A CONSTITUTIVE THEORY FOR METAL CREEP

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SUMMARY

The objective of this research has been to establish a constitutive theory for the elevated temperature behavior of metals capable of predicting the inelastic strain response for an arbitrary stress history. To determine the phenomenological description of a specific material, a series of load rate, strain rate, and creep tests were performed on wrought Udimet 700 at 1700°F. The analysis of the data showed the quasi-static creep strain rate response was bounded from above and below. The upper bound corresponded to the locus of maximum primary creep rates, and was observed in the higher of the quasi-static load tests. The lower bound was the minimum creep rate, and was observed in the creep tests, strain rate tests, and at the low load rate tests. Since the maximum primary creep rate was the same for high loading rates and the minimum creep rate was a function of stress, the creep curve was found to be time-translation invariant in these cases. The concept of transition from one creep rate to another was shown to depend on a material stress rate coordinate which was derived from the concept of a thermally activated process time which was stress dependent.

An examination of the theories of strain and time hardening and the models of Stouffer, Valanis, and Besseling showed that each contained certain specific assumptions which prohibit them from predicting the observed load response. However, since the concept of time-translation invariance is imbedded in the single integral models of viscoelasticity, a similar approach was taken to develop the proposed constitutive theory. Based on a phenomenological analysis, a single integral model was developed which included the concept of a maximum primary creep rate, and introduced a variable called the viscoplasticity parameter. This parameter was related to the load history through the concept of a material stress rate coordinate and creep strain continuity to model the anelastic effects.

The predictions of the proposed theory, strain and time hardening, and Besseling were compared to the response data from a series of cyclic creep and relaxation experiments. The results showed that, in the majority of cases, the new theory was better than the other theories. However, based on these correlations, it was also shown that the concept of a maximum primary creep rate must include a representation for static and dynamic recovery. This concept was shown to be consistent with the findings of others and should be included in future work using the proposed constitutive model.

The experimental evidence of this paper has demonstrated the shortcomings of the classical approaches to metal creep. Based on a phenomenological approach, a new single integral constitutive theory was developed which is consistent with the observed experimental results. This theory is capable of predicting most phenomenological aspects of metal creep.

1. Introduction

The development of accurate constitutive equations to describe the deformation of materials is a fundamental concern for the design and analysis of the structural components of nuclear reactors. Without the implementation of a viable description of material response behavior in structural analysis computer codes, it is difficult to access material damage, and, hence, define the structural lifetime. Despite the fundamental advances of realistic constitutive theories, the development of elevated temperature theories remains an area of active research. The main reason for this is that at elevated temperatures the deformation of metals is time dependent. The time independent formulations (such as the plasticity formulations) form, at most, only a part of the required constitutive relationships. The variety of phenomenologically observed behavior (creep, strain rate sensitivity, recovery, environmental attack, and aging) and microscopic mechanisms (slip, climb, vacancy diffusion, cross-slip, grain boundary sliding, etc.) make constitutive modeling extremely difficult.

Despite the abundance of experimental data showing the various time-dependent effects on many materials, no single experimental program has thoroughly investigated the time-dependent response of one material to various types of loading histories [1]. Thus, the purpose of this research was to identify the inelastic response of a single alloy, wrought Udimet 700 (U700W), at a single temperature 1700°F, to a specific set of representative loading programs. The resulting response data is then analyzed so as to identify the cornerstones of a foundation for a constitutive theory. A theory is then developed and the validity is checked by predicting the response of U700W to a series of loading programs distinctly different than the programs used for material characterization. In addition, the predictions of several of the classical approaches are compared to these experiments..

