9] has developed a new relationship between response spectra and power spectra which includes in its formulation a compatibility coefficient,  $\lambda$ , the role of which is to ensure that the energy contained in the estimated power spectrum is equal to the energy content of the design response spectrum. The coefficient can be determined iteratively by the condition that the expected value of the maximum acceleration computed from the power spectrum must match the given maximum acceleration. This is done automatically in the computer program PLUSH mentioned above [10]. The new relation between absolute acceleration response spectra and power spectra is:

$$S_a(\omega_k, \beta, T) = R_a \cdot L_c \quad (14)$$

where

$$\begin{aligned}
 R_a &= \left\{ \begin{aligned} &\frac{1 + 4\beta^2}{4\beta\lambda} \pi \omega_k P^i(\omega_k) \left( 1 - \exp\left(-\frac{2\beta\omega_k T}{L_{50}}\right) \right) \\ & - \frac{\omega_k}{2} P^i(\omega_k) + \sum_{j=0}^{j=k} P^i(\omega_j) \Delta\omega_j \end{aligned} \right\}^{1/2} && ; \text{ for } \beta \neq 0 \\
 \text{or} \\
 R_a &= \left\{ \begin{aligned} &\frac{\pi \omega_k^2 T P^i(\omega_k)}{2\lambda L_{50}} - \frac{\omega_k}{2} P^i(\omega_k) + \sum_{j=0}^{j=k} P^i(\omega_j) \Delta\omega_j \end{aligned} \right\}^{1/2} && ; \text{ for } \beta = 0 \\
 \text{and} \\
 L_c &= \left\{ 2\ell\pi \left[ \frac{\nu T}{\ell n(1/c)} \right] \right\}^{1/2}
 \end{aligned}$$

T is the duration of the input random process;  $\Delta\omega_j$  is the frequency step; c is the specified level of confidence (i.e., if c = 90%, the spectrum computed from eq. (14) has a 90% probability of not being exceeded);  $\nu$  is a parameter which gives a measure of the most characteristic frequency in the random process. For low-damped single degree of freedom systems  $\nu$  may be replaced by  $\omega_k/2\pi$  [1].  $L_{50}$  is the value of  $L_c$  corresponding to c = 50%. Eq. (14) provides a means for estimating response spectra with different levels of uncertainty. Also, it has been found [9] that by using  $L_{60}$  in eq. (14) the average response spectrum is closely approximated.

Similar relations between velocity,  $S_v$ , and displacement response spectra,  $S_d$ , and, power spectra have been developed. These expressions lead to the following relations between response spectra:

$$S_d(\omega_k, T, \beta) = \frac{S_a(\omega_k, T, \beta)}{\omega^2 (1 + 4\beta^2)^{1/2}} \tag{16}$$

and

$$S_v(\omega_k, T, \beta) = \omega_k S_d(\omega_k, T, \beta) \tag{17}$$

Note that eq. (17) corresponds to the well-known relationship between the pseudo velocity spectrum and the displacement spectrum.

According to the proposed method of analysis, the first step requires that the inverse problem be solved. That is, an input power spectrum must be estimated from the average design response spectrum before the seismic analysis can proceed. Thus, writing eq. (14) in terms of the power spectrum,  $P^i(\omega_k)$ , we obtain the following expression:

$$P^i(\omega_k) = P_a \left\{ \left[ \frac{S_a(\omega_k, T, \beta)}{L_{60}} \right]^2 - \sum_{j=0}^{j=k-1} P^i(\omega_j) \Delta\omega_j \right\} \tag{18}$$

where

$$P_a = \frac{1}{\left[ \frac{1 + 4\beta^2}{4\beta\lambda} \pi \left( 1 - \exp\left(-\frac{2\beta\omega_k T}{L_{50}}\right) \right) - 0.5 \right] \omega_k} ; \text{ for } \beta \neq 0$$

or

$$P_a = \frac{1}{\left[ \frac{\pi \omega_k T}{2 \lambda L_{50}} - 0.5 \right] \omega_k} \quad ; \text{ for } \beta = 0$$

It should be noted that because of the implicit form of eq. (18) the sum is taken up to the previous frequency at which  $P^i(\omega_k)$  is being estimated. Thus, in order to start the process of computing  $P^i(\omega_k)$  over the full frequency range the term corresponding to the initial frequency is assumed to be zero. The error introduced by this assumption is negligible as long as (1) the first frequency is small compared to the highest frequency considered in the response spectra and, (2) the number of frequency steps is large enough to override the effect of neglecting the first amplitude. Eq. (18) provides the required process to accomplish step (i); conversely, the response spectra may be estimated by means of eq. (14) to accomplish step (iv).

