A RATIONAL AND ECONOMICAL SEISMIC DESIGN OF BEAM COLUMNS IN STEEL FRAMES

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SUMMARY

Beam columns in steel frames are required to resist axial forces together with biaxial moments when subjected to earthquake excitation in three directions. Design of a section of such a beam column depends on the axial force and the two moments on the section and also on the distribution of the two moments along the axis of the member.

The seismic analysis is often performed by the response spectrum method. Various responses (e.g., forces and moments) in several modes of vibration under the three components of earthquake are combined probabilistically by the SRSS (square root of the sum of the squares) method. In case of modes with closely spaced frequencies a modified double sum approach is used. In either case, one obtains the maximum probable values of individual responses, which do not occur simultaneously. In the conventional design of beam columns, maximum probable values of the axial force and the moments at various sections are used as though they were acting simultaneously. Such a procedure yields a conservative design.

In the present study, a new rational procedure is used in which simultaneous variation in various response quantities is predicted. For designing the beam column section according to the AISC Manual of Steel Construction, one has to know the values of the axial force, the moments about x and y axes at the two ends, and the maximum moments about x, y axes near the center of the beam column, which altogether constitutes seven response quantities of interest for each beam column element. Normally, seven equivalent modes will be required to represent the response. However, by designing the two end sections and the intermediate section independently one can consider three equivalent modes for each section, thus simplifying the problem a great deal.

An existing computer program is used for the implementation of the proposed method. Results for typical example problems have been presented. It is shown that savings up to 42% in the steel cross-sectional area can be obtained depending upon combination of various forces and moments. The proposed method is "exact" within the existing assumptions of the SRSS or the double sum method.

1. Introduction

Beam columns in steel frames are required to resist axial forces together with biaxial moments when subjected to earthquake excitation in three directions. Design of a section of such a beam column depends on the axial force and the two moments on the section and also on the distribution of the two moments along the axis of the member.

The seismic analysis is often performed by the response spectrum method. The analysis yields the maximum value of various responses in each mode of vibration for each of the three components of earthquake. The maximum probable value of any response (displacement, velocity, force, etc.) is obtained by combining the values of that response in all the modes under three components of earthquake by the square root of the sum of squares (SRSS) method; Chu, Amin, and Singh [1]. In case of modes with closely spaced frequencies a modified double sum approach is used; Singh, Chu, and Singh [2]. Whichever is the case, the above method yields the maximum probable value of any response. In general, the maximum probable values of various responses do not occur simultaneously. This would create a problem when the design criterion of a structural element is based on the values of more than one response occurring simultaneously. Such is the case for beam columns as described above. Design of a section not only depends upon the axial force and the moments on that section but also on the distribution of these moments along the axis of the member. In the conventional method, the maximum probable values of the maximum probable axial force and moments on the section is used as if they were acting simultaneously; the distribution of moments along the axis of the beam column is assumed based on the values of maximum probable moments such that it is most unfavorable. The result is a design which may be overly conservative.

In the present study, a new rational procedure is used based on Gupta and Chu [3] in which simultaneous variation in various response quantities is predicted. For designing a beam column section according to the AISC Manual of Steel Construction [4] one has to know the values of the axial force, the moments about x and y axes at the two ends, and the maximum moments about x,y axes near the center of the beam column, which altogether constitute seven response quantities of interest for each beam column element. To predict the simultaneous variation in all seven response quantities, seven equivalent modes will be required; Gupta and Chu [3]. However, one can consider each section independently, in which case the simultaneous variation in the axial force and moments at a section can be represented with the help of only those equivalent modes--the other four models can be implicitly so selected that they do not contribute any force or moment to the section under design.

The AISC Manual [4] has two types of interaction curves: linear and nonlinear. In the case of a linear interaction curve, the design can be performed directly without going through the equivalent modes. In the case of a nonlinear interaction curve, however, it is necessary to calculate equivalent model responses.