2. Review of the Traditional Approaches

The theory of strain and time hardening is formulated from the creep strain response, ε^c, to a step stress history; ie,

$$\varepsilon^c = f(\sigma,t) \tag{1}$$

where σ is the magnitude of the stress and t is time. Both methods can be characterized by defining the creep rate, $\dot{\varepsilon}^c$, as

$$\dot{\varepsilon}^c = \frac{\partial}{\partial\tau} f(\sigma,\tau)\Big|_{\tau = \xi} . \tag{2}$$

If $\xi = t$, the current time, equation (2) represents the time hardening rule. Since Eq. (1) is single valued, the function $f(\sigma,t)$ can be solved for the time $t_e = f^{-1}(\sigma,\varepsilon^c)$ to obtain a given strain. Hence the strain hardening rule is defined by setting $\xi = t_e$ in equation (2). These approaches have been found to have moderate success for certain materials and loading profiles [2,3,4,5] provided the material is largely time independent [6].

Another useful approach can be found in the theories of viscoelasticity [7,8] where the representation is derived directly from the creep function of Eq. (1). However, implicit in these formulations is the assumption that all primary creep is anelastically recovered, which usually is not true for metals at elevated temperature.

The Endochronic theory proposed by Valanis [9] is based on the concept of a "material clock" that monotonically advances with strain and time for viscoplastic materials. Using this assumption, it can be shown [6] that the theory predicts complete stress relaxation for a constant strain history. A result which is not true for metals.

The sub-volume theory of Besseling [10], has been used successfully for plastic flow. However, for viscoplastic materials difficulty arrises in determining the material functions for each of the sub-volumes [6]. Once this has been accomplished the Besseling theory is capable of predicting anelastic recovery and stress induced anisotropy.

3. The Basic Experimental Program

The experimental program consisted of two parts. Part one, described in this section, consisted of determining the basic material characteristics. The constitutive theory was developed using this data. Part two consisted to running several arbitrary histories, much different than those of Part one, to test the predictive capability of the constitutive theory.

Preliminary tests were conducted to determine that the material was very nearly isotropic and homogeneous. Also the elastic modulus and elastic Possion ratio were determined for the test temperature of 1700°F. It was also found that the plastic deformations were isochoric, thus requiring the plastic Possion ratio to be 0.5.

Creep tests were conducted at engineering stress levels of 50, 60, 70, and 80 ksi. In almost all cases the initial stress rate was controlled at 120 ksi/min. The results were typical of most metals showing three stages of creep. Of importance, was the observation that the initial primary creep rate was finite. This fact is often overlooked, and is not accounted for in several analytical creep expressions.

In addition to the creep tests, further tests were conducted to determine the effect of initial loading rate on the creep curve. Test samples were loaded to 60 ksi at an initial loading rate of 12 ksi/min. and 35 ksi/min. The observed response is shown in Figure 1. The translated axes are defined by subtracting the initial creep strain, ε_o^c, from the instantaneous creep strain and initial time, t_o, from the running time. Though significant scatter in the data exists, it appears that these data would describe one creep curve upon correction for the initial loading. This has immense consequences on constitutive equations since the creep response appears to be time-translation invariant. Hence the assumptions of strain or time hardening are questionable for this material and a viscoelasticity approach appears consistent.

Several monotonically increasing diametral strain tests were conducted over the range of strain rates $-2.5\%/\text{min.} \leq \dot{\varepsilon}_d \leq -.01\%/\text{min.}$ A large strain rate sensitivity is present as shown in Figure 2. Each test appears to define one true stress level (engineering stress drops off in this range). A comparison of the true creep strain rates (for Figure 2) and the minimum creep rate from the creep experiments showed that these data defined the same stress-creep rate relationship and establish the minimum creep rate of the material.

Constant engineering stress rate tests were conducted to failure. These experiments covered the range of 1 ksi/min. to 120 ksi/min. Similar to the strain rate tests described in the previous section, a strong rate effect was observed on the stress-strain curve. The data of the mechanical response from these experiments displayed in Figure 3, is given in the form of true creep strain rate versus true stress. Two curves are plotted on Figure 3, one being the minimum creep rate curve, $\dot{\varepsilon}^c_{min}(\sigma)$. The other is the primary (maximum) creep rate, $\dot{\varepsilon}^c_{max}(\sigma)$, which coincides with the creep rate of the faster stress rate tests. Further, at the higher stress levels, the tendency is for the creep rate response to transfer from the maximum creep rate to the minimum creep rate. At all stress levels, the low stress rate data appear to be on the minimum creep curve as shown in Figure 3. These observations demonstrate two important factors: (1) the maximum and minimum creep rates define a characteristic time-dependent mechanical response, (2) that stress rate is a necessary independent variable.

