ANALYTICAL PROCEDURE IN ASEISMIC DESIGN
OF ECCENTRIC STRUCTURE USING RESPONSE SPECTRUM

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SUMMARY

As the design response spectrum enveloping the response spectra derived from the actual earthquake waves generally gives conservative responses, it is utilized as safe and time-saving procedure for the seismic design.

However, when the design response spectrum is used for the torsional dynamic analysis of the auxiliary building etc. in PWR nuclear power plant, of which sectional plan and the cross section are complicated and which have eccentricities between the centers of mass and rigidity, the responses perpendicular to the input earthquake direction and torsional responses caused by the root mean square technique are generally too large compared with the results of the time history analysis using actual earthquake waves.

As a reason of these phenomenon, it is considered that the participation modes perpendicular to the input earthquake direction and torsional direction in the multi-degree of freedom system do not have predominant values.

In this paper, the responses are evaluated by the following two methods by the use of the typical torsional analytical models in which masses, rigidities, eccentricities between the centers thereof and several actual earthquake waves are taken as the parameters;

(1) the root mean square of responses by using the response spectra derived from the earthquake waves,
(2) the time history analysis by using the earthquake waves.

The earthquake waves used are chosen to present the different frequency content and magnitude of the response spectra.

The typical results derived from the study are as follows;

(a) the response accelerations of mass center in the input earthquake direction by the (1) method coincide comparatively well with those by the (2) method,
(b) the response accelerations perpendicular to the input earthquake direction by (1) method are 2 to 3 times as much as those by the (2) method,
(c) the amplification of the response accelerations at arbitrary points distributed on the spread mass to those of center of the lumped mass by the (1) method are remarkably large compared with those by the (2) method in both directions respectively.

These problems on the response spectrum analysis for the above-mentioned eccentric structure are discussed, and an improved analytical method applying the amplification coefficients of responses derived from this parametric time history analysis is proposed to the actual seismic design by the using of the given design ground response spectrum with root mean square technique.
1. Introduction

In the seismic design of nuclear power plants, a design response spectrum which covers the response spectra obtained from earthquake waves having different frequency characteristics has been used for evaluating the responses of structure and investigating the stress thereof from the viewpoint of safety design and for convenience, for the mode superpositions on the response analysis and the root mean square method (hereinafter called R.M.S.) [1], [2] has been generally applied for the evaluation of the response behavior.

It has become necessary to consider the amplification of torsional response for the design of auxiliary buildings attached to the reactor containment facilities of PWR type nuclear power plants in which plans and sections are complicated in shapes. However, when R.M.S. method described in chapter 2 is applied to such complicated structures, great amount of response acceleration is derived, especially in the orthogonal direction to the base acceleration as compared with those obtained by the time history analysis (hereinafter called T.H.).

In this paper, the responses are evaluated by the following two methods for the typical torsional analytical models in which masses, rigidities, eccentricities between the centers of masses and rigidities and several actual earthquake waves are adapted as parameters; (1) R.M.S. of responses using the response spectra derived from the earthquake waves. (2) T.H. using the earthquake waves.

From those results, response value at the center of rigidity and arbitrary points on a floor of the structure will be expressed statistically by the ratio thereof to the result of T.H.. Such response values are necessary for the seismic design of equipments and pipings or the stress analysis of structures using design response spectrum.

2. Basic Equations and R.M.S. Method

The equations of motion and R.M.S. method using response spectrum are summarized below. The equation of motion is the base of our computer program of torsional dynamic analysis which has the degree of freedom of two horizontal components \((x, y)\) and one rotational \((\theta)\).