In the present study, an existing computer program is used for designing the steel sections. The theory as applied to the steel beam columns has been presented and then applied to a set of example problems. The theory predicts a possible saving in steel area of up to 42 percent. The maximum achieved in the example problem was 41 percent. In reality the saving will vary between zero and 42 percent. Conversely, the conventional method may overdesign a section between zero and 73 percent. The proposed method is "exact" within the existing assumption of the SRSS or the double sum method.

2. Review of Design Procedures of Steel Beam Columns

Figure 1 illustrates a typical beam column subjected to seven design forces and moments including an axial force, P, and two bending moments, M_x, M_y at the top end, bottom end and at an intermediate position, respectively. The axial force in the column results from the dead load of floor members, live load, and due to the overturning effect of wind and seismic forces. The bending moments at two ends result either from eccentric vertical loads or from the restrained end effect of lateral forces. The columns may be single span or sometimes continuous over one or more intermediate supports which can be either horizontal bracings or mezzanine floors. The bending moments, M_{x3}, M_{y3} at the intermediate position as shown in Figure 1 are the maximum values within the column unbraced length. These moments usually result from the application of lateral forces such as wind loads or lateral earthquake loads.

The general considerations of beam column stability and strength have been extensively treated by Massonnet [5] and Austin [6]. Current design procedures of beam columns are well covered by Salmon and Johnson [7]. Among the available design approaches, the elastic analysis working stress design method such as Part 1 of AISC Specification [4] is most commonly used in industry. The initial column sizes are usually selected based on considerations of dead loads and floor live loads only. The effects of wind and earthquake loads determined from the lateral load analysis are investigated in the final design stage. Where columns are used in Category 1 nuclear plant structures the effects of tornado, pipe whip and thermal stress are also considered in the final column design.

The AISC equations for beam columns are of the interaction type. When bending is combined with axial tension, the possibility for column instability is reduced and usually yielding governs the design. The AISC linear interaction equation (1.6-1b) reproduced below should be satisfied for all points along the column length.

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \qquad (1)$$

where f_a, f_{bx}, f_{by} are the computed axial and bending stresses, and F_a, F_{bx}, F_{by} are the corresponding allowable stresses. F_a is taken to be 0.6 F_y (yield stress) for $f_a/F_a \geq 0.15$. The computed bending compressive stress, taken alone should not exceed the permissible values which are determined from the consideration of lateral torsional buckling and local buckling of compression elements of the column section.

In members subjected to combined bending and axial compression, the secondary bending moments caused by P-Δ effect increases the chance of column instability. Besides, the shape of bending moment diagram, if not uniform along the column unbraced length, may overestimate the magnitude of secondary moments. To account for the P-Δ induced secondary moment, the AISC equation (1.6-1a) requires that the calculated bending compressive stress f_b be amplified by a factor of $(1 - f_a/F'_e)^{-1}$, where f_a/F'_e is the ratio of actual axial compression and permissible Euler buckling stress in the plane of bending. Also, a reduction factor C_m is used to handle the moment gradient effect. The AISC nonlinear interaction equation, considering the moment magnification effects must be satisfied for combined stress at all locations.

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - f_a/F'_{ex})F_{bx}} + \frac{C_{my} f_{by}}{(1 - f_a/F'_{ey})\, F_{by}} \leq 1 \qquad (2)$$

where all the variables are as defined previously. The combined stresses at the braced point (or at support) have no moment magnification effect and are checked only by the yielding consideration, i.e., eq.(1). If the maximum moment occurs away from the supports, the stability criterion will usually govern. The yielding criterion becomes the controlling factor when the member is subjected to moments causing double curvature or when the axial load is less than 15 percent of the permissible axial stress.

The values of moment reduction factor C_m depends on the braced condition of frame and presence of transverse loading. For unbraced frames, C_m is taken as 0.85. For a braced frame, C_m is given by

$$C_m = 0.6 \pm 0.4\, M_1/M_2 \geq 0.4 \tag{3a}$$

when transverse loading is not present, and

$$C_m = 1 + \psi(f_a/F'_e) \tag{3b}$$

with transverse loading between supports. The value of ψ depends on transverse loading condition and support condition and can be obtained from AISC Table 1.6.1.2.

