RESPONSE SPECTRUM ANALYSIS OF COUPLED STRUCTURAL RESPONSE TO A THREE COMPONENT SEISMIC DISTURBANCE

J. A. M. BOULET
Oak Ridge National Laboratory, Bldg. 9201-3, MS-8,
P.O. Box Y, Oak Ridge, Tennessee 37830, U.S.A.

T. G. CARLEY
Department of Engineering Science and Mechanics,
The University of Tennessee, Knoxville, Tennessee 37916, U.S.A.

SUMMARY

The work discussed herein is a comparison and evaluation of several response spectrum analysis (RSA) techniques as applied to the same structural model with seismic excitation having three spatial components.

The structural model includes five lumped masses (floors) connected by four elastic members. The base is supported by three translational springs and two horizontal torsional springs. In general, the mass center and shear center of a building floor are distinct locations. Hence, inertia forces, which act at the mass center, induce twisting in the structure. Through this induced torsion, the lateral ($\alpha$ and $\gamma$) displacements of the mass elements are coupled.

The ground motion components used for this study are artificial earthquake records generated from recorded accelerograms by a spectrum modification technique. The accelerograms have response spectra which are compatible with U.S. NRC Regulatory Guide 1.60.

Lagrange's equations of motion for the system were written in matrix form and uncoupled with the modal matrix. Numerical integration (fourth order Runge-Kutta) of the resulting modal equations produced time histories of system displacements in response to simultaneous application of three orthogonal components of ground motion, and displacement response spectra for each modal coordinate in response to each of the three ground motion components.

Five different RSA techniques were used to combine the spectral displacements and the modal matrix to give approximations of maximum system displacements. These approximations were then compared with the maximum system displacements taken from the time histories. The RSA techniques used are the method of absolute sums, the square root of the sum of the squares, the double sum approach, the method of closely spaced modes, and Lin's method.

The vectors of maximum system displacements as computed by the time history analysis and the five response spectrum analysis methods are presented. The absolute sum method, Lin's method and the closely spaced modes method are rather conservative; that is, they overestimate most or all of the maximum displacements. The results of the square root of the sum of the squares method and the double sum method are closer to the time history results but underestimate several displacements. The double sum method, on the whole, gave more accurate results than did the square root of the sum of the squares approach.

A noticeable feature of these results is that in reading down the displacement vector, the pattern of variation of the absolute value of the percentage error is roughly the same for all methods. Since the various response spectrum methods are simply different ways of combining the same numbers, this is not surprising.

The high percentage errors for some system displacements serve to emphasize the fact that system coordinate maxima are not functionally related to modal coordinate maxima. For any combination of structural model and excitation, one can expect to find certain displacements for which response spectrum analysis does not give good approximations.
1.0 Introduction

Dynamic structural analysis generally requires a well-defined structural model and a time history of the excitation for which the structure is to be designed. Given these, the maximum structural response can be found. In the case of seismic analysis, however, the design excitation is usually expressed as a ground motion response spectrum. The only information provided by a response spectrum is the maximum responses of a family of single degree of freedom systems. Therefore, in order to use a response spectrum for analysis of a multiple degree of freedom system, the equations of motion must be put into the form of independent, single degree of freedom equations. Under certain conditions, this can be accomplished through a coordinate transformation — from system coordinates to modal coordinates. Response spectrum analysis then gives the maximum values of the modal coordinates, but no functional relationship exists between the maximum values of the modal coordinates and the maximum values of the system coordinates. Hence, the maximum structural response cannot be computed.

Although response spectrum analysis does not permit the calculation of the maximum system response, it can be used to approximate that response. Many methods for doing this approximation are discussed in the literature [1], [2]. The work discussed herein is a comparison and evaluation of several of these methods, as applied to the same structural model with seismic excitation having three spatial components.

2.0 Analysis

The structural model includes five lumped masses (floors) connected by four elastic members arranged as shown in Fig. 1. The base is supported by three translational springs and two horizontal torsional springs. A typical mass element is shown in Fig. 2 with the origin of the coordinate system located at the shear center (elastic axis). The mass center and shear center of a building floor are distinct locations. Hence, inertia forces, which act at the mass center (see Fig. 2), induce twisting in the structure. Through this induced torsion, the lateral (x and y) displacements of the mass elements are coupled.

The elastic elements are prismatic beams, characterized by bending, transverse shear, and torsional stiffnesses. The beams are connected to the mass elements at the latter's shear centers.

