

## DESIGN OF MILD STEEL STRUCTURES UNDER UNEQUAL CYCLIC LOADS

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### SUMMARY

Non-static loads applied to engineering structures are generally rule rather than the exception. However, regardless of the method of estimating these loads, the ultimate interest of the design engineer is to design and predict accurately the behaviour of a structure or component. It is therefore necessary to establish a relation between loads and deformations. Such relations, in some cases, could be highly non-linear and quite complex. Since the response of a structure to excitation or non-static loads could also be a function of its deformation history, the need and value of a simplified approach is quite evident.

In this paper a method is proposed to investigate the behaviour and life of structural components under unequal cyclic loading conditions. Appropriate cyclic moment-curvature relations and life information, in the form of life versus extreme fiber strain, are developed from tests on beams under pure bending conditions. Theoretical predictions of behaviour are based on structural geometry and the cyclic moment-curvature relations used in association with the simple curvature-area method. Structural life is also predicted using the life information developed and the theoretical strain history at the critical section in conjunction with a linear damage summation criterion. Theoretical predictions of behaviour and life compare reasonably well with the experiments.

Based on this study, a design procedure is proposed for mild steel components subjected to unequal cyclic loading conditions. The loads on the tested components were such that they failed due to low cyclic fatigue (i.e., at less than  $10^5$  cycles).

## 1.0 INTRODUCTION

Cyclic loads are recognized as a cause of failure in structural elements such as aircraft components, pressure vessels, turbines, etc. These loads usually produce nominal (global) elastic stresses and highly localized plastic flow leading to conventional fatigue failure. Low-cycle fatigue, on the other hand, deals with failure of components where the stress and strain are in excess of the yield value and the number of cycles necessary to promote failure ranges from one quarter of a cycle (monotonic failure) to about  $10^5$ .

Most of the early work involving cyclic loads on structures above the elastic limit can be grouped into two classes of problems: shakedown and alternating plasticity [1]. Various experiments [2-4] were conducted to establish the shakedown load leading to the proposition that theoretical predictions of shakedown loads are, in general, conservative. The main reason for such divergence between theory and experiment is that the section behaviour is assumed invariant, ie, it has an elastic-plastic or strain-hardening moment-curvature relation regardless of cyclic history. Alternating plasticity, on the other hand, implies alternating global compressive and tensile plastic deformations in every load cycle, at least in one section of the component being investigated. Failure of such a section under cyclic loading constitutes component fatigue failure.

More recent investigations have dealt with the changes in material response, which are functions of the nature of the non-static loading conditions to which the material is subjected. Since these loading conditions may be quite random, they could be divided into two basic types involving load and deformation control. However, in practice, constant limit load or deformation control is not very common since loads and deformations are independent in most engineering situations. Therefore, investigations under either strictly load or deformation control are essentially a means of understanding the behaviour of elements subjected to common engineering cyclic loadings.

Royles [5] and Sherbourne and Krishnasamy [6] investigated the flexural behaviour of mild steel rectangular sections under constant symmetrical deformation limit control. In contrast with the approach of Royles, which dealt with stable behaviour only, the theory developed by Sherbourne and Krishnasamy could predict the response of a structure at every stage of loading. Krishnasamy, et al [7] studied the behaviour under constant symmetrical load control; the suggested cyclic moment - curvature model incorporated cyclic strain accumulation effects (cyclic creep).

The current investigation is limited to the study of miniature flexural sections and determinate and indeterminate structures made of structural mild steel under cyclic unequal deformation control. At the section level, the experimental behaviour of pure bending rectangular sections under constant limit deformation control was investigated. It

was possible to express transient behaviour and relaxation of mean moment by cycle dependent relations between moment range and mean moment, on the one hand, and curvature range and mean curvature on the other. These relations were later used in making structural predictions. The memory concept, as regards its application to structures, was also investigated concurrent with investigations of structural behaviour; fatigue life was also examined.