(a) Equations of motion

From the equilibrium of forces at the mass point, the equations of motion of multi-mass model can be given as follows:

\[
[M] \dddot{z} + [C] \ddot{z} + [K] \dot{z} = \{P(t)\}
\]

where \((z, \dot{z}, \ddot{z})\); displacement, velocity, acceleration vector respectively

\([M]\); mass matrix
\([C]\); damping matrix
\([K]\); stiffness matrix
\([P(t)]\); load vector

Since the center of rigidity does not coincide the center of mass, \([K]\) is derived as follows:

Assuming that \(z'\) is the displacement at the center of rigidity and \(P'\) is the nodal force at the same point, following equation can be obtained for the multi-mass model.

\[
\{P'(t)\} = [K'] \{z'(t)\}
\]

\([K']\) can be given directly from the stiffness of vertical members.
\([T_1]\) and \([T_2]\) are assumed as follows:

\([T_1]\); \([T_1]\) is the matrix connecting the displacement between the center of rigidity and center of mass on a rigid plane.

\([T_2]\); \([T_2]\) is the matrix connecting the nodal force between two points.

\[
\begin{align*}
(\mathbf{w}') &= [T_1] (\mathbf{x}) \\
(\mathbf{p}') &= [T_2] (\mathbf{p})
\end{align*}
\]  
\((3)\)  
\((4)\)

Substituting eqs. \((3)\) and \((4)\) to \((2)\)

\[
\sum (\mathbf{p}) = [T_2]^{-1} [K'] [T_1] (\mathbf{x})
\]  
\((5)\)

From eq. \((5)\), eq. \((6)\) is derived

\[
[K] = [T_2]^{-1} [K'] [T_1]
\]  
\((6)\)

(b) Eigenvalue, eigenvector

From eq. \((1)\) a linear, non-damping free-vibration is expressed as follows:

\[
[M] (\mathbf{x})'' + [K] (\mathbf{x}) = 0
\]  
\((7)\)

Using above-mentioned stiffness matrix and mass matrix, the following \([H]\) matrix is defined:

\[
[H] = [M]^{-1} [K]
\]  
\((8)\)

The eigenvalue problem is expressed by the following equation.

\[
[H] (\mathbf{v}) = \lambda (\mathbf{v})
\]  
\((9)\)

where \(\lambda\); eigenvalue

\([\mathbf{v}]\); eigenvector

Jacobi-method is employed to resolve this equation.

(c) Modal-analysis using response spectrum

The probable maximum acceleration at the center of mass is expressed by the R.M.S. as follows:

\[
\ddot{\mathbf{x}}' = \sqrt{\frac{\mathbf{H}}{\dddot{\mathbf{x}}'_{\text{MAX}}} (\dot{\mathbf{\Phi}} \cdot \dddot{\mathbf{y}} \cdot \dddot{\mathbf{y}}_{\text{MAX}})^2}
\]  
\((10)\)

where \(\dddot{\mathbf{x}}'\): number of mode

\(\dot{\mathbf{\Phi}}\): eigen vector

\(\dddot{\mathbf{y}}\): participation factor

\(\dddot{\mathbf{y}}_{\text{MAX}}\): response value taken from response spectrum

The relation of acceleration vectors at the center of mass and arbitrary point is expressed as follows; as well as eqs. \((3)\) and \((4)\):

\[
\dddot{\mathbf{x}}' = [T_1]^{-1} (\dddot{\mathbf{x}}')
\]  
\((11)\)

In such a case, \(\dddot{\mathbf{x}}_x\) (component of \(x\) direction) at the mode number \(\dddot{\mathbf{x}}_x\) is expressed:

\[
\dddot{\mathbf{x}}_x = \begin{bmatrix} T_{1-1} & T_{1-12} & T_{1-13} \end{bmatrix} \begin{bmatrix} \dddot{\mathbf{\Phi}}_x \\ \dddot{\mathbf{\Phi}}_y \\ \dddot{\mathbf{\Phi}}_z \end{bmatrix}
\]  
\((12)\)

Consequently a general equation at the arbitrary point is given as follows:

\[
\dddot{\mathbf{x}}_x = \sqrt{\frac{\mathbf{H}}{\dddot{\mathbf{x}}'_{\text{MAX}}} (\dot{\mathbf{\Phi}} \cdot \dddot{\mathbf{y}} \cdot \dddot{\mathbf{y}}_{\text{MAX}})^2}
\]  
\((13)\)
3. Investigation

The investigation is divided into the following two cases. Case 1: the investigation is carried out by using one mass model to obtain the general characteristics of torsional response, in which the eccentricity between the center of mass and the center of rigidity and the ratio of horizontal stiffness to the rotational stiffness of each direction and the natural period of structure are used as parameters. Case 2: the investigation is carried out by using analytical model having five masses which is the model of the actual reactor auxiliary buildings. The relations of the center of rigidity to the center of mass is used as parameters. The purpose of investigation in Case 1 is subdivided into Step 1, 2 and 3 in accordance with the various combination of parameters.