The inertia forces generated by the independent rocking (rotation about horizontal axes) of the upper four mass elements are considered insignificant relative to the other forces in the system resulting in a structural model having 21 degrees of freedom. The coordinates used are the displacements of the element mass centers and the element torsional rotations for the floor masses and the displacements and rocking rotations of the base mass.

2.1 Equations of Undamped Free Vibration — Lagrange's Equations

Lagrange's equations of motion for a conservative system are

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_4} \right) - \frac{\partial L}{\partial q_4} = 0,$$  \hspace{1cm} (1)

where

- $L = T - U$,
- $T =$ system kinetic energy,
U = system potential energy,
\( q_i \) = typical system coordinate,
and the overdot indicates differentiation with respect to time.

The kinetic energy of a typical mass element

\[ T_i = \frac{1}{2} m_i \left( \dot{r}_i^2 + \dot{\mathbf{v}}_i^2 + \ddot{t}_i^2 \right) + \frac{1}{2} I_i \dot{\theta}_i^2, \]

(2)

where

\( m_i \) = mass of the element,
\( r_i, \mathbf{v}_i, t_i \) = x, y, and z components, respectively, of the mass center displacements,
\( \theta_i \) = rotation about the z axis (elastic axis) of the element,
\( I_i \) = mass moment of inertia of the element about its z axis.

The kinetic energy of the system is

\[ T = \frac{1}{2} m_0 \left( \dot{r}_0^2 + \dot{\mathbf{v}}_0^2 + \dot{r}_0^2 \right) + \frac{1}{2} I_0 \dot{\theta}_0^2 \\
+ \frac{1}{2} \sum_{i=1}^{n} m_i \left[ \dot{t}_i^2 + \dot{\mathbf{v}}_i^2 + \dot{\mathbf{v}}_i \right] + \left[ \dot{\mathbf{v}}_i + \dot{\mathbf{v}}_i - h_i \dot{\mathbf{v}}_i \right]^2 + \left[ \dot{t}_i + \dot{t}_i \right]^2 \]

(3)

\[ + \frac{1}{2} \sum_{i=1}^{n} I_i \dot{\theta}_i^2 \]

in which

\( m_0 \) = mass of the base,
\( r_0, \mathbf{v}_0, t_0 \) = x, y, and z components of base displacement,
\( \mathbf{v}_x, \mathbf{v}_y \) = rotation of the base about its x and y axes,
\( h_i \) = height of a typical mass element above the base,
\( I_i^x, I_i^y \) = mass moments of inertia of a typical mass element about the x and y axes of the element.

The potential (strain) energy of a typical mass element is

\[ U_i = \frac{1}{2} \sum_{j=1}^{n} k_{ij} \dot{q}_j^2 + \frac{1}{2} \sum_{j=1}^{n} k_{ij} \dot{\mathbf{v}}_j^2 + \frac{1}{2} \sum_{j=1}^{n} k_{ij} \dot{t}_j^2 + \frac{1}{2} \sum_{j=1}^{n} k_{ij} \dot{\theta}_j^2, \]

(4)

The potential energy of the system is

\[ U = \frac{1}{2} \sum_{i=1}^{n} k_{x0} \dot{r}_0^2 + \frac{1}{2} \sum_{i=1}^{n} k_{y0} \dot{r}_0^2 + \frac{1}{2} \sum_{i=1}^{n} k_{z0} \dot{r}_0^2 + \frac{1}{2} \sum_{i=1}^{n} k_{x0} \dot{r}_0^2 + \frac{1}{2} \sum_{i=1}^{n} k_{y0} \dot{r}_0^2 + \frac{1}{2} \sum_{i=1}^{n} k_{m} \dot{r}_0 \dot{e}_0 \theta_0 \dot{e}_0 \theta_0 \\
+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} k_{ij} \left[ \dot{r}_j^2 + \dot{\mathbf{v}}_j^2 + \dot{\mathbf{v}}_j \right] + \left[ \dot{\mathbf{v}}_j + \dot{\mathbf{v}}_j - h_i \dot{\mathbf{v}}_j \right]^2 + \left[ \dot{r}_j + \dot{r}_j \right]^2 \]

(5)

where

\( k_x, k_y, k_z, k_m, k_{ij}, k_{x0}, k_{y0}, k_{z0}, k_{m0} = \) stiffnesses associated with deflection of the springs which support the base.