2.0 EXPERIMENTAL ARRANGEMENT

All specimens were machined from 3/8" x 1" hot-rolled semi-killed, 1020 mild steel bars, the chemical composition of which is given below:

|   |      |      |      |      |      |      |      |      |      |     |           |
|---|------|------|------|------|------|------|------|------|------|-----|-----------|
| % | Si   | S    | P    | Mn   | C    | Ni   | Cr   | Mo   | V    | Cu  | Fe        |
|   | .210 | .025 | .010 | .710 | .203 | .063 | .050 | .010 | .003 | .16 | Remainder |

All specimens were roughly machined and then stress relieved by retreating at 1600°F for half an hour and cooling to room temperature in air. The final machining was carried out with care to minimize the possibility of residual stresses.

The test specimens are shown in Figure 1 and the equipment used is described in detail elsewhere [8].

3.0 SECTIONAL BEHAVIOUR

Figure 2 shows two typical moment range versus cyclic state plots for a constant curvature or extreme fibre strain range. The moment range decreased with life for small strain ranges and remained practically constant or even decreased for larger strain ranges. Hence the section as an aggregation of fibres, may be softening, stable or hardening depending upon the cyclic strain ranges involved in the individual elements [8]. However, for a given strain range, the moment range varied little with the change in the mean strain value. Hence only an average moment range-strain range relation will be adopted for any cyclic stage regardless of mean strain ratio. This will cause some predictive error only in the early stages of structural life. For all strain ranges, the mean moment, in general, is a function of mean strain. Life span, on the other hand, did not seem to be significantly affected by mean strain.

For a particular cyclic state, the aggregation of moment range versus strain range data yields a typical cyclic moment range-strain range relation as that shown in Figure 3. Similarly, the compilation of mean moment versus strain range information yields the corresponding cyclic mean moment - strain range relation.

A two phase model of the type

$$\Delta\epsilon = \frac{\Delta M}{EI} \cdot \frac{d}{2} \quad \text{for elastic behaviour} \tag{1}$$

$$\Delta\epsilon = \frac{\Delta M}{EI} \cdot \frac{d}{2} + \alpha \left( \frac{\Delta M}{\Delta M_e} - 1 \right)^\beta \quad \text{for inelastic behaviour} \tag{2}$$

is proposed as a fit to the moment range - strain range data

- where  $\Delta\epsilon$  = strain range
- $\Delta M$  = moment range
- $\Delta M_e$  = proportional moment range
- $EI$  = flexural stiffness
- $d$  = depth of section
- $\alpha, \beta$  = section and material constants.

This model, as shown in Figure 3, fits the data reasonably well. For each cyclic stage, a set of values for  $\alpha, \beta$  and  $\Delta M_e$  were established from a least squares curve fitting analysis.

Conversely, the mean moment versus strain range relationship poses a more difficult problem. The curve required to fit the data for any mean curvature ratio (Figure 3) should have three regions: an initial straight line, an intermediate convex curve and finally a concave region where the curve asymptotically approaches zero for larger values of curvature range. The slope of the initial straight part is simply equal to that of the initial portion of the  $\Delta M - \Delta\epsilon$  relation times the mean strain ratio (ratio of mean strain over strain range). The latter region of the experimental data is modelled by an equation of the form:

$$M_m = \frac{\theta_1}{(\Delta\epsilon)^{\theta_2}} \tag{3}$$

where  $M_m$  is the mean moment and  $\theta_1$  and  $\theta_2$  are constants to be established by least squares analysis. The intermediate range is assumed to have the form:

$$M_m = C_1 + C_2 \Delta\epsilon + C_3 \Delta\epsilon^2 + C_4 \Delta\epsilon^3 \tag{4}$$

This is a third degree polynomial with four arbitrary constants. From the continuity of the function and its slope at the transition points between the regions four equations will be obtained. Solving these equations simultaneously will establish the constants  $C_1$  to  $C_4$ . Some approximation was used to locate these two transition points and this concept is developed in more detail elsewhere [8]. Figure 3 shows the experimental data and the fitted curves.

