THIN CIRCULAR CYLINDER UNDER AXISYMMETRICAL THERMAL AND MECHANICAL LOADING

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SUMMARY

To assess structural integrity of components subjected to cyclic thermal loadings one must look at thermal ratchetting as a possible failure mode. Considering a thin circular cylinder subjected to constant internal pressure and cyclically varying thermal gradient through the thickness Bree, J. Strain Analysis 2 (1967) No. 3, obtained a diagram that serves as a foundation for many design rules (e.g.: ASME code). The upper part of the French LMFR main vessel is subjected to an axisymmetrical axial thermal loading and an axial load (own weight). Operation of the reactor leads to cyclic variations of the axial thermal loading. The question that arises is whether or not the Bree diagram is realistic for such loading conditions.

A special purpose computer code (Ratch) was developed to analyse a thin circular cylinder subjected to axisymmetrical mechanical and thermal loadings. The Mendelson’s approach of this problem is followed. Classical Kirchoff-Love hypothesis of thin shells is used and a state of plane stress is assumed. Space integrations are performed by Gaussian quadrature in the axial direction and by Simpson’s one third rule throughout the thickness. Thermoelastic-plastic constitutive equations are solved with an implicit scheme (Nguyen). Thermovisco-plastic constitutive equations are solved with an explicit time integration scheme (Treonor’s algorithm especially fitted).

A Bree type diagram is obtained for an axial step of temperature which varies cyclically and a sustained constant axial load. The material behavior is assumed perfectly plastic and creep effect is not considered. Results show that the domain where no ratchetting occurs is reduced when compared with the domain predicted by the Bree diagram.

To investigate the effect of material hardening we verify Halphen’s Theorem which states that a structure made of a material with kinematic hardening behavior and constant properties with temperature will always shake down to a periodical behavior.
1. **Introduction**

In a reactor, the thermal ratchet behaviour has to be carefully evaluated. The use of a general classical finite element computer code for the inelastic analysis of structures is indeed very expensive.

Pure uniaxial models have been proposed to predict ratchetting \[1 - 2 - 3 - 4\]. From a numerical point of view it was shown how the results of these simple models could be used for a more complex structure like a nozzle [5].

But, in order to design the main vessel of a LMRBR, it is fundamental to be sure to apply precise rules. This is the reason why we wrote an inexpensive computer code, RATCH, which is only available for the special problem of a thin circular cylinder subjected to any axisymmetrical thermal and mechanical loadings and made of any thermoelasticplastic or viscoplastic material.

The purpose of this paper is to describe its results for some special thermal cyclic loadings.

2. **RATCH code**

The equations for a thin circular cylinder under axisymmetrical loadings and the procedure to solve them were taken from Mendelson [6].

Classical Kirchhoff-Love hypothesis are used and a plane stress state is assumed.

For a known distribution of plastic strains and thermal strains (initial strains) the displacements field is explicitly expressed. The same relations are applied for the rate problem.

If a plastic behaviour is assumed, the time integration is performed according to the implicit Nguyen's algorithm [7]. For a viscoplastic behaviour, the special Treanor's algorithm is used [8].

Numerical spatial integrations are performed using Gauss points in the axial direction and Simpson's rule throughout the thickness of the cylinder.

Execution time on a CDC-6400 is around 20 to 30CP seconds per loading cycle for a usual case.

3. **Perfectly plastic behaviour**

The main vessel of the French LMRBR is subjected to a dead load (weight) and a severe thermal transient when quick start-up occurs after an emergency shut-down. This transient can be visualized as a step temperature distribution that moves forward along the vessel and increases at the same time. The temperature is uniform throughout the thickness. The maximum temperature is around 400 °C so that creep effects can be neglected. At first a simplified case is studied.
3.1 A simplified loading case

The metal is assumed elastically perfectly plastic. This special behaviour is only
considered for a comparison with the Miller's and Bree's results. The material data are
assumed to be temperature independent. The yield criterion is that of Von Mises.

a - Geometrical parameters:

R : radius of the long cylinder
b : thickness

Material properties:

E, ν : Young's modulus, Poisson's ratio
α : coefficient of linear thermal expansion
σy : tensile yield stress

Mechanical loading:

N : constant axial load

Cyclic thermal loading:

Alternating hot and cold shocks consisting in a step variation ΔT along the
meridian as shown in Fig. 1.

b - The elastic solution of the problem is obtained in closed form and shows dependency on
the two unique loading parameters σl = Eα ΔT and σp = N/b; hoop and meridional str-
estreses are given by

\[ \sigma_{\theta} = \frac{E}{2} \alpha_{l} E_{E} \cos \xi + \eta \beta \sigma_{l} E_{E} \sin \xi \]

\[ \sigma_{\xi} = \sigma_{p} + \eta \beta \sigma_{l} E_{E} \sin \xi \]

where \[ \beta = \sqrt{3} / (1 - \nu^2) \]

\( \xi \) is the distance along the meridian divided by the characteristic shell parameter
\( \omega = \sqrt{Rb / \sqrt{3}(1 - \nu^2)} \), the origin being at the temperature discontinuity.

\( \eta \) is the distance across the thickness, \( 0.5 \leq \eta \leq 0.5 \).

\( \varepsilon = +1 \) if \( \xi \) is negative and \( \varepsilon = -1 \) if \( \xi \) is positive.

The elastic range of secondary stresses (thermal loading) as defined in \( \delta \) is equal
to 1.015 \( \sigma_{l} \) (when \( \nu = 0.3 \)) and is located on the outer skin of the vessel at the
section which is 0.12 \( \omega \) far from the temperature discontinuity.

We set

\[ y = \sigma_{l} / \sigma_{y} \]

\[ x = \sigma_{p} / \sigma_{y} \]

y will be considered as the elastic secondary stress range parameter as uncertainties
are involved in the numerical elastic plastic calculations, \( 1.015 \approx 1 \).
c - For the numerical iselastic analysis, length of $5\omega$ was considered on both sides of the temperature discontinuity.

In the implicit time integration scheme internal iterations were stopped when the plastically admissible stress field was different from the statically admissible stress field by less than 0.1%.

Calculations have been performed for various $X$ and $Y$ and in every case during ten cycles. The ratchet values between the tenth and the ninth cycle are taken to be the steady state incremental strain growth, this approach being conservative.

Tables 1 and 2 summarize the results for the maximum incremental growth in the axial and hoop directions. The largest one is in the axial direction and its variation with respect to $X$ and $Y$ is shown in Fig. 2 and Fig. 3. It is seen that the effect of the primary loading is the most important.

An elastic shakedown domain S (Fig. 4) is obtained by extrapolating the curves in Fig. 3; several cycles will generally be required to obtain purely elastic behaviour. Of course a plastic shakedown occurs when $X$ is zero.

In domain R of Fig. 4 ratchetting occurs and there is always yielding in both tension and compression. The ratchetting mechanism results in necking, thinning and elongation in the axial direction. Typical plastic strain and stress behaviours are shown in Fig. 5 to Fig. 8.

d - The maximum incremental growth is found to lie between the Bree and Miller uniaxial calculations (Fig. 9).

Considering a succession of hot shocks the elastic secondary stress parameter $Y$ is half the previous one. Two runs were performed and the same maximum steady state incremental growth as in the previous case was found.

3.2 A more realistic loading case

The same cylinder is now subjected to a thermal loading consisting in a step of temperature moving or a distance $h$ along the meridian and back to its original position. There are now three parameters $\sigma_1^t$, $\sigma_2^t$ and $h$.

Higher ratchet increments were obtained. In particular the incremental growth in the hoop direction is much greater than in the previous case. This can be explained by the wide extend of the plastic regions that are cyclically yielded. Table 3 gives the maximum ratchet growth after three cycles for three values of $h$.