The results from these experiments can be collectively used to determine the constants in the Marin-Pao equation [11],

$$\varepsilon^c(\sigma,t) = A(\sigma)[1.0 - e^{-\beta(\sigma)t}] + \dot{\varepsilon}^c_{min}(\sigma)t \qquad (3)$$

where $\varepsilon^c(\sigma,t)$ represents the uniaxial creep response to a step stress history of magnitude σ. As shown in [6,12] the time constant, $\beta(\sigma)$, can be determined from creep tests. The function $A(\sigma)$, is found from equation (3) by setting the creep rate equal to the primary (maximum) creep rate at $t = 0$; thus

$$A(\sigma) = \frac{\dot{\varepsilon}^c_{max} - \dot{\varepsilon}^c_{min}}{\beta} . \qquad (4)$$

Thus using a representation for $\dot{\varepsilon}^c_{min}$ and $\dot{\varepsilon}^c_{max}$ as shown in Figure 3, the material functions $A(\sigma)$ and $\beta(\sigma)$ can be determined.

4. The Development of a Constitutive Theory

As shown in Section 2 most of the conventional methods of predicting creep (strain hardening, time hardening, Besseling's theory, and simgle integral viscoelasticity) contain predetermined assumptions on how a material will respond to load. As such, each approach is capable of modeling certain specific aspects of metal creep, but are incapable of modeling all creep effects for various types of loadings. This and other considerations suggest the following approach for developing a successful viscoplasticity constitutive law:

(1) The formulation should, at least in part, be developed directly from some important experimentally determined function. This is to avoid material functions with no physical meaning.

(2) Since elevated temperature material response is highly rate (time) dependent, a constitutive formulation similar to viscoelasticity is appropriate; however, the approach must be modified to predict the correct anelastic recovery properties for metals.

(3) Establish and experimentally verify a one dimensional constitutive theory. Then, if the material is isotropic, homogeneous, and isochoric a three dimensional model can be theoretically developed with a minimum of one scalar material function. (It is unlikely that such a model could predict all of the material memory effects that would be observed in (multiaxial) testing. However, once developed, such a model would help identify which material memory effects need additional representation.)

To begin, assume that the response characteristics described by the constant load creep test, Eqs. (1) or (3), must be contained in any general constitutive representation for a time varying stress history. Further, thermodynamic coordinates q_α, can be introduced into the formulation to account for the history and memory effects. Furthermore, it can be assumed that the time argument should be replaced by a more general "material clock" measure, ζ. Valanis [9] has proposed a clock measure which is related to the arc length of the ten-dimensional space defined on the components of the strain tensor, $\underset{\sim}{\varepsilon}$, and time, t. In Reference [6], a complete development was given which included the q_α and ζ. However it was found that it was not necessary to include these terms in order to represent the material response examined herein. Therefore, the simpler representation is outlined below.

The representation given in Eq. (1) or (3) can be extended to include transient stress histories, $\sigma(\tau)$ for $\tau\varepsilon(-\infty,\infty)$. This extension rests on the <u>assumption</u>:

The amount of creep that occurs in some infinitesimal increment of time $[\tau,\tau+\Delta\tau]$ depends only on the mean value of stress and temperature present during that increment of time.

This assumption allows a representation for transient stress histories to be established by partitioning $\sigma(t)$ for $t\varepsilon[t_o,t]$ into N subintervals, evaluating the response in each interval, and integrating to obtain the total response. Let $\tau_i (i = 1,2,...N)$ be the time at the beginning of the (i^{th}) time interval and let σ_i be the average values of stress during the (i^{th}) time interval.