3. Design for Three Components of Earthquake

Both interaction equations (eq.(1) and (2)) can be written in the following form

$$R \leq 1 \tag{4}$$

where

$$R = \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \tag{5a}$$

for eq.(1) and

$$R = \frac{f_a}{F_a} + \frac{C_{mx}\, f_{bx}}{(1 - f_a/F'_{ex})\, F_{bx}} + \frac{C_{my}\, F_{by}}{(1 - f_a/f'_{ey})\, F_{by}} \leq 1 \tag{5b}$$

for eq.(2), where R can be called 'Response Factor'. If the axial force on the section is P, and the moments about x and y axes are M_x and M_y, respectively, eq.(5a) and (5b) can be expressed as

$$R = C^1 P + C^2 M_x + C^3 M_y \tag{6}$$

where C^1, C^2 and C^3 are coefficients with appropriate definitions. In many cases C^1, C^2 and C^3 are constants and do not depend upon moment values--this will be the case when design of the section is either governed by eq.(1) or by eq.(2) with transverse loadings. However, when the design is governed by eq.(2) and there is no transverse loading, the value of C_m is given by eq.(3a), rendering C^2 and C^3 functions of moment ratios M_{x1}/M_{x2} and M_{y1}/M_{y2}, where subscripts refer to the two ends of the member under consideration, 'end 1' being under consideration. In the latter case eq.(6) becomes nonlinear. In the following subsections design methods are discussed for both cases, i.e., when eq.(6) is linear or when it is nonlinear. More detailed theory of the method is given by Gupta and Chu [3].

3.1 Linear Interaction Equation

Let us rewrite eq.(6) using index notations

$$R = C^r F^r \qquad (r = 1 \text{ to } 3) \tag{7}$$

where $F^1 = P$, $F^2 = M_x$, $F^3 = M_y$. In eq.(7) and elsewhere in this paper, a repeated sub- or superscript denotes implied summation; (compare eq.(6) and (7)).

For mth mode of vibration, under ith component of excitation

$$R_{im} = C^r F^r_{im} \qquad (8)$$

The effective response factor is given by [3].

$$R_e^2 = \varepsilon_{mn} R_{im} R_{in}$$
$$= C^r C^s \varepsilon_{mn} F^r_{im} F^s_{in}$$

or

$$R_e^2 = C^r C^s G^{rs} \qquad (9)$$

where

$$G^{rs} = \varepsilon_{mn} F^r_{im} F^s_{in} \qquad (10)$$

In the above equations ε_{mn} is the coupling matrix for closely spaced modes, Singh, Chu, and Singh [2]. If all the modes are reasonably apart

$$\varepsilon_{mn} = \delta_{mn}$$

Equation (10) then becomes

$$G^{rs} = F^r_{im} F^s_{im}$$

If other forces and moments denoted by F^r_o are also present, then the corresponding response factor is calculated using eq.(7)

$$R_o = C^r F^r_o$$

The total response factor is given by

$$R = R_o \pm R_e \qquad (11)$$

For safe design the value of R from eq.(11) should satisfy eq.(4).

3.2 Nonlinear Interaction Equation

Equation (7) still defines the response factor R with $F^2 = M_{x1}$, and $F^3 = M_{y1}$. As indicated previously, C^2 is now a function of M_{x1}/M_{x2} and C^3 is a function of M_{y1}/M_{y2}. The method used in the previous subsection cannot be used because in that method it is necessary that C^r,s be constant. C^r's will be constant only when ratios M_{x1}/M_{x2} and M_{y1}/M_{y2} remain constant at all times during an earthquake motion; this is something which is usually not likely to happen.