Application of Lagrange's equation yields

\[ \ddot{\mathbf{v}}_0 \left[ m_0 + \frac{4}{i=1} m_i \right] + \ddot{\mathbf{v}}_0 \left[ \sum_{i=1}^{n} h_i m_i + \frac{4}{i=1} m_i \right] + K_{x0} \dot{\mathbf{v}}_0 = 0, \]

(6a)
\[ \begin{align*}
\ddot{\psi}_o \left( m_o + \sum_{i=1}^4 m_i \right) - \dot{\psi}_o \sum_{i=1}^4 h_i m_i + \sum_{i=1}^4 m_i \dot{\psi}_1 + K_y \dot{\psi}_o &= 0, \\
\ddot{\tau}_o \left( m_o + \sum_{i=1}^4 m_i \right) + \dot{\tau}_o \sum_{i=1}^4 m_i \dot{\psi}_1 + K_z \tau_o &= 0, \\
\ddot{\psi}_x \left[ \dot{I}_{x}^2 + \sum_{i=1}^4 \left( I_{x}^1 + h_i^2 m_i \right) \right] - \dot{\psi}_o \sum_{i=1}^4 h_i m_i - \frac{4}{3} \sum_{i=1}^4 h_i m_i \dot{\psi}_1 + \psi_x \dot{G}_x &= 0, \\
\ddot{\psi}_y \left[ \dot{I}_{y}^2 + \sum_{i=1}^4 \left( I_{y}^1 + h_i^2 m_i \right) \right] + \dot{\tau}_o \sum_{i=1}^4 h_i m_i + \frac{4}{3} \sum_{i=1}^4 h_i m_i \dot{\psi}_1 + \psi_y \dot{G}_y &= 0, \\
\ddot{\psi}_m^x + \dot{\tau}_o m_1 + \ddot{\psi}_x \dot{h}_1 m_1 + \frac{4}{3} \sum_{i=1}^4 k_{ij} \left( r_j + e_y \theta_j \right) &= 0, \\
\ddot{\psi}_m^y + \dot{\tau}_o m_1 - \ddot{\psi}_x \dot{h}_1 m_1 + \frac{4}{3} \sum_{i=1}^4 k_{ij} \left( v_j - e_x \theta_j \right) &= 0, \\
\ddot{\tau}_m^z + \dot{\tau}_o m_1 + \frac{4}{3} k_{ij} z_j &= 0, \\
\ddot{\theta}_j I_1 + \sum_{i=1}^4 \left( k_{ij} e_y y_i e_y + k_{ij} e_x e_x \right) + K_{ij} \theta_j + \frac{4}{3} \sum_{i=1}^4 \left( k_{ij} e_y y_i e_x - e_y e_x \right) &= 0. 
\end{align*} \]  

Putting equations (6) in matrix form, we have

\[ [m] \{q\} + [k] \{q\} = \{0\}, \]

in which

\[
[k] = \begin{bmatrix}
K_x & 0 & 0 & 0 & 0 \\
0 & K_y & 0 & 0 & 0 \\
0 & 0 & K_z & 0 & 0 \\
0 & 0 & 0 & G_x & 0 \\
0 & 0 & 0 & 0 & G_y \\
\end{bmatrix}
\]

\[
[k_{ij}] = \begin{bmatrix}
k_{ij}^{X} & 0 & 0 & k_{ij}^{X} \\
0 & k_{ij}^{Y} & 0 & 0 \\
0 & 0 & k_{ij}^{Z} & 0 \\
0 & 0 & 0 & k_{ij}^{Y} \\
\end{bmatrix}
\]

\[
k_{ij}^{X} = [k_{ij}^{X} e_{y_i}, k_{ij}^{X} e_{x_j}], \quad k_{ij}^{Y} = [-k_{ij}^{Y} e_{x_j}, k_{ij}^{Y} e_{y_i}], \quad k_{ij}^{Z} = [k_{ij}^{Z} e_{x_j}, k_{ij}^{Z} e_{y_i}], \quad k_{ij}^{Y} = [k_{ij}^{Y} e_{x_j}, k_{ij}^{Y} e_{y_i}]
\]
and

\[
[m] = \begin{bmatrix}
[m_{11}] & [m_{12}] \\
[m_{21}] & [m_{22}]
\end{bmatrix}
\]

\[
[m_{12}] = \begin{bmatrix}
1^t \{m_1\} & 1(0) & 1(0) & 1(0) \\
1(0) & 1^t \{m_1\} & 1(0) & 1(0) \\
1(0) & 1(0) & 1^t \{m_1\} & 1(0) \\
1(0) & 1(0) & 1(0) & 1(0)
\end{bmatrix}
\]