The increment of strain at any time τ due to a stress pulse σ_i during the time interval $[\tau_i, \tau_i+\Delta\tau_i]$ is assumed to given by the Eq. (1) or (3) applied at time τ_i and subtracted at time τ^*, where

$$\tau^* = \tau_i + \alpha\Delta\tau_i \qquad (5)$$

for $0 \leq \alpha \leq 1$. The variable α is a material function that allows for varying amounts of anelastic recovery to be included in the model. In general, α will be a path and time dependent variable. Proceeding to construct an integral using a method similar to linear viscoelasticity, [6], gives

$$\varepsilon(t) = \int_o^t \{\alpha \frac{\partial}{\partial t} \varepsilon^c[\sigma(\tau),t-\tau] + (1-\alpha)\frac{\partial}{\partial \xi} \varepsilon^c[\sigma(\tau),\xi]\Big|_{\xi=o} \} \, d\tau \; . \qquad (6)$$

The total strain, $\varepsilon^T(t)$, is then given by

$$\varepsilon^T(t) = \frac{\sigma(t)}{E} + \varepsilon(t) \qquad (7)$$

where E is the elastic modulus.

Let us consider the effect of the material parameter α on the range of values $0 \leq \alpha \leq 1$. If $\alpha = 1$, equations (6) and (7) corresponds to the viscoelasticity theory of Stouffer, [7], where all primary creep is anelastic and therefore recoverable. Also, for a constant stress history, $\alpha=1$ yields the creep equation (1) or (2). If $\alpha = 0$, then eq. (6) becomes

$$\varepsilon\ (t) = \int_0^t \left. \frac{\partial \varepsilon^c[\sigma(\tau),\xi]}{\partial \xi} \right|_{\xi=0} d\tau \,. \qquad (8)$$

In this case the model predicts that the creep is permanent for all time (ie, non-recoverable) as in plasticity. The result for $\alpha = 0$ has another physical interpretation. Substituting Eq. (3) and (4) into (5) gives

$$\varepsilon(t) = \int_0^t \dot{\varepsilon}^c_{max}\,[\sigma(\tau)]d\tau, \qquad (9)$$

which corresponds to the response to a high stress rate loading as shown in Figure 3. Thus α is a viscoplasticity parameter tha controls the relative contribution the viscoelastic and plastic components to the inelastic strain.

Thus, as shown through the previous discussion, Eq. (6) is sufficiently general to model the spectrum of deformation response features characteristic of materials at elevated temperatures. However, during an arbitrary stress history, a method is needed which will translate the current stress condition into a value of α. For example, if the stress is constant, then α should be unity in order to predict the creep curve response. Conversely, if the material is experiencing a rapid change in stress, then the response should follow the maximum creep rate curve (as shown by the experimental data in Section 3) and should approach zero. However, if anelastic recovery occurs in the material during a rapid stress transient, then a method of accounting for this type of response is needed and α cannot be exactly zero. These considerations are used to develop a representation for α. It is shown in [6], that α is a functional of the history of the stress rate and that α must also satisfy certain continuity requirements.

5. Evaluation of the Constitutive Equation

To evaluate the constitutive theory, a total of twelve cyclic creep and relaxation experiments were conducted. These experiments demonstrate the potential advantages and shortcomings of the proposed theory. Due to space limitations only the most pertinent test results will be reviewed. All experiments were uniaxial tension cycling only.

Figure 4 shows the results of an interrupted creep test. After 10 minutes at 50 ksi, the stress is cycled to 70 ksi and back to 50 ksi. The results demonstrate that after several cycles a distinct difference emerges between the other theories and the data. The proposed theory matches the data quite well.

Figure 5, demonstrates the results for a creep test with two distinct creep stress levels, 50 and 70 ksi. The results demonstrate the value of the proposed theory for this type of cycling. Figure 6 shows the results for continuously varying stress between 70 and 50 ksi. In this case the results from Eq. (6) overpredict the experimental results to a greater extent than the other theories. However, the darkened symbols show the results of the proposed theory when it is modified for this type of cycle [6]. Further extensions to the theory are found in [13] which give this result without recourse to specific consideration of the cycle type.