In order to achieve a better understanding of the dynamic response of a soil-structure system, maximum values of accelerations, strains, moments, etc., should be determined. Mean maximum responses may be estimated by means of the expression:

$$E[\Delta] = L_{60} \cdot \sigma \tag{19}$$

where  $\sigma$  is the root mean square value of the quantity of which the maximum value is to be estimated and  $E[\Delta]$  is the mean value of the variable  $\Delta$  being estimated. Uncertainty levels, i.e. 90% upper and lower bounds may be calculated simply by replacing  $L_{60}$  by  $L_{90}$  or  $L_{10}$  in eq. (19).

#### 4. Application of the Method of Analysis

A new computer code, PLUSH [10], was written to perform the operations indicated in the previous sections. In this program the finite element method is used to obtain the mass and stiffness matrices of eqs. 9 and 13. The equations of motion are solved in the frequency domain and the damping characteristics of the system are introduced by means of the complex response method [6].

In order to illustrate the capabilities of the proposed method of analysis, parallel computations were performed by the probabilistic procedure, PLUSH, and the equivalent deterministic procedure developed by Lysmer et. al. [6], [7], FLUSH, for the symmetric single-structure model shown in Fig. 3. The finite element model used contained 106 elements and 134 nodal points. Only some of the more important nodal points at which the motions were monitored are shown in Fig. 3 which also shows the material properties of the structure.

The soil material properties were taken equal to the corresponding iterated, strain compatible, properties in the free field. The resulting shear moduli and damping ratios for the free field, which consisted of 18 layers, are shown in Fig. 4.

A synthetic earthquake with wide band frequency content was used as control motion for the deterministic analysis. The control point was specified at the ground level in the free field. The effective time duration of the excitation was 20.48 sec. and its maximum acceleration was 0.25 g.

The seismic environment for the probabilistic analysis was defined by the two percent damping acceleration response spectrum of the synthetic time history mentioned above. The input average response spectrum is shown in Fig. 5.

Distributions of maximum accelerations along three vertical sections (see Fig. 20) within the soil-structure system are shown in Table I. It may be seen that the agreement between deterministic and probabilistic results is good at all points considered.

Acceleration response spectra for horizontal motions determined at selected nodal points of the finite element mesh by both the deterministic and the probabilistic methods of analysis are compared in Fig. 6. The deterministic spectrum generally follows the average probabilistic spectrum and falls within the 90% confidence limits for the set of random motions.

The results presented in Table I and Fig. 6 indicate that the individual members of motions which fit a given power spectrum may lead to a wide band of results even though all of these motions have the same frequency, and thus energy, content. An ensemble of motions fitting the same acceleration response spectrum is likely to lead to an even wider distribution since such motion in addition to phase differences would also have different energy contents.

This observation is of considerable importance because most seismic analyses of nuclear power plants are currently accomplished using a single time history which fits one of the design response spectra specified by the U.S. Nuclear Regulatory Commission in its Regulatory Guide 1.60. Thus in the currently used design procedure it has been tacitly assumed that when any one time history with a response spectrum enveloping the design response spectrum is used as the control motion, the computed response of the system will be essentially the same. In view of the results presented herein this procedure could lead to serious over- or under- estimates of the motions which are predicted for a given seismic environment. The adoption of the probabilistic method presented in this paper would eliminate this problem. Furthermore, since the specification of a seismic environment is essentially a specification of energy input the authors believe that a specification of the seismic environment in terms of a power spectrum as employed in the probabilistic method presented is intrinsically better than specification in terms of a response spectrum.

In addition, the probabilistic method requires less computational effort than the corresponding deterministic procedure. The computer time used by program PLUSH is from 50% to 80% lower than the time used by program FLUSH. The time saving comes mainly from: a) fast computation of response spectra and, b) efficiency in estimating the maximum values of strains, accelerations, etc.