\[
[m_{21}] = [m_{12}]^T
\]

\[
[m_{22}] = \begin{bmatrix}
\{m_1\} & [0] & [0] & [0] \\
[0] & \{m_1\} & [0] & [0] \\
[0] & [0] & \{m_1\} & [0] \\
[0] & [0] & [0] & \{1_1\}
\end{bmatrix}
\]

\[
[m_{11}] = \begin{bmatrix}
0 & 0 & 0 & m_t \\
0 & m_1 & 0 & -m_t \\
0 & 0 & m_1 & 0 \\
0 & 0 & 0 & m_{44}
\end{bmatrix}
\]

\[
m_t = r_1 + \frac{4}{\pi} m_1 \\
m_{44} = r_1 + \frac{4}{\pi} (r_1 + m_1 h_1^2) \\
m_t = \sum_{i=1}^{4} m_i h_i \\
m_{55} = r_1 + \frac{4}{\pi} (r_1 + m_1 h_1^2)
\]

All of the indicated submatrices are of order 4. The solution of eq. (7) leads to an eigenvalue problem which can be solved for the natural frequencies (square roots of the eigenvalues) and mode shapes (eigenvectors) of the system. And, for certain kinds of damping, the matrix whose columns are the mode shapes (referred to as the modal matrix) can be used to uncouple the equations of damped forced vibration.

The eigenvalue problem resulting from the solution of eq. (7) is

\[
[k] \{\phi\}_i = \omega_i^2 [m] \{\phi\}_i ,
\]

where
\( \{\phi\} \) = typical mode shape,
\( \omega \) = typical natural frequency.

2.2 Seismic Response

The equations of motion for the dynamic response of the structure to a seismic disturbance characterized by three orthogonal components of ground acceleration \( x_g, y_g, \) and \( z_g \) are

\[
[m] \ddot{\{q\}} + [c] \{q\} + [k] \{q\} = -\{P\},
\]

in which \([c]\) is the damping matrix and \([P]\) is the vector of seismic inertia forces applied at the mass center of each element.

2.3 Modal Analysis

Equation (9) represents a system of 21 coupled equations of damped forced vibration. To uncouple these equations, first define modal coordinates, \( \{\eta\} \), such that

\[
\{q\} = \{\phi\} \{\eta\}.
\]

The columns of the matrix \([\phi]\) are the eigenvectors resulting from the solution of eq. (8). With eq. (10), equations (9) become

\[
\ddot{\eta} + 2\xi \omega \ddot{\eta} + \omega^2 \eta = -\Gamma^x \ddot{\eta}_x - \Gamma^y \ddot{\eta}_y - \Gamma^z \ddot{\eta}_z.
\]

The modal participation factors are given by

\[
\Gamma^x = \frac{1}{M_{ii}} \left\{ n_1 \phi_{1i} + \sum_{j=1}^{4} n_j \phi_{ji} + \sum_{j=5}^{9} n_j \phi_{ji} \right\},
\]

\[
\Gamma^y = \frac{1}{M_{ii}} \left\{ n_1 \phi_{2i} - \sum_{j=1}^{4} n_j \phi_{ji} + \sum_{j=11}^{13} n_j \phi_{ji} \right\},
\]

\[
\Gamma^z = \frac{1}{M_{ii}} \left\{ n_1 \phi_{3i} + \sum_{j=14}^{17} n_j \phi_{ji} \right\},
\]

and

\[ M_{ii} = \text{typical mass coefficient}, \]
\[ \eta_i = \text{typical modal coordinate}, \]
\[ \phi_{ji} = \text{element } j \text{ of mode shape } i. \]

2.4 Time History Analysis

Because they are uncoupled, the 21 modal equations represented by eq. (11) can be solved independently to give \( \{\eta\} \) as functions of time. The values of \( \{q\} \) at any time can be found from the values of \( \{\eta\} \) for that time and eq. (10). Once the time histories of \( \{q\} \) are developed, the maximum values of each of the system coordinates is available by inspection. For purposes of this study, time history analysis was performed for simultaneous application of three spatial components of ground motion. The maximum values of the system coordinates resulting from this analysis were used as a standard for evaluating the various response spectrum analysis techniques discussed below.
2.5 Response Spectrum Analysis

Unfortunately, it is not possible to express the ground motion of future seismic disturbances as functions of time. Nor is it prudent to use but one earthquake record for structural design excitation. Rather, a structure should be designed to survive, with little or no damage, all seismic disturbances it is likely to encounter. To this end, the structural designer turns to response spectrum analysis.

