COMPUTATIONAL TECHNIQUES FOR INELASTIC ANALYSIS AND NUMERICAL EXPERIMENTS

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SUMMARY

A number of formulations have been proposed for inelastic analysis, particularly for the thermal elastic-plastic creep analysis of nuclear reactor components. In the elastic-plastic regime, which principally concerns with the time independent behavior, the numerical techniques based on the finite element method have been well exploited and computations have become a routine work. However, the establishment of the constitutive relations is still an important task. Therefore, a computer program should be designed and developed with sufficient generality so that it allows for the novel constitutive equations which may be proposed from the side of material scientists interested in the micro-mechanisms of plastic deformation.

With respect to the problems in which the time dependent behavior is significant, it is desirable to incorporate a procedure which is workable on the mechanical model formulation as well as the method of equation of state proposed so far. A computer program should also take into account the strain-dependent and/or time-dependent micro-structural changes which often occur during the operation of structural components at the increasingly high temperature for a long period of time.

Special considerations are crucial if the analysis is to be extended to large strain regime where geometric nonlinearities predominate. First of all, the proper choice of stress and/or stress-rate in constitutive equations is mandatory, and the computer program should enable the solution of linear algebraic equations which may include the nonsymmetric geometric stiffness matrices in some occasions. Incorporation of an eigen solution routine augments the applicability of the program to the problems including instability and/or bifurcation phenomena.

The present paper introduces a rational updated formulation and a computer program under development by taking account of the various requisites stated above. Numerical examples include 1) thermal elastic-plastic creep deformation of a thick walled cylinder, 2) numerical experiments on the tensile instability of uniaxial test specimens and 3) typical examples from the activities of JSME (Japan Society of Mechanical Engineering) Committee on Research and Development of Inelastic Analysis Programs, the second phase of the project being sponsored by PNC (Power Reactor and Nuclear Fuel Development Corporation), Japan.

1. INTRODUCTION

The success of the finite element method in linear continuum mechanics and the stringent requirement for safety of nuclear reactor components operating at increasingly higher temperature have been stimulating the development and/or use of the nonlinear analysis computer programs. A number of formulations have been proposed and incorporated in the general purpose programs.

There are two significant nonlinear behaviors to be considered in performing the rational design of critical components, material and geometric nonlinearities. Material nonlinearities concern with plasticity as well as time dependent phenomena such as creep and relaxation, and plasticity was the first which was exploited in the history of nonlinear finite element analysis. Time dependent behavior emerges into the design of reactor system components particularly when the considerations on the limits on deformation-controlled quantities are crucial. It is well known that there has been a dispute of long standing on the appropriateness between 'equation of state' and 'memory theory' formulations for creep analysis. Earlier computer programs for 'plastic analysis' and 'thermal stress and creep' were reviewed by Armen [1] and Nickell [2] respectively. 'Time dependent materials' were the topics of a review article of Yamada [3] as well. Reference should also be made here to the heuristic and comprehensive works by Argyris, Balmer et al. [4,5], Zudans et al. [6,7] and Rashid [8,9] on elastic-plastic and creep analyses.

Geometric nonlinearity is important factor when the possibility of buckling and structural instability should be taken into account in the design. It is interpreted as the effect of nonzero 'initial' stress existing at each incremental stage of deformation on the local or overall equilibrium of the structure. Theoretically the buckling and instability can take place at infinitesimally small strain in perfect systems, i.e. without imperfections, but it is usual in practical situations that the prebuckling finite strain or deformation affects the occurance of these phenomena. In case of thermal reactor, deformations due to creep may enhance the possibility of instability, thus giving rise to the creep buckling. A through overview of the solution procedure for the static problem with geometric nonlinearities was given by Yamada [10]. Further, Bathe et al.[11] and Stricklin and Haisler [12] extended the scope to dynamic analysis.

