

INELASTIC ANALYSIS OF OVERALL SHAKEDOWN PHENOMENON IN NOZZLES SUBJECTED TO ONE OR TWO LOADING PARAMETERS

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SUMMARY

For a cyclically loaded structure, we shall say that there is "strict shakedown" if during the first unloading the behavior is purely elastic. If the loading is defined by one parameter, p , it is known that as p_{\max} increases the strict shakedown limit, Δp_s , decreases, and thus is not uniquely defined. The usual shakedown rule $\Delta p = 2p_0$ (p_0 being first-yield-load) can therefore be non-conservative with respect to the strict shakedown. Moreover high stress concentrations in the structure may lower the Δp_s limit and not to exceed it would lead to a serious handicap. Therefore it is admitted to exceed the Δp_s limit provided that the alternately yielding zones are localized. It is this qualitative concept that we call "overall shakedown", and we want to give it a quantitative acceptance in order to define a precise Δp_g limit not to be exceeded. Indeed with higher Δp , the notch effect in low-cycle fatigue will seriously increase, without speaking of the risk of ratcheting. It is this notion of overall shakedown that the ASME code tries to ensure by the $3S_m$ rule because in this limitation the peak stresses are eliminated. If the overall shakedown is ensured the elastic strain invariability method of S. S. Manson applies: the total true strain amplitude $\Delta \epsilon_{tp}$ can be correctly approximated by a fictitious purely elastic strain amplitude $\Delta \epsilon_e$. It is what the ASME code is using in its fatigue analysis requirements. Let $K_e = \Delta \epsilon_{tp} / \Delta \epsilon_e$. The aim of the ASME code is therefore to ensure a value of K_e closed to 1.

By an elastic perfectly plastic calculation using the F.E. method, we study on two nozzles this concept of overall shakedown. We give a definition of the Δp_g limit and compare it with the $3S_m$ rule.

In the first part we analyze a radial nozzle in a spherical shell with only one loading parameter (internal pressure p). The K_e ratio was calculated for eight types of cycles corresponding to a wide range of p_{\max} and Δp . The results show that Δp_s corresponding to $K_e = 1$ is dependent on p_{\max} , but as soon as K_e becomes greater than 1 there is a unique relationship between K_e and Δp . This feature allows us to define the Δp_g limit by $K_e \leq 1 + \alpha$, where α will be given function of the accuracy desired in the fatigue analysis on an elastic basis. With $\alpha = 0.1$ Δp_g is 23 to 35% higher than Δp_s . The $3S_m$ rule of ASME code is more conservative.

In the second part we analyze in the same way an other nozzle geometry reproducing a PWR configuration.

In the third part we analyze the first nozzle under a loading depending on two parameters: the internal pressure p and an axial load f pulling or pushing the nozzle. The K_e factor is calculated for radial, triangular and rectangular loading paths in the p, f plane, admissible with respect to the ASME $3S_m$ rule. For a given Δp , Δf , independency of K_e with respect to loading path is checked.

1. Concept of overall shakedown of a structure subjected to cyclic loadings

Consider a structure subjected for clarity to one loading parameter P which increases from zero to P_{max} and then varies cyclically between P_{max} and P_{min} . There is "strict shakedown" if after plastic loading the unloading is elastic at every point of the structure [1 - 2] : the subsequent behavior is thus purely elastic everywhere. The maximum value of $\Delta P = P_{max} - P_{min}$ for which this condition is verified is called the strict shakedown limit ΔP_s .

It is well known that ΔP_s is a function of P_{max} (or P_{min}) and that, if P_0 is the first yield load, $\Delta P_s \leq 2 P_0$ because of the rotation of the stress vector on the yield locus during plastic loading (assuming perfectly plastic material behavior). In [3] we had $1.82 P_0 \leq \Delta P_s \leq 2 P_0$ depending upon the value of P_{max} .

High stress concentrations result in low ΔP_s limit and it may be a serious handicap for in service needs to remain below it. Therefore it is admitted to exceed ΔP_s provided that low - cycle fatigue cracking is avoided in stress and strain concentration zones which now yield alternately at each loading and each unloading. In that purpose it is necessary to limit the corresponding number of cycles allowable during the life of the structure.