By definition, a response spectrum is, for a given damping ratio and excitation, the maximum responses of a family of damped, single degree of freedom oscillators plotted against the natural frequencies of the oscillators. The response spectrum provides only the maximum values of the modal coordinates.

Although the maximum system response cannot be calculated precisely from response spectrum analysis, it can be approximated. As indicated in the section on modal analysis (Section 2.3), the system and modal coordinates are related by eq. (10). From response spectrum analysis, we have only the maximum modal coordinate values given by

\[
\begin{align*}
\{\eta^x\}\text{max} &= \{(r^x)_{\text{max}}\}, \\
\{\eta^y\}\text{max} &= \{(r^y)_{\text{max}}\}, \\
\{\eta^z\}\text{max} &= \{(r^z)_{\text{max}}\},
\end{align*}
\] (12)

where the \(\{r\}\) are response spectrum values, and the superscripts indicate the spatial component of the ground motion. Since the \(\{\eta\}\text{max}\) are not functionally related to the \(\{q\}\text{max}\), it is desired to find an equation that approximates the \(\{q\}\text{max}\) in terms of the \(\phi_{ij}\) and the \(\{\eta\}\text{max}\). The equations discussed below were all evaluated for this study. The response spectra used were generated from the three spatial components of ground motion used in the time history analysis.

**Absolute sum** [1]. In this method, the absolute values of the modal responses are added. That is,

\[
(q_i^s)\text{max} = \sum_{j=1}^{21} |\phi_{ij} (\eta_i^s)\text{max}|,
\] (13)

where the superscript refers to the component of excitation considered. This equation implies that all the modal maxima occur at once, and represents an upper bound for \(q_i^s\text{max}\).

After applying eq. (13) for each of the three components of excitation, the results are combined by

\[
(q_i^s)\text{max} = \sqrt{(q_i^x)^2_{\text{max}} + (q_i^y)^2_{\text{max}} + (q_i^z)^2_{\text{max}}},
\] (14)

**Square root of sum of squares** [3]. As is clear from its name, the equation is

\[
(q_i^s)\text{max} = \sqrt{\sum_{j=1}^{21} \phi_{ij} (\eta_j^s)\text{max}^2}.
\] (15)

As in the absolute sum method, eq. (15) is used to combine the results for the three components of ground motion.
Double sum \([4]\). This method attempts to take into account the mutual reinforcement that occurs between two modes if their natural frequencies are nearly equal. The equation is

\[
\begin{aligned}
(q_{I}^{\alpha})_{\text{max}} = \left[ \sum_{j=1}^{21} \sum_{k=1}^{21} \phi_{ij} (n_{j})_{\text{max}}^{\alpha} \phi_{ik} (n_{k})_{\text{max}}^{\alpha} e_{jk} \right]^{1/2},
\end{aligned}
\]  

(16)

where

\[
\begin{aligned}
e_{jk} &= \left[ 1 + \left( \frac{\omega_{j}^{2} - \omega_{k}^{2}}{\omega_{j} \omega_{k} + \beta_{j} \omega_{k}} \right)^{2} \right]^{-1},
\end{aligned}
\]

\[
\begin{aligned}
\omega_{j}^{2} &= \omega_{j}^{2} \left( 1 - \beta_{j}^{2} \right)^{1/2},
\end{aligned}
\]

\[
\begin{aligned}
\beta_{j} &= \varepsilon_{j} + \frac{2}{c_{d} \omega_{j}}.
\end{aligned}
\]

d = duration of ground motion.

Note that eq. (15) is a special case of eq. (16) with

\[
\begin{aligned}
e_{jk} = 0 \quad \text{for} \quad j \neq k.
\end{aligned}
\]

As in the previous methods, eq. (14) is used to combine the results from the three ground motion components.