## 5. Conclusions

A new probabilistic procedure for evaluating the seismic response of soil-structure systems has been developed by combining elements from the theory of random vibrations, the finite element method and the complex response method. This approach has the advantage that all possible time histories of motion fitting a specified power spectrum may be considered in a soil-structure interaction analyses and that potential effects of variations in time history may conveniently be considered in the design studies.

Results obtained by the new method are in generally excellent agreement with corresponding results obtained by the deterministic method and the new method has the following additional advantages:

1. The probabilistic method operates directly from design response spectra. Hence, the need for generation of specific "matching" time histories is eliminated.



2. The probabilistic method implicitly considers all possible motions which match a given design power spectrum. This is important since a single deterministic analysis may lead to a non-conservative design.
3. The probabilistic method provides confidence limits on all results (output response spectra, maximum accelerations, maximum stresses etc.) Hence, the results of the analysis are potentially more useful for the design of critical components.
4. The probabilistic method invariably leads to considerably lower computer costs than the deterministic method. Accordingly, it is believed that the probabilistic approach described above provides a valuable supplementary design tool for evaluation of soil-structure interaction effects.

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TABLE I - COMPARISON BETWEEN MAXIMUM ACCELERATIONS (g)

Nodal Point	Probabilistic Method			Deterministic Method
	Lower Level 90%	Mean Value	Upper Level 90%	
1	0.405	0.477	0.542	0.538
5	0.151	0.182	0.210	0.178
17	0.123	0.146	0.167	0.151
20	0.156	0.182	0.206	0.193
89	0.420	0.493	0.561	0.568
93	0.154	0.187	0.216	0.186
105	0.130	0.154	0.175	0.159
108	0.156	0.182	0.206	0.198
111	0.161	0.194	0.224	0.191
116	0.148	0.179	0.206	0.167
123	0.141	0.165	0.188	0.184
126	0.156	0.181	0.205	0.198

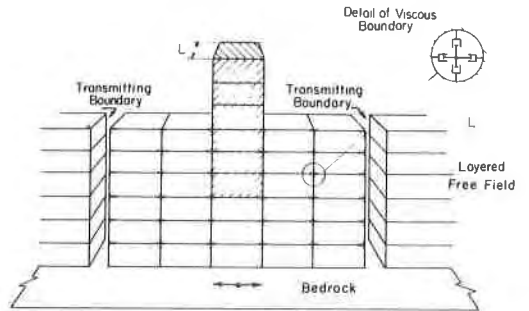
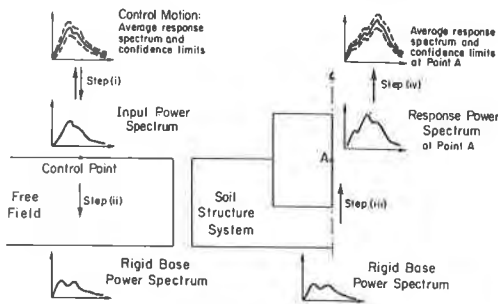


Fig. 1 Schematic Representation of Seismic Analysis.

Fig. 2 Soil-Structure Model.

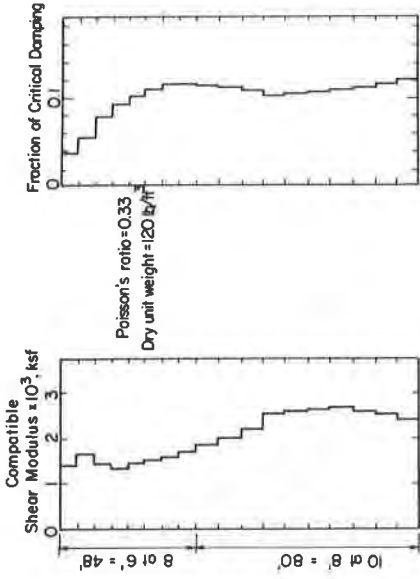


Fig. 4 Free Field Material Properties.