However ΔP_s must not be exceeded "too much" in order to allow an important simplification in the now necessary low-cycle fatigue analysis and to avoid the risk of progressive deformation. When ΔP_s is exceeded, the elastic-plastic strain range $\Delta \epsilon_{ep}$ in the stress and strain concentration regions must be calculated because it is an essential parameter in initiation of low-cycle fatigue cracks [4]. But if ΔP_s is not exceeded "too much", alternately yielding zones are very localized and restrained by the remaining of the structure that behaves elastically. Then it is possible to avoid the elastic-plastic calculation of $\Delta \epsilon_{ep}$ by applying the "elastic strain invariability method" of Manson [5] which states that, in these conditions, $\Delta \epsilon_{ep}$ is correctly approximated by the elastic fictitious strain range $\Delta \epsilon_e$ given at the same point by a purely elastic calculation.

The ASME code section III has the same views : shakedown is not required in stress concentration zones since the $3Sm$ limit (to be compared to $2Sy$) does not apply on true stresses but on so-called primary plus secondary stresses obtained by eliminating peak stresses. On the other hand the low-cycle fatigue analysis is performed on a purely elastic basis.

To conclude ΔP_s may be exceeded but to allow simplification in low-cycle fatigue analysis, it is necessary that alternately yielding zones be very much confined : then we say that an "overall shakedown" has been achieved and it is this qualitative and subjective concept that we want to quantify. We define objectively an overall shakedown limit ΔP_g and we study its properties in case of a radial nozzle on a sphere subjected to one or two loading parameters.

The ASME limit ΔP_{3Sm} is compared to our ΔP_g .

2. One loading parameter

2.1 Internal pressure alone

The studied nozzle is shown on Fig.1. The material behavior is assumed elastic perfectly plastic ; Von Mises yield criterion and Hill maximum work principle are used. Material properties are listed on Fig.1..The finite element calculation, axisymmetrical, incremental with respect to plastic solution, uses the eight nodes isoparametric element.

Numerical integration is performed within an element with nine Gauss points. The design pressure is $P_d = 17.8$ MPa and the opening is uniformly reinforced in accordance with the ASME code NB - 3332 to 3337. For more details on the mesh and on the elastic - plastic results see [3] .

Point N (Fig.1) is the point of first yield for a pressure $P_o = 19.8$ MPa. Let $p = P/P_o$. The limit pressure is $p_1 = 1.88$.

The characteristics of the eight types of cycles studied in [3] , the relative shift s and the relative thickness r of successive loops, are defined in Table 1. Conventionally stabilization was defined by s less than 1 %. Cycles n° 9 to 13 are checking calculations.

On stabilized loops we compute at each point and for each cycle the true elastic - plastic strain range $\Delta \epsilon_{ep}$ with Tresca or Mises measures as defined on Fig. 2 [3 - 4] .

We also compute independently the purely elastic fictitious strain range $\Delta \epsilon_e$ (with the two measures, for the same cycle and same point). Then we deduce $Ke = \Delta \epsilon_{ep} / \Delta \epsilon_e$.

The maximum value of Ke in the structure is at point C of Fig. 1 (as soon as Ke_{max} is greater than 1.04). $\Delta \epsilon_{ep}$, $\Delta \epsilon_e$, Ke values at point C for Mises and Tresca measures are given in Table 1 for each cycle. We found that Ke_{Tresca} and Ke_{Mises} were identical within 4 %. In the following we take $Ke = Ke_{Tresca}$.

We noticed an interesting property of this Ke_{max} ratio : it seems (within the accuracy we need here, roughly speaking 1 %) that Ke_{max} depends only upon Δp and not upon p_{max} or p_{min} , at least in the range of values of Ke_{max} we are interested in here (1.1, to 1.2). We have found again this feature in case of two independent loading parameters (see 3.). Recall that Δp_g depended much upon p_{max} .

Figure 3 displays Ke at point C versus p for all cycles ; notice that p_{max} and p_{min} are widely varying.

The simplification of an elastic fatigue analysis is allowed as far as Ke_{max} is close to 1 : the degree of proximity is given by the accuracy wanted in the fatigue analysis. It is then possible to define quantitatively and objectively an overall shakedown limit Δp_g by a conventional limit for Ke_{max} . For instance we take here 1.14 (see 2.2.), hence $\Delta p_g = 2.53$.