#### 4.0 BEHAVIOUR OF CANTILEVER BEAMS

The behaviour of the cantilever beam shown in Figure 1 is predicted theoretically and investigated experimentally in the present study. A single cyclic load is applied at its tip to cause cyclic deflections confined to two predetermined values; both symmetrical and unsymmetrical deflection limits were considered. Since the cantilever beam is under deflection control, the sections are under neither moment control nor curvature control but under some mixed control condition. Nonetheless, it

seems reasonable to predict cyclic structural behaviour, in this case, from a set of moment-curvature relationships (Equations 1-4) based on controlled fixed curvature limits [9]. The method of predicting the response of a cantilever beam under unequal cyclic deflection control is explained in detail elsewhere [10].

Nine tests were conducted on cantilever beams, four pairs and a single. Each pair of beams had a single deflection range; one beam had no mean deflection and the other had a mean deflection equal to about one eighth of the deflection range, providing examples of complete and partially reversed deflections. The deflection ranges were 0.17", 0.207", 0.242", 0.393" and 0.750".

The influence of mean deflection on the experimental load range (Figure 4) was small, the maximum discrepancy being about 9%. The difference between the theoretical load range and the averaged experimental values, on the other hand, compared fairly well, the maximum difference being no more than 4% for any deflection range. It can be seen that the theoretical predictions are closer, in general, to the experimental results for non-zero mean rather than zero mean deflection. This can be explained with reference to Figure 3 which shows that the average moment range - strain range curve fits the partially reversed or repeated strain cases better than the completely reversed case. But, since the error involved is fairly small, neglect of the effect of mean strain on moment range under pure moment, and on load range under deflection control, can be easily justified.

#### 5.0 INDETERMINATE BEAMS UNDER DEFLECTION CONTROL

This section deals with the investigation of a three span continuous beam, loaded under deflection control conditions. A concentrated load is applied at the centre of the middle span, hereafter referred to as the central section. The beam, shown in Figure 1(c) is analyzed theoretically and experimentally under either symmetric or unsymmetric deflection limits.

Ten continuous beam specimens were tested under central section deflection control [11]. Five were subjected to completely reversed deflections and the others cycled between unequal deflections. The deflection ranges were 0.048", 0.075", 0.100", 0.192" and 0.217". Typical results are plotted in Figure 5.

When two beams were tested under the same deflection range, where one was subjected to mean deflection while the other was not, the experimental load ranges differed slightly. The difference between these two load ranges will be referred to as the discrepancy. Similarly the difference between theoretical values of load range will be called the variation. The experimental mean load, under completely reversed deflection, was found to be zero or insignificant for all the deflection ranges considered. The comments made with regard to the cantilever beam concerning the effects

of mean deflection on load range, the difference between predicted and average experimental load range, predicted and experimental mean load, are also tenable for the continuous beam. In contrast with the cantilever beam, however, it was possible for the continuous beam to sustain higher mean loads as the mean deflection increased, although the critical section could not sustain any higher mean moments in continuing to behave as a mean moment "cyclic plastic hinge" [8]. It followed that the increase in mean load was being carried by relatively unstrained segments of the beam adjacent to the critical section. This phenomenon is analogous to elastic-plastic moment redistribution.

#### 6.0 MEMORY MODELS AND BEHAVIOUR UNDER RANDOM DEFORMATION CONTROL

A theory for the prediction of structural behaviour under cyclic random loads, from a knowledge of structural behaviour under deterministic loading conditions, is presented. It is based upon a set of "memory rules", which collectively constitute a "memory model". Such a model simulates the case whereby a structure "remembers" its previous loading history while responding to a newly applied loading cycle. Predictions are compared with experiments on cantilever beams, random loads being defined as those which vary arbitrarily between an initial unloaded condition and a complete collapse condition. The model is discussed elsewhere in detail [8].