The alternative is to use the theory presented by Gupta and Chu [3], for evaluating the simultaneous variation in various responses to cause the extreme probable effect. First, the responses of interest are represented in terms of equivalent modes. Since in the present problem, there are three primary responses of interest (P, M_{x1}, M_{y1}), there will be three equivalent modes, F^r_α ($\alpha = 1$ to 3). In each of these equivalent modes, one must know five responses: three primary responses (P, M_{x1}, M_{y1}, or F^r, r = 1 to 3), and two secondary responses (M_{x2}, M_{y2}, or F^r, r = 4,5). These modal responses can be calculated from

$$F^r_\alpha F^s_\alpha = G^{rs} = \varepsilon_{mn} F^r_{im} F^s_{in} \qquad (12)$$

where r = 1 to 3, s = r to 5, $\alpha = 1$ to 3; m and n denote structural modes and i denotes the component of earthquake. Equation (12) actually represents 12 unique equations for 15 unknown values of F^r_α (r = 1 to 5, $\alpha = 1$ to 3). Therefore, those responses in the equivalent modes can be chosen arbitrarily. Table I gives the values of responses in the equivalent modes. Note the three zero elements are the ones chosen arbitrarily.

The values of F^r expected to occur simultaneously to cause the extreme probable effect can now be written in terms of the equivalent modal responses

$$F^r = K_\alpha F^r_\alpha \tag{13}$$

All the possible values of K_α are given by

$$K_\alpha K_\alpha = 1 \tag{14}$$

which is the same as

$$K_1^2 + K_2^2 + K_3^2 = 1$$

If other forces and moments denoted by F^r_o are also present, then eq.(13) is modified as follows

$$F^r = F^r_o + K_\alpha F^r_\alpha \tag{15}$$

With the theory presented above, a design procedure evolves which is outlined below.

1. Using the results of the response spectrum analysis calculate G^{rs} (eq.(10)).
2. Calculate the equivalent modal response using the above value of G^{rs} (Table I).
3. Choose several reasonably close spaced values of K_α's which satisfy eq.(14).
4. For each set of K_α calculate F^r from eq.(15).
5. Calculate C^r's from the calculated value of F^r for the trial section under review.
6. Calculate the response factor R from eq.(7).
7. If none of the values of R exceed one for the assumed trial section, and there are some reasonably close to one, an optimum design has been reached. Otherwise, select a new section and repeat steps 5 and 6.

The above steps are straightforward and can be applied to any analysis-design program as was done for the example problems presented in this paper.

4. Economy by Using the Proposed Method

Two methods have been proposed: one for linear interaction and another one for non-linear interaction. All the equations presented for nonlinear interaction also held good for the linear interaction. For the purpose of estimating the economy, let us assume that a non-linear problem can be treated as a locally linear problem.

The response factor for an equivalent mode is given by eq.(7)

$$R_\alpha = C^r F^r_\alpha, \qquad (r = 1 \text{ to } 3,\ \alpha = 1 \text{ to } 3) \tag{16}$$

The effective response factor is given by [3]

$$\begin{aligned} R_e^2 = R_\alpha R_\alpha &= C^r C^s F^r_\alpha F^s_\alpha \\ &= C^1C^1 F^1_\alpha F^1_\alpha + C^2C^2 F^2_\alpha F^2_\alpha + C^3C^3 F^3_\alpha F^3_\alpha \\ &\quad + 2\, C^1C^2 F^1_\alpha F^2_\alpha + 2\, C^1C^3 F^1_\alpha F^3_\alpha + 2\, C^2C^3 F^2_\alpha F^3_\alpha \end{aligned} \tag{17}$$

However by definition

$$(F^1_e)^2 = F^1_\alpha F^1_\alpha, \quad (F^2_e)^2 = F^2_\alpha F^2_\alpha, \quad (F^3_e)^2 = F^3_\alpha F^3_\alpha \tag{18}$$