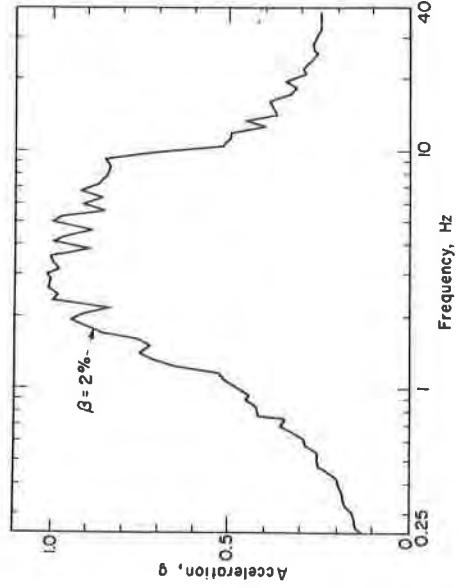


Fig. 5 Response Spectrum for Control Motion.

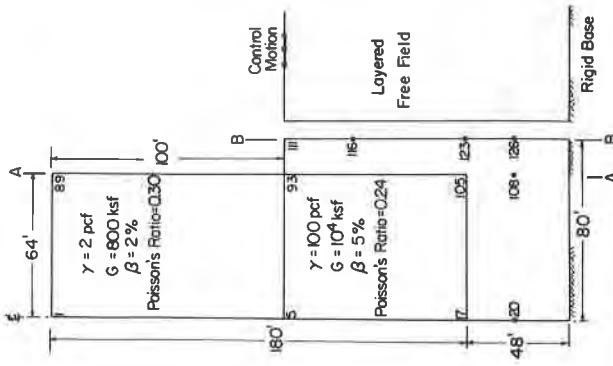


Fig. 3 Soil-Structure Model and Free Field.

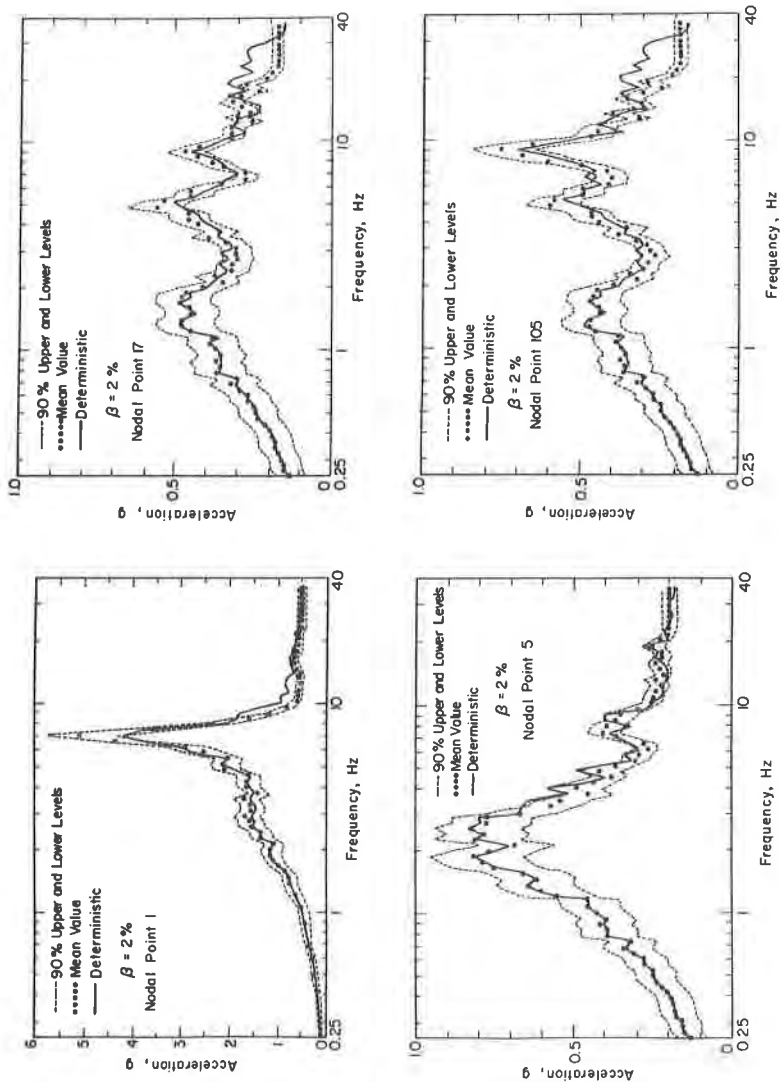


Fig. 6 Comparison of Probabilistic and Deterministic Response Spectra.