Three cantilever beams were investigated under pseudo-random tip deflection control. The first beam was subjected only to blocks of cyclic deflection (Figure 6(a)) whereas the second beam was tested under a combination of block reversals and single reversals, Figure 6(b). The chosen random deflection programmes were designed, as such, to test the model under a variety of conditions. Good agreement was obtained, the maximum difference between predicted and measured load range being about 12%.

#### 7.0 FATIGUE LIFE UNDER CONSTANT LIMIT AND PSEUDO - RANDOM DEFLECTION CONTROL

Concurrently with behaviour investigations, fatigue life was also examined. For pure bending specimens, tested under constant limit curvature control, life was unaffected by mean curvature and the associated mean moment in the low-cycle fatigue range, Figure 2. This is due to the fact that mean moment is carried principally by the core of the section as the mean stress relaxes rapidly at the extreme fibre; in consequence, mean curvature little affects crack initiation at these fibres. Within the low-cycle fatigue domain, the relation between experimental numbers of cycles to failure and total extreme fibre strain range may be represented by a straight line (Figure 7) on a log-log scale.

The fatigue life of the cantilever and continuous beams under constant limit deflection control was considered, for purpose of theoretical prediction, equal to the life of their critical sections under pure bending

subjected to the same strain history. The theoretical predictions of the behaviour of these structures provided the extreme fibre strain history at discrete cyclic states. But, since the strain range changed with cycling as a result of material softening and hardening, it was not possible to apply the life-strain range relation directly. Rather, a linear damage summation was used. The prediction of life for both types of beams was in good agreement with experiments Figure 8.

The theoretical life predictions of the cantilever beam discussed above were related to the tip deflection range by a power function. This relation was employed to forecast cantilever beam life under pseudo-random tip deflection control. Since, generally, there are no hysteresis loops when cycling under random control, another damage measure has to be devised. This was the damage accruing along each load-deflection trace (or reversal) between the two limiting load reversal points. The life power function can, therefore, be interpreted in terms of damage accumulation over reversals rather than cycles. Appropriate consideration was made to account for the effects of reversal sequence. The subsequent predicted lives agreed well with experiment (Figure 6); the maximum and minimum ratios of predicted life over the experimental values were 105% and 72% respectively.

#### 8.0 SUMMARY AND CONCLUSIONS

In the earlier part of the investigation, low-cycle fatigue behaviour and life of mild steel flexural sections, cantilever and continuous beams, was investigated. The work was confined to deformation control where the deformation limits were maintained constant throughout the history of the member. The limits were either symmetrical or unsymmetrical giving rise, in the latter case, to the presence of mean loads. The behaviour and life of the two types of beams was predicted from geometry and sectional behaviour.

At the start, the behaviour and life of rectangular sections in pure moment were investigated experimentally under constant limit deformation control. For any one specimen the curvature limits were identical and opposite in one case, and were unsymmetrical in others. The resulting mean curvature was associated, in general, with a mean moment that did, or did not, relax with cycling depending upon the ambient test conditions. The behaviour was observed and failure recorded.

Assembling and fitting a curve to the moment range versus curvature range data, obtained from the various experiments at any given cyclic state, yields a relation similar to the monotonic moment-curvature relation. The mean moment data also was compiled for various combinations of curvature range and ratio of mean curvature to total curvature range. The expression "cyclic moment-curvature relation" represents both of the cyclic moment range-curvature range and mean moment-curvature range-mean curvature ratio relations. Both relations were modelled mathematically for

consecutive cyclic states.

A cantilever beam loaded at its tip was tested where the tip was cycled between two fixed deflection limits. Five deflection ranges were examined. The mean deflection was either zero (completely reversed), one-eighth of the deflection range (partially reversed) or one-half of the deflection range (repeated deflection). A three span continuous beam was also examined. The central section of the beam was loaded by a point force and the deflection at the same point was cycled between constant limits. Again, these limits were symmetrical or unsymmetrical and mean load was associated with the latter case.