The major objective of the present paper is to introduce a procedure for nonlinear analysis which the author and his colleagues have been pursuing for several years. It adopts the updated incremental formulation and features a unified compact way of processing the combined effects of material and geometric nonlinearities. Two pilot programs EPIC(Elastic-PlastIC Analysis Program) and MAGNAP(Material And Geometric Nonlinear Analysis Program) have been developed for the verification of formulation as well as the elaboration of various subroutines in connection with the nonlinear analysis. The process is now going on for incorporating of these into the general purpose program COMPOSITE (COMputer Program Overture to SImulation of CompositTE). Typical numerical examples obtained by these programs and comparison of solutions are given in the second part of the paper.

It is worth noting here that verification and qualification of programs by benchmark calcuations are essential in the area of inelastic analysis. In parallel to the computer programs verification activities in the U.S. [13] and as a continuation of the previous report [14] on our efforts, the present paper summarizes the recent status in Japan of cooperative work for establishment of program verification reference by way of the cross-checking of computer solutions.

2. MATERIAL NONLINEARITIES AND CONSTITUTIVE EQUATION

To describe the plastic and viscoelastic behaviors, we adopt the mechanical model of generalized Voigt (or Kelvin) type shown in Fig.1. The constitutive equation relevant to this model can be written as

$$\{\dot{\sigma}\} = [D_V^p]\{\dot{\varepsilon}\} - [D_V^p]\{\dot{\varepsilon}^\theta\} - [D_V^c]\{\dot{A}\} - \{\dot{\sigma}^a\} \tag{1}$$

where

$$\{D_V^p\} = [D_g^e] - \frac{[D_g^e]\{\partial g/\partial\sigma\}\lfloor\partial f/\partial\sigma\rfloor[D_g^e]}{H_V'} = [D_V^c][D_g^e] \tag{2}$$

$$[D_V^c] = [I] - \frac{[D_g^e]\{\partial g/\partial\sigma\}\lfloor\partial f/\partial\sigma\rfloor}{H_V'} \tag{3}$$

$$\{\dot{\sigma}^a\} = -\frac{[D_g^e]\{\partial g/\partial\sigma\}(\partial\bar{\sigma}/\partial T)\dot{T}}{H_V'} \tag{4}$$

with

$$H_V' = (H'+H_k')/c + \lfloor\partial f/\partial\sigma\rfloor[D_g^e]\{\partial g/\partial\sigma\} \tag{5}$$

$$\{\dot{A}\} = [D_g^e]([\eta_g]^{-1} + \Sigma[\eta_i]^{-1})\{\sigma\} - [D_g^e]\Sigma[\eta_i]^{-1}[D_i^e]\{\varepsilon_i\} \tag{6}$$

$$c = \bar{\sigma}/\lfloor\sigma\rfloor\{\partial g/\partial\sigma\} \tag{7}$$

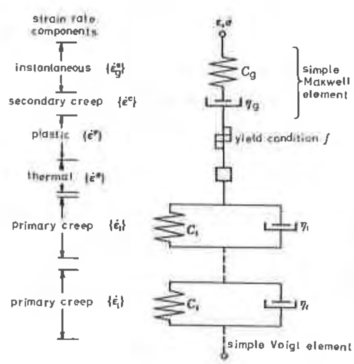

Fig.1 The Generalized Voigt (Kelvin) Model

Although the derivation of above equations is detailed in ref.[10], we assume here the plastic potential $g(\sigma_{ij})$ which is not necessarily coincident with the yield criterion $f(\sigma_{ij})$. Thus the plastic strain rate $\{\dot{\varepsilon}^p\}$ is given by

$$\{\dot{\varepsilon}^p\} = \dot{\lambda}\{\partial g/\partial \sigma\} \tag{8}$$

where

$$\dot{\lambda} = c\dot{\bar{\varepsilon}}^p = \frac{\lfloor \partial f/\partial \sigma \rfloor ([D_g^e]\{\dot{\varepsilon}-\dot{\varepsilon}^\theta\}-\{\dot{A}\})-\frac{\partial \bar{\sigma}}{\partial T}\dot{T}}{H_V'} \tag{9}$$

In order to check whether the plastic element keeps on the yielded state or not, the positiveness of $\dot{\lambda}$ or the equivalent plastic strain rate $\dot{\bar{\varepsilon}}^p$ is used in our programs.