4. Kinematic hardening behaviour

4.1 Justification of this hypothesis

The domain of functioning of the main vessel for a perfectly plastic material behaviour is very restrictive if no ratchetting is allowed. Austenitic stainless steels exhibit a significant hardening. Even if no attempt is made to include the effect of cyclic changes of the stress-strain curve which would require sophisticated constitutive relations, the linear kinematic hardening as proposed by Prager is at least a first step.

Happhen [10] have shown that such a structure will elastically or plastically shake down to any cyclic loading.

Zarka [11] have also given some recommendations and useful bounds; in particular there is an elastic shakedown independently of the strain hardening modulus if the elastic secondary stress range is lower than twice the uniaxial yield stress.
4.2 Description of a loading case

In order to look at some plastic shake-dowm states, we run a loading case where the cylinder is subjected to alternating hot and cold shocks as shown on Fig. 1. The loading parameters are \( \textbf{x} = 0.5 \) and \( \textbf{y} = 3.6 \) (for these values the perfectly plastic analysis gave high ratchet values, Fig. 3).

For two uniaxial hardening modulus \( \textbf{M/E} = 0.11 \) and \( \textbf{M/E} = 0.25 \), typical plastic strains behaviour is shown in Fig. 10 and Fig. 11.

We obtain a periodic solution (plastic shake-down) in a finite number of cycles (about 20 cycles for \( \textbf{M/E} = 0.11 \)). For lower values of \( \textbf{M} \) this number of cycles would increase quickly.

5. Conclusion

An inexpensive computer code was written to analyse thin cylinders.

For a perfectly plastic material, we have seen how our results (bidimensional calculations) for this special structure are situated with respect to uniaxial models.

We have verified that plastic shake-down ultimately occurs for a kinematic hardening material. It is the writers' opinion that for design such a behaviour has to be considered.

But, only comparisons with foreseen experiments will corroborate this hypothesis.

References


Table 1. Hot and cold shocks. Maximum axial plastic strain
ratchet $E \frac{\delta e_x}{\sigma_y}$ (perfect plasticity)

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<tr>
<th>$x$</th>
<th>0.1</th>
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Table 2. Hot and cold shocks. Maximum hoop plastic strain
ratchet $E \frac{\delta e_\theta}{\sigma_y}$ (perfect plasticity)

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Table 3. Moving step of temperature $X = 0.3$, $Y = 3.0$. Maximum
ratchet strains between the third and the second cycle. Perfectly
plastic material behaviour.

<table>
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<tr>
<th>$k$</th>
<th>$\omega$</th>
<th>$2.5\omega$</th>
<th>$5\omega$</th>
<th>Hot and cold shocks</th>
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<td>1.32</td>
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Fig. 1. Cylinder subjected to alternately axial hot and cold shocks.

Fig. 2. Cylinder subjected to axial hot and cold shocks (perfect plasticity). Maximum axial plastic strain ratchet versus primary load parameter $x$.

Fig. 3. Cylinder subjected to axial hot and cold shocks (perfect plasticity). Maximum axial plastic strain ratchet versus secondary load parameter $y$.

Fig. 4. Bree type diagram for a cylinder subjected to alternately hot and cold axial shocks.
E : elastic domain, S : elastic shakedown domain, R : ratchetting domain, P : plastic shakedown domain.
Fig. 5. Hot and cold shocks. Typical plastic strains behaviour (perfect plasticity).

Fig. 6. Hot and cold shocks. Stresses behaviour on skins of section O. (perfect plasticity)

Fig. 7. Hot and cold shocks. Typical plastic strains behaviour (perfect plasticity).

Fig. 8. Hot and cold shocks. Stresses behaviour on skins of section O. (perfect plasticity)
Fig. 9. Hot and cold shocks (perfect plasticity). Maximum plastic strain ratchet; comparison with Bree's and Miller's results ($y = 2.4$).

Fig. 10. Hot and cold shocks. Kinematic hardening behaviour, $H/E = 0.11$. Typical plastic strain behaviour (10 cycles).

Fig. 11. Hot and cold shocks. Kinematic hardening behaviour, $H/E = 0.25$. Typical plastic strains behaviour (10 cycles).