A quasi-static analytical technique, based on the curvature-area method, employing the "cyclic moment curvature" relations, was used for the theoretical predictions of load range and mean load. The predicted loads are in reasonable agreement with the measured values.

A "memory model" was established to predict the behaviour of structural elements under random deformation control conditions by simultaneously incorporating the effects of cyclic softening, hardening, relaxation of mean load and effects of prior straining history. The model was applied to predict the response of a cantilever beam under an imposed pseudo-random tip deflection control. Good agreement was obtained between prediction and experiments.

Along with behaviour investigations, fatigue life was also examined. A relation between life of pure bending specimens, tested under constant limit curvature control, and total extreme fibre strain range was established within the low-cycle fatigue domain. The relationship, coupled with a linear damage summation criterion, was applied for the prediction of life of cantilever and continuous beams, and compared with experiments.

The theoretical life predictions of the cantilever beam were related to the tip deflection range by a power function and then employed to forecast cantilever beam life under pseudo-random tip deflection control. The predicted lives correlated well with experiment.

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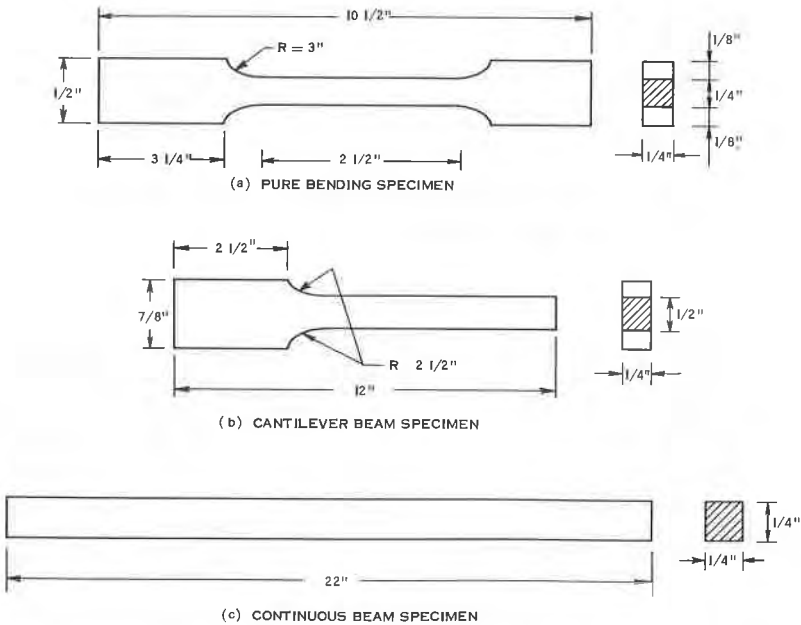