Fig. 4 shows the regions which yield alternately at each loading and unloading for some cycles : we see how confined they must be in order to profit of the simplification.

We have here $\Delta p_{3Sm} = 1.72$ (the 3Sm limit is reached on inner skin of section N). This low value comes from the fact that pressures are divided by P_o , first yield load with Mises criterion, whereas the ASME code uses Tresca yield criterion, and also from the fact that linearization of elastic stresses throughout the thickness leads here to a slightly higher stress intensity on inner skin of section N.

When Δp exceeds Δp_{3Sm} the code prescribes to multiply in fatigue analysis the elastic results ($\Delta \epsilon_e$) by a coefficient Ke_{ASME} given in NB 3228-3. This coefficient is shown

in fig. 5 for the three families of materials given in the code and in Table 1 for carbon steels. We see that the code is here highly conservative.

2.2 Proportional loadings

The opening is subjected to an internal pressure P and to an axial force F tensile (positive) or compressive (negative). When the force is applied alone first yield occurs at point C for a force $F_0 = 38750 \text{ N/rd}$. Let $f = F/F_0$. We consider proportional loading $f = kp$.

From elastic calculations for different values of k we deduce the point M_0 in plane (p, f) of first yield as well as the point of the structure where it occurs (see Table 2). Locus (C_0) of points M_0 in plane (p, f) is shown in Fig. 5. (C_0) is symmetrical with respect to 0.

Linearizing elastic stresses throughout thickness for several sections, we obtain in plane (p, f) , for each value of k , the point M_{3Sm} , for which the 3Sm limit of the ASME code is just reached, and corresponding section and skin. It means that we consider a radial cycle between 0 and M_{3Sm} . Table 2 summarizes these results and locus (C_{3Sm}) of points M_{3Sm} is plotted on Fig. 5. (C_{3Sm}) is symmetrical with respect to 0.

Elastic - plastic radial cycles were performed between M_{max} and M_{min} for each value of k . Table 3 gives the characteristics of these cycles, the point of the structure where Ke is maximum and the corresponding value Ke_{max} . Ke_{max} being a function of ΔM ($\Delta p, \Delta f$) only and not of M_{max} or M_{min} , we have slightly interpolated or extrapolated when necessary to obtain ΔM_g ($\Delta p_g, \Delta f_g$) that gives $Ke_{max} = 1.14$. Table 2 gives these results and Fig. 5 shows the locus (C_g) of couple of points M_g such that $OM_g = \Delta M_g$. This curve is then symmetrical with respect to 0.

We see on Fig. 5 that a part of (C_{3Sm}) is almost identical to the constant $Ke_{max} = 1.14$ locus (C_g) : this is the reason why we choose 1.14 as limiting value for Ke_{max} . (C_g) is then the overall shakedown limit curve for proportional loading. The remaining part of (C_{3Sm}) is conservative with respect to (C_g) . This conservatism is as high as $OM_g/OM_{3Sm} = 1.5$ for $k = -1$.

For comparison we plot on Fig. 5 the curve $2M_0$ (obtained by a similarity of center 0 and ratio 2 from the curve (C_0)) which gives an upper bound for strict shakedown and the curve $2.4 M_0$ which is close to (C_g) for a large part and is conservative for the remaining part (conservatism as high as 1.25).

3. Two independent loading parameters

3.1. Results

Among all possible cyclic paths in plane (p, f) , we study cyclic loadings for which p and f vary one after the other or proportionally (triangular and rectangular paths). The cycle characteristics and results (Ke_{max} and corresponding point) are given in Table 3.

We noticed that at point of Ke_{max} , strain loops closed and were stabilized within the accuracy we need here (about 1 % on Ke_{max}) after one to three descriptions of the load cycle. This may be brought together with the convergence theorem [6] and the strong elastic restraint.

Here the points of interest are located on both skins. For them the principal directions of stress and strain are therefore fixed. We plotted the strain loops in the

principal strains space, ϵ_1 radial, ϵ_2 meridional, ϵ_3 hoop, projected in the deviatoric plane $\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$ [3].