It can also be derived from Gupta and Chu [3], that

$$0 \leq 1\ (F^r_\alpha F^s_\alpha)/(F^r_e F^s_e)\ \leq 1, \quad r \neq s \tag{19}$$

From eq.(17) through (19) the minimum possible value of R_e is given by

$$R_M^2 = (C^1F^1_e)^2 + (C^2F^2_e)^2 + (C^3F^3_e)^2 \tag{20}$$

The conventional value of R_e can be derived by using the maximum probable values of F^r's (F^r_e) and substituting them in eq.(7); let us denote this value by R_c, then

$$R_c = C^r\ F^r_e \tag{21}$$

In eq.(20) and (21), substitute

$$C^1F^1_e = \ell\ R_M, \quad C^2F^2_e = m\ R_M, \quad C^3\ F^3_e = n\ R_M \tag{22}$$

which gives

$$\ell^2 + m^2 + n^2 = 1 \tag{23}$$

and

$$R_c = R_M\ (\ell + m + n) \tag{24}$$

The value of R_c will be maximum when

$$\ell = m = n = 1/\sqrt{3} \tag{25}$$

which is given by

$$R_c = \sqrt{3}\ R_M \tag{26}$$

Equation (26) signifies that the maximum value of the response factor, R_c, obtained from the conventional method (eq.(21)) is $\sqrt{3}$ times the minimum possible value of the response factor, R_M, obtained from the proposed procedure. The maximum possible reduction in the response factor is by a factor of $(1 - 1/\sqrt{3})$ or approximately 42 percent, conversely, the conventional method may overestimate the response factor by a factor of $(\sqrt{3}-1)$ or approximately 73 percent. Now, assuming that the section modulii of a section are proportional to area of cross section only, the total required area for a given seismic loading may be considered proportional to the response factor R. Therefore, the proposed method may achieve savings in the area of cross section of up to 42 percent over the conventionally designed area.

The savings indicated above are the maximum possible savings under most favorable circumstances. In general actual savings can be anywhere from zero to 42 percent calculated above. This particular aspect is illustrated in the example problems illustrated in Section 5.

5. Examples of Applications of the Proposed Method

To keep variables within comprehension, for the example problems it is assumed that all the modal responses can be represented in terms of the maximum probable values of the force and moments at the section under design, viz., P_e, M_{xle}, M_{yle}, and in terms of two variables, α and β. The equivalent modal responses for this case are given in Table II.

It is assumed that there is no coupling between the axial force and the bending moments, therefore, any mode of vibration which causes axial force in this beam column member will not cause any bending moments, and vice versa. The factor α represents the coupling between the M_{x1} and M_{y1} moments. When α is zero, M_{x1} and M_{y1} moments can be considered to be completely independent, and when it is one, the two moments will vary proportionately--their maximum values will occur simultaneously. There is no assumption involved in the introduction of the factor α. As far as M_{x2} and M_{y2} are concerned, it is assumed here that they vary proportionately with M_{x1} and M_{y1}, the coefficient of proportionality being β.

Let us consider a column which is 20 ft high in a braced frame and the effective length factor, $K=1$. A W14 section has to be designed for the following maximum probable values of force and moments at the critical end section.

P_e = 750 kips, M_{x1e} = 4800 kips-inch, M_{y1e} = 2160 kips-inch

Appropriate values of α and β are assumed in the various cases solved. Two yield strengths are considered, F_y = 36 or 50 ksi. Design for all the column sections was done using the method described in Section 2.2. This method was implemented on a design computer program, Chu [8], which finally gave the required column section for each case.

5.1 Effect of the Factor α

The difference between the conventional method and the proposed method will be maximum when $\alpha=0$ which means that there is no coupling between the two moments and the conventional method cannot recognize it. Three designs are performed here, one by the conventional method, one for $\alpha=0.1$ and one for $\alpha=0.5$. The factor β is taken to be 1.0 which gives $C_m=1.0$. The yield stress is 36 ksi. The results of the three designs are given in Table III.