6. Conclusions

From the results of this paper several conclusions can be drawn. The most important are:

(1) The experimental results indicate that a general theory of metal creep is possible without regard to loading profile.

(2) The fact that the initial primary creep rate is finite has several ramifications on creep curve expressions, and determines the creep response during transient loadings.

(3) A new constitutive theory has been formulated, and can reproduce many of the commonly observed response features of metal creep.

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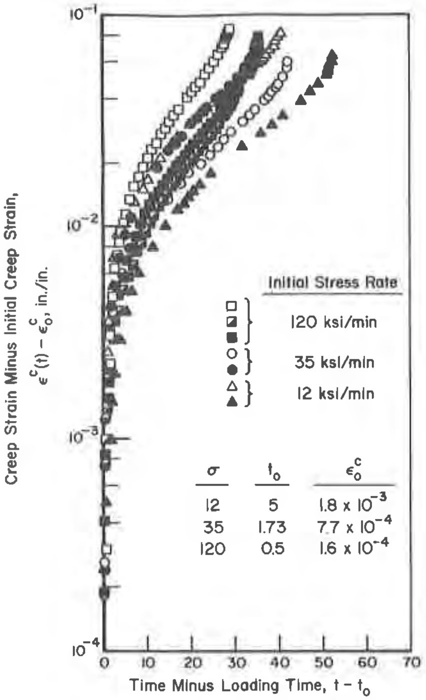

Figure 1. Time translated creep response of U700W at 1700°C.

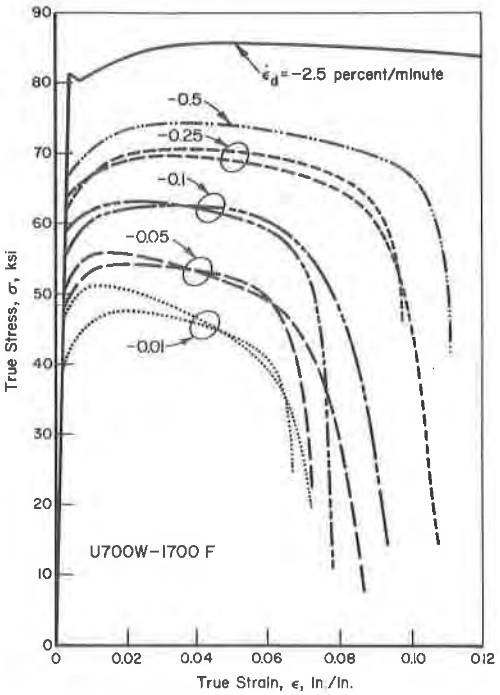

Figure 2. True stress vs. true strain of U700W at 1700°F for different constant diametial strain rate histories.

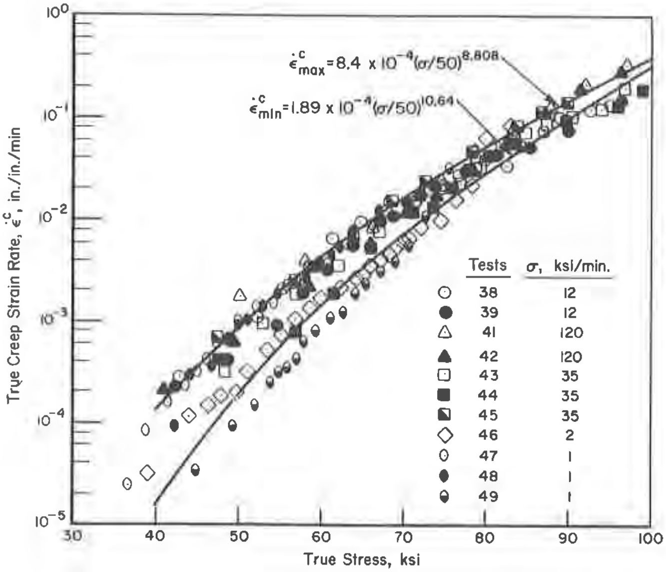

Figure 3. Strain rate response to different constant stress rate histories for U700W at 1700°F.