For the straight, triangular, rectangular and hexagonal loading paths considered here, the elastic - plastic strain loops had their sides almost straight (strong elastic restraint) and the maximum on couples (P, Q) of Fig.2 giving the strain amplitude $\Delta\epsilon_{ep}$ was obtained between two apexes (A, B) for Tresca as for Mises measures. For each loading cycle, these two apexes are given in Table 3 for the point of Ke_{max} . Here they are the same for $\Delta\epsilon_e$ and $\Delta\epsilon_{ep}$.

It turns out (Table 3) that the ratio Ke_{max} is remarkably "stable" at least in the range of our interest (1.1, 1.2) and for our accuracy needs. It seems that Ke_{max} depends only on the shape and size of the loading path and is independent of :

- the initial state when cycling starts (that is the way of arrival on the loop). This may be brought together with the convergence theorem [6] and the strong elastic restraint.
- the position of the loading cycle in plane (p,f) (translation, provided not to exceed the limit load curve) ;
- the direction of description of the loading cycle (clockwise or counterclockwise).

Ke_{max} depends only on the shape and dimensions of the loading cycle.

3.2. Admissibility with respect to overall shakedown for any loading cycle

3.2.1. ASME code

One must verify that for any couple (A, B) of points on the loading cycle, the difference between the two corresponding stress fields does not exceed the $3Sm$ limit (after elimination of peak stresses). It is equivalent to check if translating AB in plane (p, f) to put A (or B) in O, B (or A) is inside the (C_{3Sm}) curve.

Therefore one checks the $3Sm$ limit by translating the loading cycle in plane (p, f) in such a way that point O describes the entire cycle. The cycle is admissible if the covered area S is included in the (C_{3Sm}) curve. (We consider only the repeated cycle and not the arrival path from O because this path is not significant for overall shakedown).

3.2.2. Present concept of overall shakedown

Let us try the same process as for ASME admissibility ; the covered area S is now compared with curves of equal Ke_{max} derived from proportional loadings (Fig.6). The maximum value reached by the covered area S, say Ke^* , is obtained for point B of the cycle when the origin O is at point A of the cycle (A and B can be interchanged).

If we describe the proportional loading AB, by construction we find $Ke_{max} = Ke^*$ at a point T of the structure. If now we describe the cycle, we have noticed that for all triangular and rectangular cycles studied here :

- (1) The obtained value of Ke_{max} is identical to Ke^* .
- (2) The point of Ke_{max} is still T.
- (3) The apexes of the loading cycle giving the amplitude $\Delta\epsilon$ of the strain loop described at T ($\Delta\epsilon_e$ and $\Delta\epsilon_{ep}$) were precisely A and B.

We tried then the hexagonal path given in Fig.6 and Table 3 to see if these results apply to a path less "sharp" than a triangle or a rectangle. The "extremal" points A and B are shown in Fig.6 and K_e^* was calculated to be 1.16, the point of $K_{e_{max}}$ being C. The calculations for this hexagonal cycle gave $K_{e_{max}} = 1.20$ at point C, the apexes giving $\Delta \epsilon_e$ and $\Delta \epsilon_{ep}$ being A and B. So above properties (2) and (3) are maintained but $K_{e_{max}}$ is 3.5 % greater than for the radial path AB.

4. Conclusion

The usual concept of overall shakedown is qualitative and subjective : the structure behaves subsequently elastically almost everywhere except in small stress concentration regions that yield alternately.

The purpose of this study was to quantify this concept to be able to recognize whether a loading cycle meets this requirement.

The admissibility ratio $K_{e_{max}}$ had been naturally introduced and it turned out to be a good parameter to characterize the effect of a loading cycle because of its great "stability" : $K_{e_{max}}$ seems to depend only on the shape and size of the loading cycle and to be independent of the way of arrival in the cycle, of the position and of the direction of description of the cycle.

Furthermore considering only proportional loadings one can derive a limit curve (Cg) for overall shakedown, $K_{e_{max}} = 1.14$ for example. (This is not expensive because of the small amount of plasticity involved at low $K_{e_{max}}$). It seems that this (Cg) curve allows to recognize whether any loading cycle is admissible within a few percents in $K_{e_{max}}$.