Walls and other structural members resisting seismic force are designed not to produce much torsional response and from these experiences, we have selected the values of eccentricity as parameters.

1) Case 1;

One mass model is used. The purpose is to investigate the results by T.H. and the those by R.M.S. and to compare one to the other by changing $\Theta_e$ (eccentricity between rigidity and mass), $K_r/Kx$ (Kx: rotational stiffness, Kx: horizontal stiffness of the direction of base acceleration) and the natural period of structure. The analytical model is illustrated in Fig.-1. Mass, mass rotational inertia, horizontal stiffness, rotational stiffness, etc. used herein are found generally in our design of PWR type nuclear power plants. Analysis of the case where the center of rigidity can be found in point 3 in Fig.-1 (hereinafter called C.R.3) is omitted because there is no eccentricity in the direction of base acceleration and consequently no torsional effect is found.

(a) Step 1

The purpose of Step 1 is to obtain the following ratios of the specific accelerations to the acceleration at the center of mass. Specific accelerations are:

1) the response acceleration at the extreme end of model in the direction to base acceleration as indicated in Fig.-1.

2) the response acceleration at the center of mass in orthogonal to the base excitation.

3) the response acceleration at the extreme end of model in orthogonal to the base excitation.

Parameters and properties used in Step 1 are shown in Table - I.

(b) Step 2

The above-mentioned ratios are studied for $K_r/Kx$ value as parameter instead of eccentricity in Step 1.

Parameters and properties used in Step 2 are shown in Table - II.

(c) Step 3

The ratio of response acceleration in earthquake wave direction at the extreme end of model and center of mass for $K_x$ values as parameters.

Parameters and properties used in Step 3 are shown in Table - III.
2) Case 2;
In Case 1 the statistical tendency in the results by T.H. and R.M.S. is studied using one mass model.

In Case 2 fine lumped mass model is used. The purpose of this case is to obtain statistical tendency by T.H. and R.M.S. changing the centers of the upper masses or lower masses.

Among the values as parameter the mass, mass rotational inertia, horizontal stiffness, rotational stiffness, etc. are set at fixed value (shown in Table - IV) considering general tendency of structural characteristics of A/B to simplify analytical procedure. The analytical model is illustrated in Fig.-2.

4. Earthquake Waves
Following four earthquake waves are used which have different frequency characteristics.

1) EL CENTRO 1940 NS (CIT. REPORT VOL. II)
2) GOLDEN GATE 1957 S80E ("")
3) TAFT 1952 EW ("")
4) SENDAI 501 NS

The response acceleration spectrum for 5% damping ratio is illustrated in Fig.-3.
The above-mentioned four earthquake waves show their peak values at the following periods,

1) EL CENTRO at 0.13, 0.18 and 0.25 seconds
2) GOLDEN GATE at 0.14 and 0.22 seconds
3) TAFT at 0.22 and 0.33 seconds
4) SENDAI 501 at 0.13 and 0.28 seconds

These waves cover from the first to third natural periods of reactor containment facilities and auxiliary buildings, and are, therefore, considered reasonable for our study.

5. Results of Investigation
1) Case 1
Results of analysis of Step 1 through 3 of Case 1 are illustrated in Fig.-4 (a) through (f).