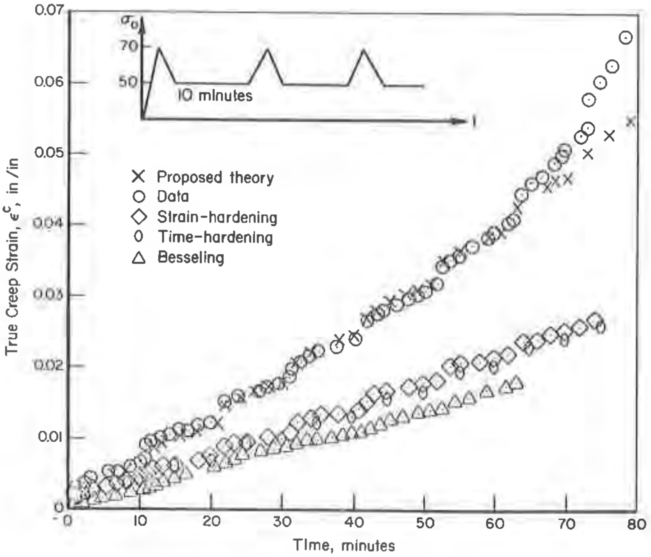

Figure 4. Comparison of preditions to experimental data for a periodically interrupted creep test of U700W at 1700°F.

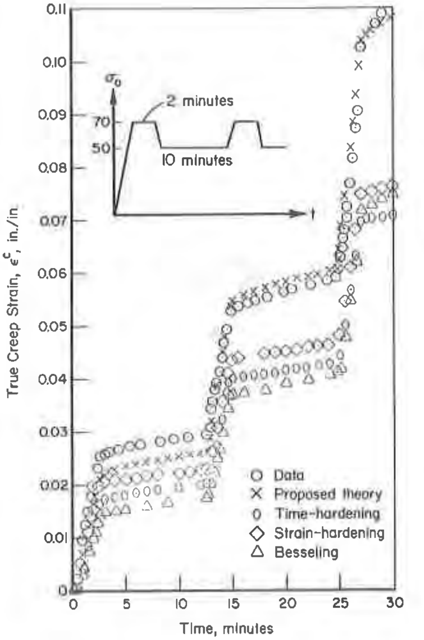

Figure 5. Comparison of predictions to experimental data for a cyclic stress history with dwell periods for U700W at 1700°F.

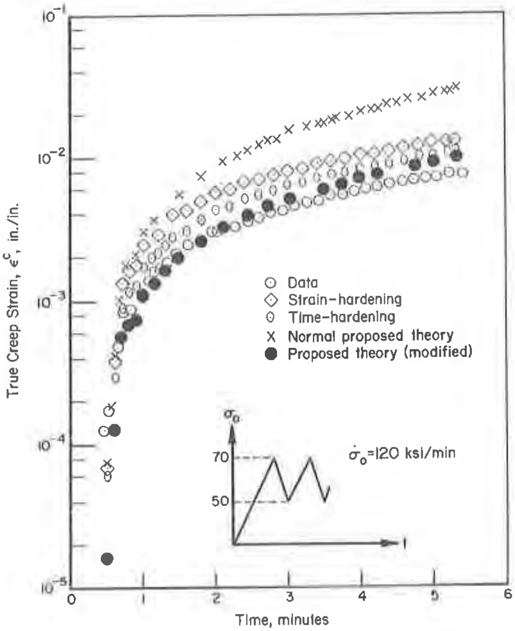

Figure 6. Comparison of prediction and experimental data for the first 15 cycles of a cyclic stress history for U700W at 1700°F.