Figure 1. TEST SPECIMENS

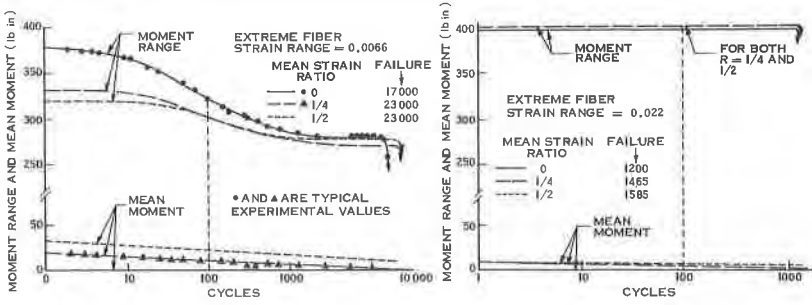


Figure 2. CYCLIC VARIATION OF MOMENT RANGE AND MEAN MOMENT UNDER CONSTANT STRAIN RANGE: TYPICAL RESULTS

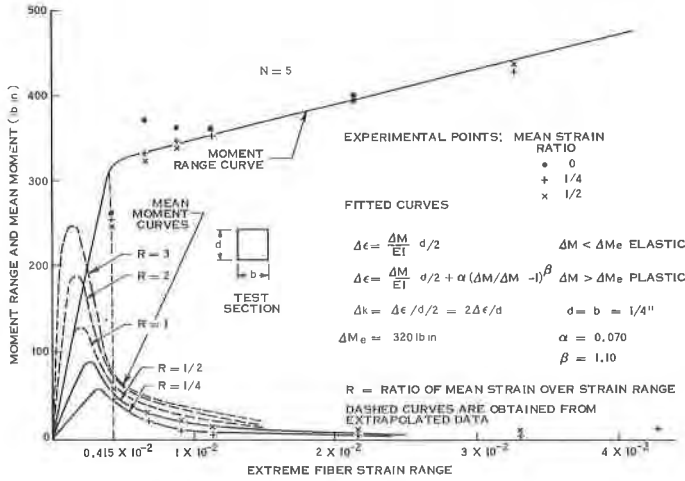


Figure 3. CYCLIC MOMENT RANGE-STRAIN RANGE RELATION: TYPICAL RESULTS

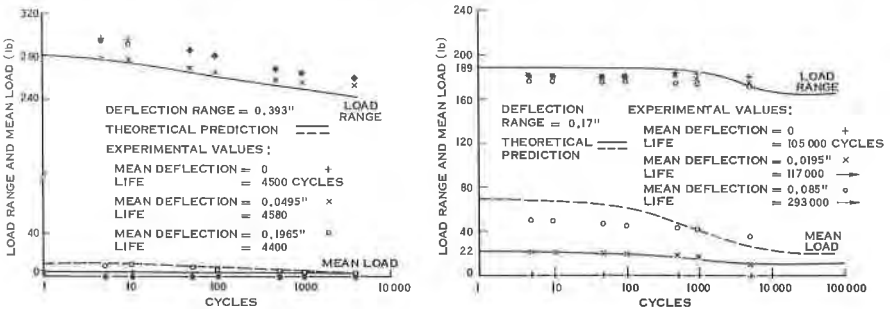


Figure 4. CYCLIC VARIATION OF LOAD RANGE AND MEAN LOAD ON CANTILEVER BEAMS UNDER CONSTANT DEFLECTION RANGE: TYPICAL RESULTS

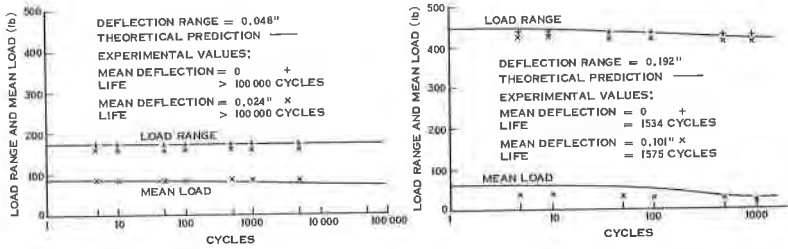


Figure 5. CYCLIC VARIATION OF LOAD RANGE AND MEAN LOAD RANGE ON CONTINUOUS BEAMS UNDER CONSTANT DEFLECTION RANGE: TYPICAL RESULTS