1-1) Step 1
As indicated in Figs.-4 (a) and (b), the result in Step 1 shows that $a_1$ and $a_2$ increase as $c_s$ in case the center of rigidity is at 1 (hereinafter called C.R.1) and at 2 (hereinafter called C.R.2) with minor exception. As shown in Fig.-5 specific translational accelerations are defined as follows:

$A_0$ : acceleration at the center of masses longitudinal to the base acceleration.
$A_1$ : acceleration at the extrem end of the model parallel to the base acceleration.
$A_2$ : acceleration at the center of mass of orthogonal to the base acceleration. $A_3$ : acceleration at the extrem end of the orthogonal to the base acceleration.

The ratios of $A_1$, $A_2$ and $A_3$ to $A_0$ are represented by $a_1$, $a_2$ and $a_3$ respectively as in Fig.-6. The exceptional results mentioned before are the characteristics of $a_1$ and $a_2$ of C.R.1 by SENDAI 501 and $a_1$ of C.R.2 by TAFT. The response acceleration by R.M.S. are bigger than those by T.H. with one ex-
ception of C.R.1 by SENDAI 501. In the result of C.R.2 the response accelerations parallel and orthogonal to the base acceleration are identical at center of mass and along the extreme ends, respectively.

1-2) Step 2

The result of investigation in Step 2 is illustrated in Figs.-4 (c) and (d). The result shows that the values of \( a_1 \), \( a_2 \) and \( a_3 \) indicate their maximums at \( 1.0 \times 10^4 \) m/s²/Rad. of \( K_x/K_x \) and decrease at \( K_x/K_x \) increase or decrease. It is indicated that the values of analytical results by R.M.S. are bigger than those by T.H. excluding the one of C.R.2 by SENDAI 501. In the result of analysis of C.R.2 the relations between parallel and orthogonal response accelerations to the base acceleration are identical Step 1.

1-3) Step 3

The result of investigation in Step 3 is shown in Figs.-4 (e) and (f). Any significant difference of the value of \( a_3 \) is not seen for the various stiffnesses of whole building or its natural period in C.R.1 and C.R.2 by T.H. and R.M.S. In the result of Step 3 \( a_1 \) by R.M.S. is greater than that by T.H. as seen Step 1 and Step 2.

From the results of steps 1 through 3, the comparison of the ratios of response accelerations to the one at the center of mass parallel to the base acceleration by T.H. and R.M.S. is studied and depicted graphically in Figs.-7 (a) through (c). In order to compare the torsional effects by T.H. and R.M.S. following ratios are studied. These are:

1. \( \alpha_{1E}/\alpha_{1S} \); the ratio of translational components at the extreme end of model parallel to the base acceleration.
2. \( \alpha_{2E}/\alpha_{2S} \); the ratio of translational components at the center of mass orthogonal to the base acceleration.
3. \( (\alpha_{3E} - \alpha_{2E})/(\alpha_{3S} - \alpha_{2S}) \); the ratio of torsional components orthogonal to the base acceleration.

where: \( \alpha_{1E} \), \( \alpha_{2E} \) and \( \alpha_{3E} \); \( \alpha_{1S} \), \( \alpha_{2S} \) and \( \alpha_{3S} \) by T.H. respectively, \( \alpha_{1S} \), \( \alpha_{2S} \) and \( \alpha_{3S} \) by R.M.S. respectively.

The observations can be arrived at from Figs.-7 (a) through (c).

1. Most values of \( \alpha_{1E}/\alpha_{1S} \) are shown from 0.77 to 0.93. This indicates the result by T.H. is a little smaller than that by R.M.S.

2. Most values of \( \alpha_{2E}/\alpha_{2S} \) are shown from 0.01 to 0.04. This indicates the result by T.H. is generally much less than that by R.M.S.

3. With respect to \( (\alpha_{3E} - \alpha_{2E})/(\alpha_{3S} - \alpha_{2S}) \) values are shown over a wide range, and most values are shown from 0.47 to 0.65. This indicates the result by T.H. is approximately half of the value by R.M.S.