As expected, the proposed method achieves a saving of 38 percent over the conventional method when $\alpha=0.1$. This saving goes down to 32 percent when $\alpha=0.5$. The effect of α is also studied simultaneously with other parameters in the following subsections.

5.2 Effect of Factor β

The same problem which was solved in Section 5.1 is solved here with $\beta=0.75$. The corresponding C_m is 0.9. Design was again performed by conventional method and by the proposed method for $\alpha=0.1$ and 0.5. The results are given in Table IV. Because of the lower moment reduction factor (C_m), all the sections for $\beta=0.75$ are lighter than the corresponding sections for $\beta=1.0$, except when $\alpha=0.5$. The latter is because a standard section of appropriate area for $\beta=0.75$, $\alpha=0.5$ was not available. Again, for $\alpha=0.1$ the proposed method gives a section with 38 percent less area than that required by the conventional method.

5.3 Effect of Yield Stress

Let us design a section for the same forces and moments as in Section 5.1 but with a new value of yield stress, F_y = 50 ksi. The design sections obtained by the conventional method and by the proposed method for $\alpha=0.1$ and 0.5 are given in Table V. Because of the favorable standard section availability, the savings by using the proposed method are 41 percent for $\alpha=0.1$ which is very close to the theoretically maximum possible savings predicted in Section 3.3. Savings for $\alpha=0.5$ has also gone up from Table III which was for F_y = 36 ksi. As indicated above, these increased savings are attributed more to the availability of the correct section than to the effect of increased yield stress.

5.4 Effect of Nonseismic Loads

Let us assume that the maximum probable values of the total force and moments remain the same as in Section 5.1. However, half of it now consists of forces and moments due to other nonseismic loads. The maximum nonseismic forces and moments are assumed to occur simultaneously, whereas the maximum probable seismic forces and moments in general do not. Again the design is performed by the conventional method and by the proposed method for $\alpha=0.1$ and 0.5. The new design sections are reported in Table VI. The conventional method gives the same section as when there was no nonseismic load (Subsection 5.1, Table III).

However, it can be observed that the presence of nonseismic loads reduces the savings by the proposed method. As observed before, here again the lower value of α gives higher savings.

6. Summary and Conclusions

A new rational method for designing beam columns in steel frames subjected to three components of earthquake is presented. This method is based on the theory presented by Gupta and Chu [3]. Depending upon the state of loading, etc., the interaction equation recommended by the AISC Code is either linear or nonlinear. Methods have been developed and presented for both the cases. Theoretically, it is shown that the proposed methods can achieve savings of up to 42 percent in the steel cross-sectional area over the present conventional method, or conversely, the present designs may be up to 73 percent conservative with respect to the proposed method.

The above conclusions have been checked against actual design problems. When the coupling between the two moments (M_x and M_y) is zero or small, the proposed method achieves maximum savings - up to 41 percent in the actual problem. Due to nonlinearity, and primarily due to the nonavailability of sections of exact dimensions predicted by the analysis, the maximum 42 percent savings predicted by the theory may not be achieved in the example problems. It is noted however that 42 percent is the maximum possible savings when all the conditions in the theory are satisfied. In reality, many of these conditions will not be satisfied and savings will be between zero and 42 percent in those cases.

The presence of nonseismic loads also reduces the maximum probable savings by the proposed method. The maximum possible savings will be zero when all the forces and moments are due to nonseismic loads, and will be 42 percent when all the forces and moments are due to seismic load; for the intermediate mix the savings would be between zero and 42 percent

The proposed method is straightforward and can be easily implemented with the help of a computer program as was actually done for the example problems.