H' and H_k' in eq.(5) denote the isotropic and kinematic hardening rate respectively, so that the constitutive equation (1) copes with the combined hardening effects. Softening effect is also incorporated through the apparent stress rate vector $\{\dot{\sigma}^a\}$ of eq.(4) in which $\partial\bar{\sigma}/\partial T$ represents the dependency of the equivalent stress $\bar{\sigma}$ on temperature T.

Time dependent behaviors are all contained in $\{\dot{A}\}$ of eq.(6). If no allowance is made for the simple Voigt elements in the model of Fig.1, eq.(6) degenerates to

$$\{\dot{A}\} = [D_g^e][\eta_g]^{-1}\{\sigma\}$$

The creep strain rate $\{\dot{\varepsilon}^c\}$ in this case is expressed by

$$\{\dot{\varepsilon}^c\} = [C_g^e]\{\dot{A}\} = [\eta_g]^{-1}\{\sigma\} \tag{10}$$

It is worth noting that the inverse $[\eta_g]^{-1}$ of viscosity matrix for the isotropic material with infinite volumetric viscocity $\eta_K=\infty$ is given by

$$[\eta_g]^{-1} = \frac{1}{6\eta_{Gg}}\begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix} \tag{11}$$

Then eq.(10) can be written as

$$\{\dot{\varepsilon}^c\} = \frac{1}{2\eta_{Gg}}\{\sigma'\} \tag{12}$$

where $\{\sigma'\}$ denotes the deviatoric stress.

The creep strain rate $\{\dot{\varepsilon}^c\}$ in 'equation of state' formulation is often expressed following way

$$\{\dot{\varepsilon}^c\} = \dot{\lambda}_c\{\sigma'\}, \quad \dot{\lambda}_c = 3\dot{\bar{\varepsilon}}^c/(2\bar{\sigma}) \tag{13}$$

Comparing eqs.(12) and (13), we can see that the 'mechanical model' formulation includes the 'equation of state' solution procedure as a special case by setting the coefficient of shearing viscosity η_{Gg} as

$$\eta_{Gg} = \frac{1}{2\dot{\lambda}_c} = \frac{\bar{\sigma}}{3\dot{\bar{\varepsilon}}^c} \tag{14}$$

By designing the computer program so that the choice of the yield criterion f and the plastic potential g are left to users, we can make the program more versatile. Program COMPOSITE-III adopts this policy and sanctions user's subroutine in which the functions f and/or g are identified. At the present stage, the yield criteria which can be accomodated to the program are confined to those belonging to the isotropic and kinematic hardening. Whereas the Mroz and mechanical sublayer model studied by Hunsaker et al.[15] and formulation based on micro-slip mechanism such as worked by Miyamoto et al.[16] are hoped to be incorporated in the near future.

3. GEOMETRIC NONLINEARITY

The element stiffness equation in infinitesimal or small strain regime is expressible as

$$[k]\{\dot{u}\}_e = \{\dot{\bar{p}}\}_e+\{\dot{p}^a\}_e \tag{15}$$

where [k] denotes the usual stiffness matrix defined by

$$[k] = \int[B_\varepsilon]^T[D_V^p][B_\varepsilon]dV \tag{16}$$

$[B_\varepsilon]$ is the strain rate - displacement matrix representing the internal strain rate in terms of the nodal velocity. The apparent nodal load rate $\{\dot{p}^a\}_e$ is obtainable from the apparent stress rate terms in eq.(1) as follows

$$\{\dot{p}^a\}_e = \int[B_\varepsilon]^T([D_V^p]\{\dot{\varepsilon}^\theta\}+[D_V^c]\{\dot{A}\}+\{\dot{\sigma}^a\})dV \tag{17}$$