Closely spaced modes \([5]\). Like the double sum method, this method attempts to account for intermodal coupling. But rather than include all the cross products and calculate a coupling coefficient for each one, this method mixes the first two methods. For modes in a closely spaced group, the absolute sum method is first applied. Then, the square root of the sum of the squares method is applied to the group totals and the remaining well-separated modes. A closely spaced group includes those modes whose frequencies lie between the lowest in the group and a frequency 10% higher. The equation is

\[
\begin{aligned}
(q_{I}^{\alpha})_{\text{max}} = \left\{ \sum_{j=1}^{k} \left[ \phi_{ij} (n_{j})_{\text{max}}^{\alpha} \right]^{2} + \sum_{l=1}^{m} \left[ \sum_{n=1}^{p_{l}} \phi_{ln} (n_{n})_{\text{max}}^{\alpha} \right]^{2} \right\}^{1/2},
\end{aligned}
\]  

(17)

where

\[
\begin{aligned}
k &= 21 - \sum_{l=1}^{m} p_{l}, \text{number of well-separated modes},
m &= \text{number of groups of closely spaced modes},
p_{l} &= \text{number of modes in group } l.
\end{aligned}
\]

Lin’s method (after Lin \([2]\)). Lin’s method differs significantly from the others discussed here. Rather than compute the maximum system response to each component of ground motion by absolute sums and then combine by the square root of the sum of the squares, this method first combines absolutely the modal maxima due to the three components of excitation and then computes the overall maximum system response by taking the square root of the sum of the squares. The equation is

\[
\begin{aligned}
(q_{I})_{\text{max}} = \left[ \sum_{j=1}^{21} \left[ \phi_{ij} \left( (n_{j}^{x})_{\text{max}} + |(n_{j}^{y})_{\text{max}}| + |(n_{j}^{z})_{\text{max}}| \right) \right]^{2} \right]^{1/2}.
\end{aligned}
\]  

(18)
3.0 Numerical Calculations

The structural data are given in Figure 1. The ground motion components used are artificial earthquake records (ground acceleration given numerically at 0.01 second intervals) generated by a spectrum modification technique from response spectra compatible with U.S. NRC Regulatory Guide 1.60 for a maximum ground acceleration of 1g and 20 seconds duration. A modal damping of 5% was used.

4.0 Comparison of Response Spectrum Methods

The vectors of maximum system displacements as computed by the time history analysis and the five response spectrum analysis methods, are presented in Table 1. A noticeable feature of these results is that in going down the displacement vector, the pattern of variation of the absolute value of the percentage error is roughly the same for all methods. Since the various response spectrum methods are simply different ways of combining the same numbers, this is not surprising. The high percentage errors for system displacement \( r_1 \) serve to emphasize the fact that system coordinate maxima are not functionally related to modal coordinate maxima. For any combination of structural model and excitation, one can expect to find certain displacements for which response spectrum analysis does not give good approximations. That all the response spectrum methods gave their worst results for the \( r_1 \) displacement simply marks this coordinate as a "difficult" one for the structural model and excitation used in this study. Likewise, the typically low percentage errors in the approximations of the vertical coordinates show that response spectrum analysis can give quite good results for some coordinates.

The absolute sum method, Lin's method, which is similar to the absolute sum method, and the closely spaced modes method, are rather conservative; that is, they overestimate most or all of the maximum displacements. On the other hand, the results of the square root of the sum of the squares method and the double sum method are closer to the time history results but underestimate several displacements. The double sum method, on the whole, gave more accurate results than did the square root of the sum of the squares approach. This is in agreement with the prediction of Rosenblueth and Elorduy, and with the statements by Chu, et al., to the effect that consideration of intermodal coupling leads to an improvement over the square root of the sum of the squares method.

5.0 Conclusions

Strictly, the results of this study are valid for only one particular combination of structural model and excitation, and the relative merits of the various response spectrum techniques cannot be judged in an absolute sense until many more studies of this type are done. If the results of this study are taken as representative of such studies, the following conclusions are in order:

(1) The use of the absolute sum method, Lin's method, or the closely spaced modes method, is highly conservative and could result in significant overdesigning.

(2) The use of either the double sum approach or the square root of the sum of the squares approach leads to the underestimation of some maximum displacements and cannot be recommended.
(3) To insure that no maximum displacements are underestimated, and to minimize overdesigning, it is recommended that the maximum system displacements be computed by the square root of the sum of the squares method and these results multiplied by 1.2. (This compensates for up to 15% underestimation.)

Despite the superiority of the double sum approach, the square root of the sum of the squares method would be recommended because it is computationally the simpler of the two.

6.0 List of References


<table>
<thead>
<tr>
<th>DISPLACEMENT COORDINATE</th>
<th>TIME HISTORY</th>
<th>ABSOLUTE SUM</th>
<th>SRSS</th>
<th>DOUBLE SUM</th>
<th>CLOSELY SPACED MODES</th>
<th>LTN'S METHOD</th>
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Figure 1. Structural Model

Figure 2. Mass Element