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TABLE I. EQUIVALENT MODAL RESPONSES

Response r	F^r \ Equivalent Mode α	1	2	3
1	P	$\sqrt{G^{11}}$	0	0
2	M_{x1}	G^{12}/F_1^1	$\sqrt{G^{22} - (F_1^2)^2}$	0
3	M_{y1}	G^{13}/F_1^1	$\frac{G^{23} - F_1^2 F_1^3}{F_2^2}$	$\sqrt{G^{33} - (F_1^3)^2 - (F_2^3)^2}$
4	M_{x2}	G^{14}/F_1^1	$\frac{G^{24} - F_1^2 F_1^4}{F_2^2}$	$\frac{G^{34} - F_1^3 F_1^4 - F_2^3 F_2^4}{F_3^3}$
5	M_{y2}	G^{15}/F_1^1	$\frac{G^{25} - F_1^2 F_1^5}{F_2^2}$	$\frac{G^{35} - F_1^3 F_1^5 - F_2^3 F_2^5}{F_3^3}$

TABLE II. EQUIVALENT MODAL RESPONSES - SPECIAL CASE

Response r	F^r \ Equivalent Mode α	1	2	3
1	P	P_e	0	0
2	M_{x1}	0	M_{x1e}	0
3	M_{y1}	0	αM_{y1e}	$(1-\alpha^2)^{1/2} M_{y1e}$
4	M_{x2}	0	βM_{x1e}	0
5	M_{y2}	0	$\alpha\beta M_{y1e}$	$(1-\alpha^2)^{1/2} \beta M_{y1e}$

TABLE III. COMPARISON OF DESIGN SECTIONS
$\beta = 1.0$, $C_m = 1.0$, $F_y = 36$ ksi

Design Case	Section	Area of Cross Section (sq inch)	Savings by Using the Proposed Method
Conventional	W14x426	125.0	-
$\alpha = 0.1$	W14x264	77.6	38
$\alpha = 0.5$	W14x287	84.4	32

TABLE IV. COMPARISON OF DESIGN SECTIONS
$\beta = 0.75$, $C_m = 0.9$, $F_y = 36$ ksi

Design Case	Section	Area of Cross Section (sq inch)	Savings by Using the Proposed Method
Conventional	W14x398	117.0	-
$\alpha = 0.1$	W14x246	72.3	38
$\alpha = 0.5$	W14x287	84.4	28

TABLE V. COMPARISON OF DESIGN SECTIONS
$\beta \doteq 1.0$, $C_m = 1.0$, $F_y = 50$ ksi

Design Case	Section	Area of Cross Section (sq inch)	Savings by Using the Proposed Method
Conventional	W14x342	101.0	-
$\alpha = 0.1$	W14x202	59.4	41
$\alpha = 0.5$	W14x219	64.4	36

TABLE VI. COMPARISON OF DESIGN SECTIONS WITH 50 PERCENT NONSEISMIC LOAD
$\beta = 1.0$, $C_m = 1.0$, $F_y = 36$ ksi

Design Case	Section	Area of Cross Section (sq inch)	Savings by Using the Proposed Method
Conventional	W14x426	125.0	-
$\alpha = 0.1$	W14x342	101.0	19
$\alpha = 0.5$	W14x370	109.0	13

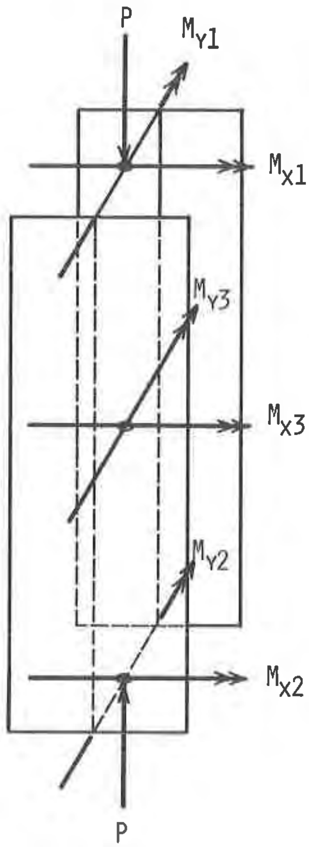

FIGURE 1 A TYPICAL BEAM COLUMN ELEMENT