When geometric nonlinear effects are taken into consideration, the element stiffness equation converts to the following augmented form

$$([k]+[k_G]+[k_C])\{\dot{u}\}_e = \{\dot{\bar{p}}\}_e+\{\dot{p}^a\}_e \tag{18}$$

$[k_G]$ and $[k_C]$ are called the geometric stiffness and load correction matrix respectively. $[k_G]$ represents the effect of nonzero 'initial' stress existing at each incremental deformation stage on the stress equilibrium and its specific form is dependent on the species of stress rate adopted in the expression of constitutive equation [10]. The load correction matrix $[k_C]$ emerges when the boudary traction rate is prescribed in terms of true rate, instead of nominal rate. Typical example is the problem where the hydrostatic pressure is the external load specified on the boundary.

It should be noted that $[k_C]$ is nonsymmetric. Further, $[k_G]$ turns out to be nonsymmetric when the intrinsic or Jaumann rate of Eulerian stress is used in the constitutive equation. Therefore, the equation solver for geometric nonlinear problems should be capable of commanding the nonsymmetric matrices. A suggested convenient way is to employ the Cholesky's decomposition of the assembled stiffness matrix $[K]+[K_G]+[K_C]$. Then the overall stiffness equation to be solved can be written as

$$[L]^T\lceil D\rfloor[U]\{\dot{u}\} = \{\dot{\bar{p}}\}+\{\dot{p}^a\} \tag{19}$$

where $[L]^T$, $[U]$ and $\lceil D\rfloor$ represent the lower and upper triangle, and diagonal matrices, respectively.

The solution proceeds through the following two steps composed of forward elimination and backward substitution.

$$[L]^T\{z\}_{n+1} = \{\dot{\bar{p}}\}_n + \{\dot{p}^a\}_n \tag{20a}$$

$$[D][U]\{\dot{u}\}_{n+1} = \{z\}_{n+1} \tag{20b}$$

The critical load points, i.e. the maximum load and bifurcation points, are characterized by the property

$$|[K]+[K_G]+[K_C]| = 0 \tag{21}$$

The Cholesky's decomposotion is also useful, since it enables the computation of the eigenmode $\{\phi\}$ at the critical load points in the following way [17]

$$[L]^T\{z\}_{n+1} = \{\phi\}_n \tag{22a}$$

$$[D][U]\ell_{n+1}\{\phi\}_{n+1} = \{z\}_{n+1} \tag{22b}$$

where ℓ_{n+1} denotes the normalizing factor for the eigenvector $\{\phi\}_{n+1}$.

4. NUMERICAL EXPERIMENTS AND RESULTS

In the course of program development several elastic-plastic, elastic-plastic-creep, and large strain problems have been solved in the order of increasing difficulties to verify the inelastic analysis capabilities of programs. The program EPIC employs the elastic-plastic analysis procedure in which the external load increment for each progressive step is determined in the process of computation so that the finite elements are brought to yielding one by one [18]. This method has been found to give accurate solutions but often time-consuming. In an alternative method the loading schedule is specified in advance and a weighting procedure is used for the element of transition region