We found that the 3Sm limit ensures overall shakedown : the limit curve (C_{3Sm}) is for a part identical to the present conventional overall shakedown curve (Cg) and is conservative for the remaining part. Table 2 gives $K_{e_{ASME}}$ (NB 3228.3) for carbon steels at various points of (Cg) (proportional loading).

All the above conclusions were obtained in the particular case studied here. They should be checked on other types of cycles, structures and loading. Influence of work hardening should also be investigated.

References

























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Table 1

Cycle																
	1	2	3	4				5	6	7	8	9	10	11	12	13
P_{max}	1.83	1.83	1.72	1.72				1.46	1.46	1.46	1.16	1.83	1.83	1.83	1.16	1.83
P_{min}	-1.52	-0.40	-0.70	-1.10				-0.94	-1.14	-1.52	-1.52	-0.7	-0.8	-0.6	-1.26	-0.5
ΔP	3.35	2.23	2.42	2.82				2.40	2.60	2.98	2.68	2.53	2.63	2.43	2.42	2.33
Loops	1	1	2	1	2	3	4	1	1	1	1.5	0.5	0.5	0.5	0.5	0.5
r (%)	34.9	0.8	2.4	2.0	10.3	9.6	8.8	1.5	3.6	13.7	5.1	2.8	4.3	1.8	1.7	1.0
s (%)	2.50	0.30	1.14	0.66	2.55	1.50	1.03	0.75	0.17	0.3	0.6	0.5				
$\Delta \epsilon_{pp}$ Mises (%)	1.104	0.262	0.309	0.477				0.300	0.367	0.571	0.401	0.340	0.377	0.308	0.305	0.283
$\Delta \epsilon_{pp}$ Tresca (%)	1.144	0.272	0.317	0.480				0.309	0.371	0.579	0.403	0.347	0.382	0.316	0.313	0.293
$\Delta \epsilon_s$ Mises (%)	0.386	0.257	0.279	0.325				0.277	0.300	0.344	0.309	0.291	0.303	0.280	0.279	0.268
$\Delta \epsilon_s$ Tresca (%)	0.402	0.267	0.290	0.339				0.288	0.312	0.357	0.321	0.303	0.315	0.291	0.290	0.279
ϵ_e Mises	2.86	1.02	1.11	1.47				1.08	1.22	1.66	1.25	1.17	1.24	1.10	1.09	1.05
ϵ_e Tresca	2.84	1.02	1.09	1.42				1.07	1.19	1.62	1.25	1.14	1.21	1.09	1.08	1.05
ϵ_e ASME carbon steels	2.90	1.59	1.81	2.27				1.79	2.02	2.47	2.12	1.94	2.06	1.82	1.81	1.71

Table 2

k		+ ∞	+ 3.50	+ 1.00	+ 0.29	0.00	-0.29	-1.00	-1.60	- 3.50
M_o	p	0.00	0.23	0.54	0.85	1.00	1.15	1.35	1.05	0.36
	f	1.00	0.81	0.54	0.24	0.00	0.33	1.35	1.67	1.26
Point of first yield		C	C	C	C	N	G	M	C	C
ΔM_{3S_m}	Δp	0.00	0.56	1.08	1.47	1.72	2.06	2.64	2.67	0.88
	Δf	2.44	1.96	1.08	0.42	0.00	0.59	2.64	4.27	3.08
Section and skin		D outer	N inner	N inner	N inner	C inner	C inner	M inner	M inner	D outer
ΔM_g	Δp	0.00	0.56	1.30	2.00	2.54	3.10	4.02	2.66	0.89
	Δf	2.40	1.96	1.30	0.57	0.00	0.89	4.02	4.25	3.10
Point of $K_{e_{max}} = 1.14$		C	C	C	C	C	N	M	C	C
Ke ASME on Cg carbon steels		1.00	1.00	1.41	1.72	1.95	2.01	2.05	1.00	1.00