2) Case 2

The ratios, \( \alpha_1 \), \( \alpha_2 \) and \( \alpha_3 \) by T.H. and R.M.S. under various conditions in Table IV are shown in Figs.-6 (a) and (b). The ratios \( \alpha_1 \), \( \alpha_2 \) and \( \alpha_3 \) in the figures represent the same content as defined in Case 1. From the result of this analysis the followings can be generally said. The results by R.M.S. show greater values as compared
with those by T.H. The difference of values obtained by analysis of both is small with respect to $\alpha_2$, but rather large with respect to $\alpha_1$ and $\alpha_3$. From the result shown in Figs.-8 (a) and (b), the values of $\alpha_{1E}/\alpha_{1S}$, $\alpha_{2E}/\alpha_{2S}$ and $(\alpha_{2E} - \alpha_{2S})/(\alpha_{3E} - \alpha_{3S})$ are obtained in the same manner as described in Case 1, and they are shown graphically in Figs.-9 (a) through (c). Definitions of $\alpha_{1E}$, $\alpha_{1S}$, $\alpha_{2E}$, $\alpha_{2S}$, $\alpha_{3E}$ and $\alpha_{3S}$ are the same as described in Case 1. From Figs.-9 (a) through (c) the following can be said:

1) Values of $\alpha_{1E}/\alpha_{1S}$ are generally closer to 1.00 than values obtained in Case 1 and most values are ranging from 0.9 to 1.00 with the highest frequency at 1.00. This indicates the result by T.H. is a little smaller than that by R.M.S.

2) In most cases the values of $\alpha_{2E}/\alpha_{2S}$ are from 0.00 to 0.05 as found Case 1 with the highest frequency at 0.00. This indicates that the results by T.H. show very small value as compared with those by R.M.S.

3) With respect to $(\alpha_{3E} - \alpha_{2E})/(\alpha_{3S} - \alpha_{2S})$, values are shown over a wide range, and most values are from 0.5 to 1.7, having average value of 0.96 and standard deviation 0.27, with the highest frequency at 1.00. This indicates that the result by T.H. is generally a little smaller than those by R.M.S.

6. Conclusion

As mentioned previously, in the design of nuclear power plants the design response spectrum which covers response spectrum yielded from earthquake waves having different frequency characteristics as illustrated in Fig.-3 is generally made and used accompanied by R.M.S. to evaluate conservative response. Most buildings have eccentricities between the center of mass and rigidity, and consequently are subjected to torsional effect. The torsional responses of buildings can be evaluated using the result obtained statistically from our investigation. To yield moderate response acceleration affected by torsional effect from responses by R.M.S. used here following analysis and modifications will be needed.

1) The response acceleration parallel to the base acceleration by R.M.S. may be used without modification at any point.

2) At the center of mass, the ratio of response accelerations orthogonal to parallel to the base acceleration, $\delta_2$, may be reduced to one-twentieth.

3) Torsional effects in orthogonal direction to base acceleration can be estimated to be $(\alpha_{3S} - \alpha_{2S})$ which takes into account one standard deviation as engineering judgement.

4) Using the estimated accelerations in 2 and 3, the translational acceleration ratio orthogonal to base acceleration shall be expressed as follow.

$$0.05 \alpha_{2S} + 1.25 (\alpha_{3S} - \alpha_{2S}) = 1.25 \alpha_{3S} - 1.20 \alpha_{2S}$$

References


Fig. 1  Model for Case 1

ELEVATION
CR: CENTER OF RIGIDITY
CM: CENTER OF MASS

Fig. 2  Model for Case 2

ELEVATION
PLAN

Fig. 3  Response Acceleration Spectrum of Earthquake Waves
Fig. 4  Ratio of Acceleration of Case 1

Fig. 5  Response Acceleration  Fig. 6  Ratios of Accelerations at each points to Longitudinal Acceleration at the Center of Mass

Fig. 7  Histogram of Acceleration Ratio of Case 1
Table II Properties and Parameters in Step 2 of Case I

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<th>$\ell_m$ (MASS ROTARY INERTIA)</th>
<th>$k_{\theta}$ (ROTATIONAL STIFFNESS)</th>
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Table III Properties and Parameters in Step 3 of Case I

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Table IV Properties and Parameters in Case 2

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Fig. 9 Histogram of Acceleration Ratio of Case 2