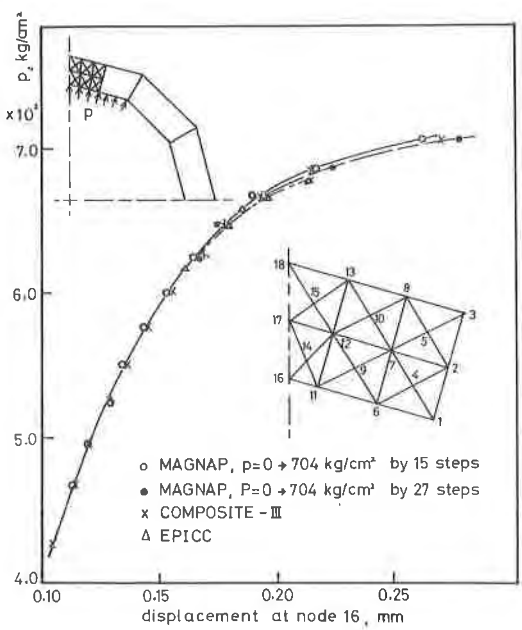

Fig.2 Elastic-Plastic Analysis of Dodecagonal Tube under Internal Pressure

that starts out as elastic and ends up by being elastic-plastic [19]. This method is incorporated with some modifications in the programs MAGNAP and COMPOSITE-III.

Above three programs which have been developed parallelly but almost independently are applied to a dodecagonal tube under internal pressure shown in Fig.2. It can be seen from the results of Fig.2 that displacement versus internal pressure curves obtained by these three programs which differ considerally in minor details agree reasonably well. Further, the growth of plastic engrave not shown here figures out an almost identical pattern. Therefore it can be concluded that the elastic-plastic analysis capabilities of the programs have been verified.

The second example shown in Fig.3 concerns with the elastic-plastic-creep analysis capabilities. This example was chosen as one of the benchmark test problems and solved on a number of computer programs during the first phase of verification and qualification activities in Japan of inelastic analysis computer programs [14].

In this example, the thick cylinder being closed at both ends is subjected first to the internal pressure p=1.5py, where p_y=244kgf/cm^2 is the incipient yielding pressure estimated by elastic analyses. Solutions for t=0 in Fig.4 compares the distribution of elastic-plastic circumferential stress σ_θ obtained by COMPOSITE-III (indicated by circles) and mean curve fit (dotted line) to the solutions of ref.[14].

The cylinder is then exposed to the constant temperature of 550°C for a period of time and it is requested to pursue the creep deformation and/or the redistribution of stress due to relazation. The law we adopted is given by

$$\bar{\varepsilon}^c = \varepsilon_t[1-\exp(-rt)]+\dot{\varepsilon}_m t \qquad (23)$$

The creep rate calculable from the above equation is

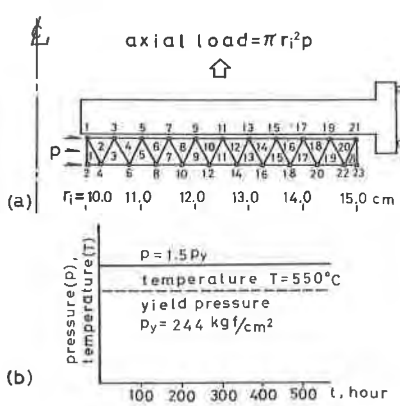

Fig.3 Thick Cylinder under Internal Pressure and Thermal Loading

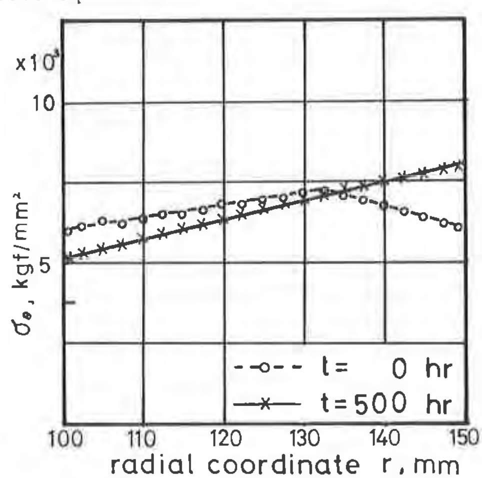

Fig.4 Comparisons of Elastic-Plastic and Redistributed Circumferential Stress

$$\dot{\bar{\varepsilon}}^c = \varepsilon_t r \exp(-rt) + \dot{\varepsilon}_m \tag{24}$$