Loading cycle	Apexes		$K_{e \max}$	Point of $K_{e \max}$	Loading cycle	Apexes				$K_{e \max}$	Point of $K_{e \max}$	Apexes giving $\Delta \xi$
	1	2				1	2	3	4			
	0.00	0.00	1.13	C		0.50	0.50	-0.06		1.14	C	1-3
	2.20	-0.20				1.75	-0.21	-0.21				
	0.50	-0.06	1.14	C		0.50	-0.06	-0.06		1.14	C	1-3
	1.75	-0.21				1.75	1.75	-0.21				
	0.65	-0.65	1.14	C		0.50	-0.06	-0.06	-0.50	1.14	C	1-3
	0.65	-0.65				1.75	1.75	-0.21	-0.21			
	1.00	-1.00	1.13	C		0.57	1.13	1.13	0.57	1.14	C	1-3
	0.28	-0.28				-2.00	-2.00	-0.04	-0.04			
	1.27	-1.27	1.14	C		0.57	1.13	1.13		1.14	C	1-3
	0.00	0.00				-2.00	-2.00	-0.04				
	1.40	-1.40	1.06	N		0.57	0.57	-0.31		1.53	C	1-2
	-0.40	0.40				-2.00	1.10	1.10				
	1.50	-1.50	1.11	N		0.57	-0.31	-0.31		1.52	C	2-3
	-0.43	0.43				-2.00	1.10	-2.00				
	-1.82	1.82	1.09	M		0.65	0.65	-0.24		1.54	C	1-2
	-1.82	1.82				0.65	-2.45	0.65				
	1.29	-1.29	1.10	C		0.57	0.57	-0.13		1.15	C	1-2
	-2.06	2.06				-2.00	0.45	0.45				
	1.34	-1.34	1.15	C		0.57	-0.09	-0.09		1.10	C	2-3
	-2.14	2.14				-2.00	0.30	-2.00				
	0.57	-0.31	1.13	C		0.65	-0.65	-0.65	0.65	1.14	C	1-3
	-2.00	1.10				0.65	0.65	-0.65	-0.65			
	0.90	-0.90	1.16	C		1.00	1.00	1.00	-1.00	1.12	C	1-3
	0.44	-0.44				0.00	-0.57	-0.57	0.00			



Loading cycle	Apexes						$K_{e \max}$	Point of $K_{e \max}$	Apexes giving $\Delta \xi$
	1	2	3	4	5	6			
	0.57	1.00	-1.00	-1.00	1.00		1.12	C	2-4
	-2.00	0.00	0.00	-0.57	-0.57				
	0.70	0.70	0.20	-1.10	-1.10	0.00	1.20	C	1-4
	-0.15	-1.20	-1.85	-1.02	-0.60	0.00			

Table 3

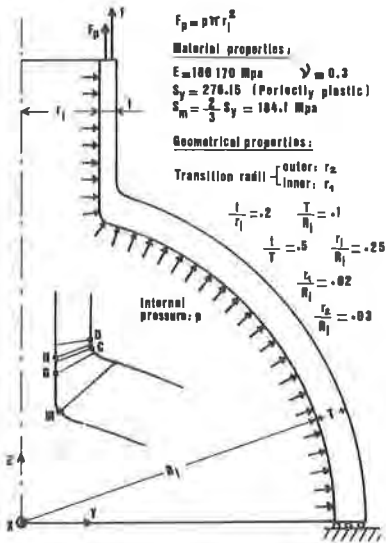
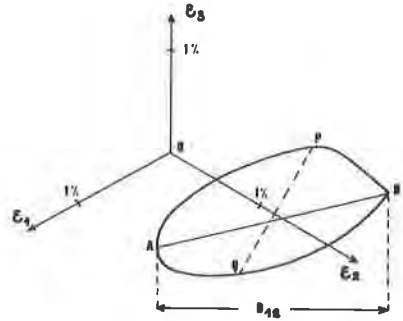


Fig. 1



$$\epsilon_{11}(P) - \epsilon_{11}(0) = \delta_{11}$$

$$(\Delta \epsilon_{\text{missa}})^2 = \frac{2}{3} \max_{(P, \theta)} \left[(\delta_{22} - \delta_{11})^2 + \dots + 0 \delta_{33}^2 + \dots \right]$$

$$\Delta \epsilon_{\text{tressca}} = \frac{2}{3} \max_{(P, \theta)} \left[\max(|\delta_1 - \delta_2|, |\delta_2 - \delta_3|, |\delta_3 - \delta_1|) \right]$$

$$\delta_1, \delta_2, \delta_3 : \text{eigenvalues of } \delta_{ij}$$

$$\text{Here : } \Delta \epsilon_{\text{missa}} = \sqrt{\frac{2}{3}} \bar{\Delta \epsilon} \text{ and : } \Delta \epsilon_{\text{tressca}} = \frac{2\sqrt{2}}{3} \bar{\Delta \epsilon}$$