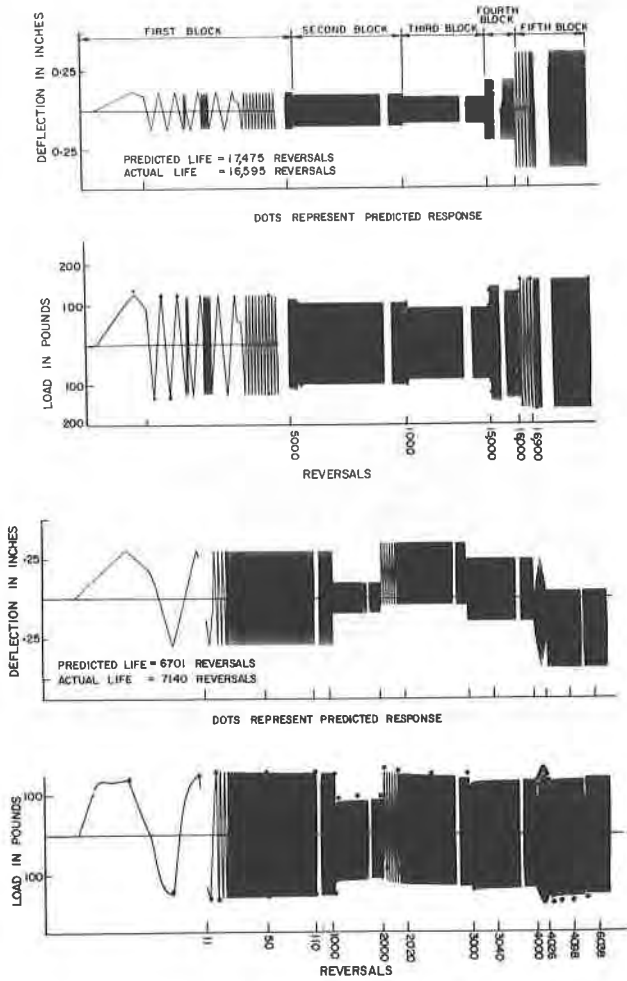


Figure 6. RESPONSE OF CANTILEVER BEAM TO PSEUDO-RANDOM (a & b) DEFLECTION CONTROL

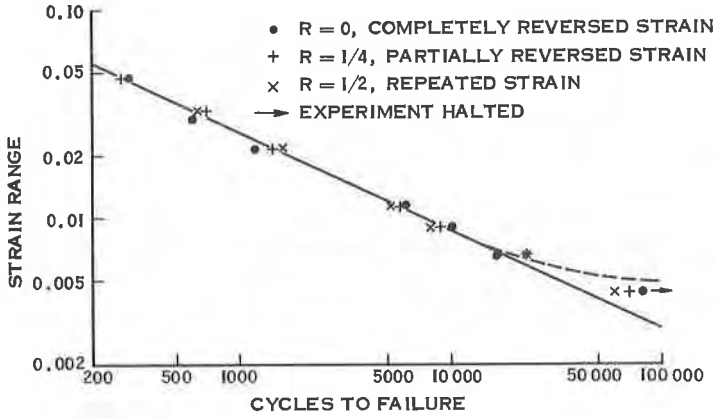


Figure 7. EXTREME FIBRE STRAIN RANGE VS CYCLES TO FAILURE

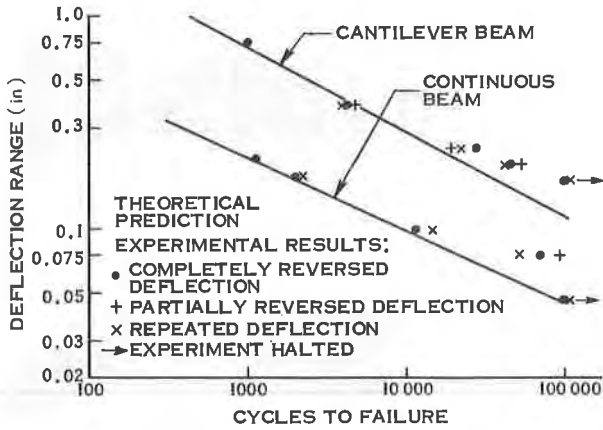


Figure 8. FATIGUE LIFE OF CANTILEVER BEAMS: THEORY AND EXPERIMENT