Two alternative approaches based respectively on the 'equation of state' and 'mechanical model' formulations have been examined on the program COMPOSITE-III. As pointed out in sec.2, computations obedient to the equation of state approach for isotropic materials are executable on this program by using the equivalence condition of eq.(14). Specifically, by introducing eq.(24) into eq.(14)

$$\eta_{Gg} = \bar{\sigma}/3[\varepsilon_t r \exp(-rt) + \dot{\varepsilon}_m] \tag{25}$$

In applying the second method, we note that the creep compliance of the generalized Voigt model of Fig.1 under uniaxial test condition is given for a four element model

$$C_E(t) = C_{Eg} + C_{E1}[1 - \exp(-\frac{t}{T_{E1}})] + \frac{t}{\eta_{Eg}} \tag{26}$$

where subscript E denotes quantities pertinent to uniaxial test. Differentiation of eq.(26) yields

$$\dot{C}_E(t) = \frac{C_{E1}}{T_{E1}} \exp(-\frac{t}{T_{E1}}) + \frac{1}{\eta_{Eg}} \tag{27}$$

Comparing eqs.(24) and (27) and noticing that $C_E(t)$ is defined as the creep strain response to the unit axial stress $\bar{\sigma}$, we have the following equivalence relations

$$\frac{C_{E1}}{T_{E1}} = \frac{\varepsilon_t r}{\bar{\sigma}}, \quad \frac{1}{T_{E1}} = r, \quad C_{E1} = \frac{\varepsilon_t}{\bar{\sigma}}, \quad \frac{1}{\eta_{Eg}} = \frac{\dot{\varepsilon}_m}{\bar{\sigma}} \tag{28}$$

In actual computation by the program COMPOSITE-III, we use the following relation for deriving the shear creep compliance $C_G(t)$ from $C_E(t)$ for isotropic materials

$$C_G(t) = 3C_E(t) - C_K(t)/3 \tag{29}$$

where $C_K(t)$ denotes the volumetric creep compliance. For four element model under consideration, $C_G(t)$ is expressible as

$$C_G(t) = C_{Gg} + C_{G1}[1 - \exp(-\frac{t}{T_{G1}})] + \frac{t}{\eta_{Gg}} \tag{30}$$

Then, from eqs.(26), (29) and (30)

$$C_{Gg} = 3C_{Eg} - \frac{C_K}{3}, \quad \frac{1}{T_{G1}} = r, \quad C_{G1} = \frac{3\varepsilon_t}{\bar{\sigma}}, \quad \frac{1}{\eta_{Gg}} = \frac{3\dot{\varepsilon}_m}{\bar{\sigma}} \tag{31}$$

It has been assumed in eq.(31) that the volumetric deformation is purely elastic, i.e. $C_K(t) = C_K = 1/K$, where K stands for the bulk modulus of elasticity.

In Fig.4, solid line and points indicated by cross give the circumferential stress σ_θ for the problem of Fig.3 after a period of 500 hr's redistribution. In this example, these solutions which have been obtained respectively on the conventional equation of state and the mechanical model formulations agree almost completely. However it should be remembered that the stress relaxation proceeds at a faster rate than the creep strain and may amounts to hundred times or more faster rate in certain circumstances [20].

The third and last example in this paper challenges to the necking instability of the tensile specimen by numerical experiments. It is known that there had been diversified discussions on this phenomena, but it is considered that the confusion stemmed principally from the mixing up of the limit (maximum) load point and the bifurcation point where the tensile diffuse neck initiates. Recently, Miles [21] succeeded in identifying the bifurcation point as well as obtaining the analytical eigenmodal solution for necking instability in rectangular specimen. This work motivated the author and his colleagues to the numerical experiments on the same problem by using the pilot geometric nonlinear analysis program MAGNAP.