Fig. 2

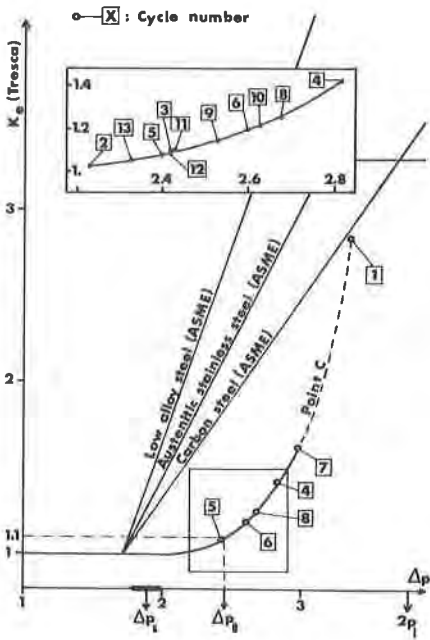


Fig. 3

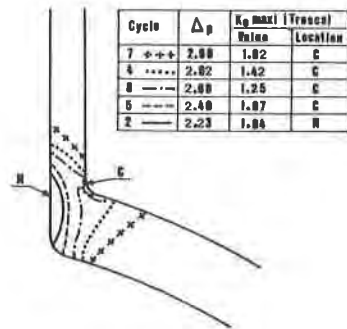


Fig. 4

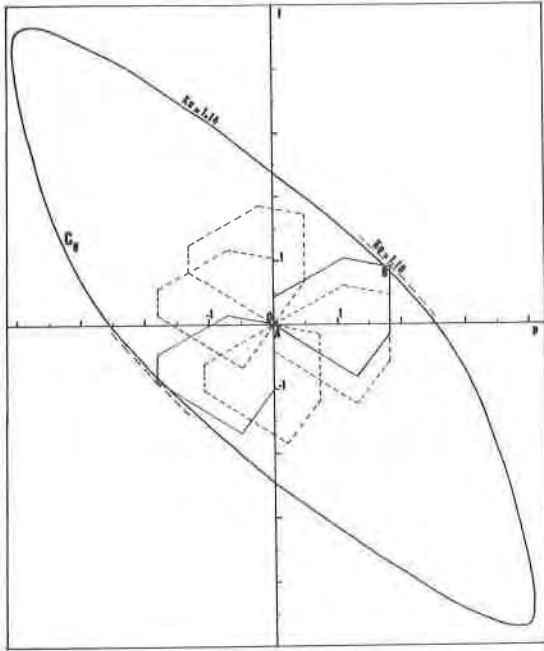


Fig. 5

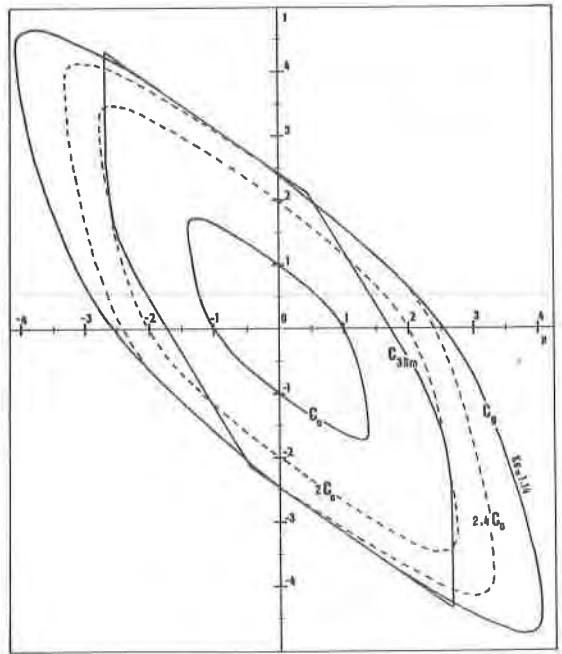


Fig. 6