Fig.5 depicts the finite element division used for the numerical experi-

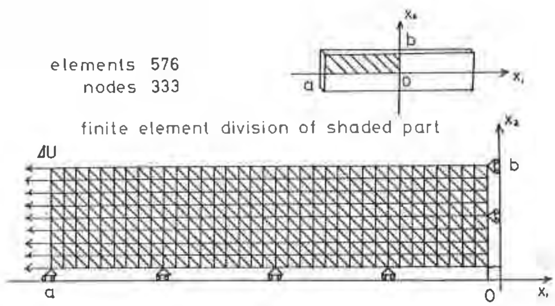

Fig.5 Finite Element Mesh used for Tensile Necking Analysis

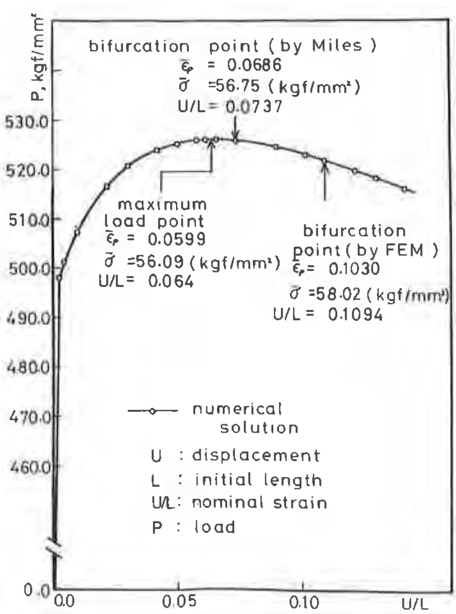

Fig.6 Comparison of Bifurcation Points in Uniaxial Tensile Test Specimen

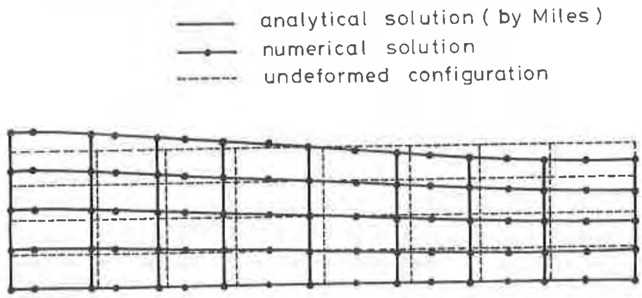

Fig.7 Comparison of Eigenmode in Tensile Instability—Diffuse Necking (Magnification: 4 times in the width direction of specimen)

ments on necking of tensile coupon specimen under plane stress condition. A rather fine mesh is needed for the computation in order to realize a state of uniform straining prior to bifurcation. Fig.6 compares the present numerical solution [22] to the analytical prediction by Miles [21]. A difference can be seen between the numerical and analytical bifurcation points. But, it is attributable to the fact that the velocity in the thickness direction is uniform within each finite element in case of the present study, while the analytical eigenmode obtained by Miles has a cosine form in thickness as well as width directions. Further, the stress-plastic strain curve used has the form

$$\bar{\sigma} = 68.37(0.02+\bar{\varepsilon}^{p})^{0.08}$$

As the hardening exponent n=0.08 is small, a minor difference in stress values leads to a large discrepancy in a comparison by plastic strains at bifurcation point. Fig.7 compares the eigenmodes (diffuse neck configuration) and shows a satisfactory agreement of numerical results with analytical prediction.

5. VERIFICATION OF INELASTIC COMPUTER PROGRAMS

Besides the basic formulations and fundamental numerical experiments, verification and qualification of computer programs by cross-checking the solutions for practical problems are essential in case of the inelastic analysis. A cooperative research in this direction has been actively under way in Japan and hoped to be expanded to international coordination. As a continuation of the first phase of such activities [14], a committee on Research and Development of Inelastic Analysis Programs has been formed. The benchmark test probelms being set up by the committee are divided into the following four categories.

a) Pressure Vessel and Reactor Components, b) Structural Application, c) Fundamental Elements, d) Conbined Problems and Others.

Detailes of the problems and example solutions will be presented at the Conference.

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